$\mathcal{ECE}$*309*
Thermodynamics & Heat Transfer

Quiz #2:

Name:	______________________
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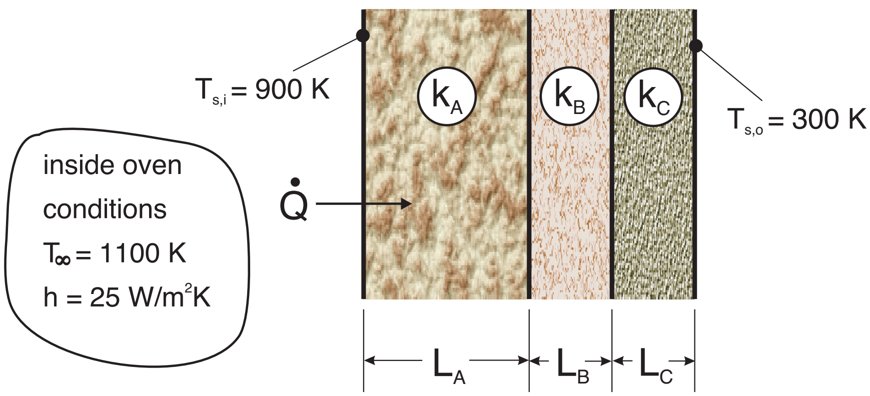
The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 20\ W/m \cdot K$ and $k_C = 50\ W/m \cdot K$, and known thickness, $L_A = 0.30\ m$ and $L_C = 0.15\ m$. The third material B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.15\ m$, but unknown thermal conductivity, k_B.

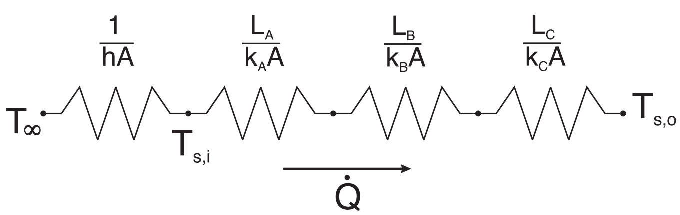
Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 300\ K$, and an inner surface temperature of $T_{s,i} = 900\ K$, and an oven air temperature of $T_\infty = 1100\ K$. The inside convection coefficient h is known to be $25\ W/m^2 \cdot K$. What is the value of k_B.



Assume:

- steady state
- 1D heat flow
- constant properties
- negligible contact resistance

The resistor network looks like



The heat flow within the wall can be expressed as

$$\dot{Q} = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A \cdot A} + \dfrac{L_B}{k_B \cdot A} + \dfrac{L_C}{k_C \cdot A}}$$

Since the cross sectional area is common for each section, we can write

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{s,i} - T_{s,o}}{\dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}}$$

$$= \frac{(900 - 300)\ K}{\dfrac{0.3\ m}{20\ W/m \cdot K} + \dfrac{0.15\ m}{k_B} + \dfrac{0.15\ m}{50\ W/m \cdot K}}$$

$$= \frac{600}{0.018 + 0.15/k_B}\, W/m^2$$

But we also know that

$$\begin{aligned} \dot{q} &= h(T_\infty - T_{s,i}) \\ &= (25\, W/m^2 \cdot K) \times (1100 - 900)\, K \\ &= 5000\, W/m^2 \end{aligned}$$

Combinining these two equations for $\dot{q}$ allows us to solve for k_B

$$\frac{0.15}{k_B} = \frac{600}{\dot{q}} - 0.018 = \frac{600}{5000} - 0.018 = 0.102$$

or

$$k_B = 1.47\, W/m \cdot K \;\Leftarrow$$