ECE309
Thermodynamics & Heat Transfer

Quiz #1:

Name:	______________
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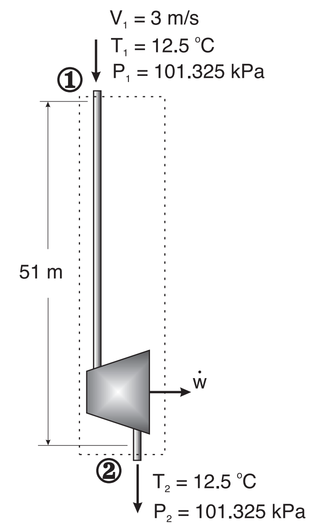

Problem: An amusement park at the bottom of Niagara Falls wants to install a water turbine to produce $100\ kW$ of power. Water (assumed to be incompressible) would enter the pipeline leading to the turbine at $12.5\ °C$ and $101.325\ kPa$ at the top of the falls, $51\ m$ above the turbine exit, with a velocity of $3\ m/s$. The water would leave the turbine at $12.5\ °C$ and $101.325\ kPa$. The pipeline and the turbine are both adiabatic.

a) Determine the mass flow rate $[kg/min]$ of the water.
b) Determine the diameter $[m]$ of the pipeline. Assume a circular cross section and uniform diameter throughout the system.

Assumptions:

- pipeline and turbine are adiabatic (given)
- pipeline is circular and uniform cross section (given)
- water is incompressible (given)
- steady state, steady flow
- quassi equilibrium

Part a)

Choose the control volume to include the inlet at the top of the falls and the outlet at the exit of the turbine.

Performing an energy balance where:

$$\dot{E}_1 = \dot{W} + \dot{E}_2 \qquad \Rightarrow \qquad \dot{m}e_1 = \dot{W} + \dot{m}e_2$$

The specific heat is calculated at the mean temperature of $12.5\ °C$ from Table A-3

$$C_p(@12.5\ °C) = 4.20\ kJ/(kg \cdot K)$$

$h_1 = h_2$ since $T_1 = T_2$ and $P_1 = P_2$
$ke_1 = ke_2$ since $\mathcal{V}_1 = \mathcal{V}_2$ ($\mathcal{V} = m/\rho \cdot A$ and mass, density and cross sectional area are constant at the inlet and the exit.)

$$\dot{m} = \frac{\dot{W}}{e_{in} - e_{out}} = \frac{\dot{W}}{(h_1 - h_2)^{\nearrow 0} + (pe_1 - pe_2) + (ke_1 - ke_2)^{\nearrow 0}}$$

$$= \frac{\dot{W}}{g(z_1 - z_2)}$$

$$= \frac{100\ kW}{(9.81\ m/s^2) \times (51\ m)} \left(\frac{1000\ m^2/s^2}{kJ/kg} \right)$$

$$= 199.88\ kg/s \Leftarrow$$

Part b)

The mass flow rate can be written as

$$\dot{m} = \rho \mathcal{V} A = \rho \times (\pi D^2/4) \times \mathcal{V}$$

At $T_{mean} = 12.5\ ^\circ C$ the density of water from Table A-3 is

$$\rho = 998.5\ kg/m^3$$

$$D = \sqrt{\frac{4\dot{m}}{\pi \rho \mathcal{V}}}$$

$$= \sqrt{\frac{4 \times (199.88\ kg/s)}{\pi \times (998.5\ kg/m^3) \times (3\ m/s)}}$$

$$= 0.2915\ m \Leftarrow$$