ECE 309
Thermodynamics & Heat Transfer

Quiz #3:

Name:	____________
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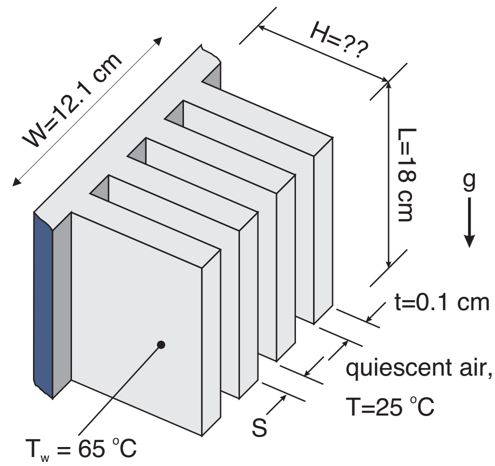
Problem: A 12.1 cm wide and 18 cm high vertical hot surface in 25 $°C$ air is to be cooled by an aluminum heat sink ($k_s = 177\ W/m \cdot K$) with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction. Determine the optimum fin height and the rate of heat transfer by natural convection from the heat sink if the base temperature is 65 $°C$.

Note: $H_{opt} = \sqrt{hA_c/pk_s}$



Assumptions

1. steady state
2. constant thermal properties
3. air is an ideal gas
4. atmospheric pressure is 1 atm

The film temperature is calculated as

$$T_f = \frac{T_w + T_\infty}{2} = \frac{65 + 25}{2} = 45\ °C$$

From Table A-22

$$\begin{aligned} k &= 0.02699\ W/m \cdot °C \\ \nu &= 1.750 \times 10^{-5}\ m^2/s \\ Pr &= 0.7241 \\ \beta &= \frac{1}{T_f} = \frac{1}{45 + 273} = 0.003145\ K^{-1} \end{aligned}$$

Step 1: Calculate the Rayleigh number

$$\begin{aligned} Ra &= \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} Pr \\ &= \frac{(9.81\ m/s^2)(0.003145\ K^{-1})(65 - 25\ K)(0.18\ m)^3}{(1.750 \times 10^{-5}\ m^2/s)^2}(0.7241) \\ &= 1.707 \times 10^7 \end{aligned}$$

Step 2: Optimum conditions
The optimum fin spacing is

$$S = 2.714\left(\frac{L}{Ra^{1/4}}\right) = 2.714\left(\frac{0.18\ m}{(1.707\times 10^{7})^{1/4}}\right) = 7.6\ mm$$

The heat transfer coefficient for this optimum fin spacing is

$$h = 1.307\frac{k}{S} = 1.307\left(\frac{0.02699}{0.0076}\right) = 4.641\ W/m^2\cdot{}^\circ C$$

The optimum fin height is calculated as

$$H = \sqrt{hA_c/pk} = \sqrt{\frac{(4.641\ W/m^2\cdot{}^\circ C)(0.18\ m\times 0.001\ m)}{2\times(0.18\ m + 0.001\ m)(177\ W/m\cdot{}^\circ C)}} = 0.00361\ m$$

The number of fins is determined as

$$n = \frac{W}{s+t} = \frac{0.121\ m}{0.0076+0.001} = 14\ fins$$

The surface area is determined as

$$A_s = 2nLH = 2\times 14\times 0.18\ m\times 0.00361\ m = 0.0182\ m^2$$

The rate of heat transfer is

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.641\ W/m^2\cdot{}^\circ C)(0.0182\ m^2)(65-25\ {}^\circ C) = 3.38\ W$$