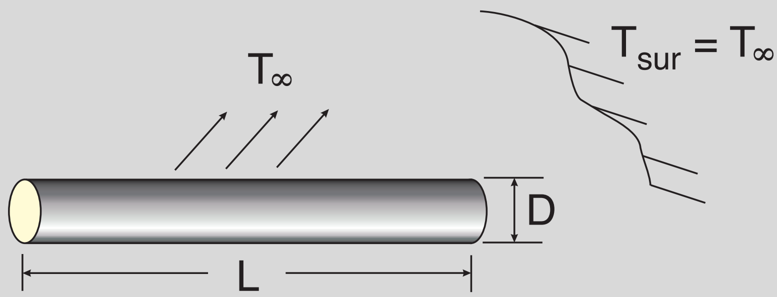
Example 5-2: Determine the time it takes a fuse to melt if a current of $3\ A$ suddenly flows through the fuse subject to the following conditions:



Given:

$$D = 0.1\ mm \qquad T_{melt} = 900\ ^\circ C \qquad k = 20\ W/mK$$

$$L = 10\ mm \qquad T_\infty = 30\ ^\circ C \qquad \alpha = 5 \times 10^{-5}\ m^2/s \equiv k/\rho C_p$$

Assume:

- constant resistance $\mathcal{R} = 0.2\ ohms$
- the overall heat transfer coefficient is $h = h_{conv} + h_{rad} = 10\ W/m^2K$
- neglect any conduction losses to the fuse support

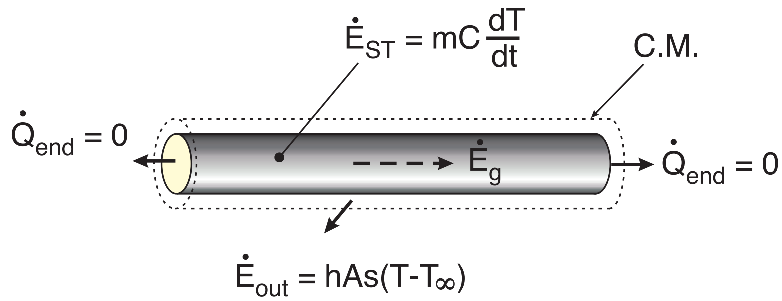
Solution: First, check the size of the Biot number

$$Bi = \frac{hV}{kA_s} = \frac{hD}{4k} = 1.25 \times 10^{-5} \ << 0.1$$

Therefore the lumped system approach is applicable and we can approximate $T \approx T(t)$ only.

Performing an energy balance over the fuse

$$\frac{dE_{ST}}{dt} = \cancel{\dot{E}_{in}}^{\,0} - \dot{E}_{out} + \dot{E}_g$$



where

$$\frac{dE_{ST}}{dt} = m\,C\,dT/dt$$

$$\dot{E}_{out} = hA_s(T - T_\infty)$$

$$\dot{E}_g = I^2\mathcal{R}$$

Therefore

$$m\,C\,\frac{dT}{dt} = -hA_s(T - T_\infty) + I^2\mathcal{R}$$

and

$$\frac{dT}{dt} = \underbrace{-\frac{hA_s}{m\,C}}_{-1/\tau}\left[\underbrace{T - T_\infty - \frac{I^2\mathcal{R}}{hA_s}}_{\theta(t)}\right]$$

Collecting terms

$$\frac{d\theta}{\theta} = -\frac{1}{\tau}\,dt \quad \Rightarrow \quad \ln\theta = -\frac{t}{\tau} + C_1$$

The initial conditions are

$$@t = 0 \quad \Rightarrow \quad T = T_i \qquad \Rightarrow \qquad C_1 = \ln\left(T_i - T_\infty - \frac{I^2\mathcal{R}}{hA_s}\right)$$

Therefore

$$T(t) = T_\infty + (T_i - T_\infty)e^{-t/\tau} + \frac{I^2\mathcal{R}}{hA_s}\left(1 - e^{-t/\tau}\right)$$

Using $T_i = T_\infty$ and $T(t) = T_{melt}$ we can determine the time, t for the fuse to blow out

$$900\,^\circ C = 30\,^\circ C + 5.73 \times 10^4\,^\circ C(1 - e^{-t})$$

Solving for t gives

$$t = 15.3\,ms \quad \Leftarrow$$