Important Concepts for ECE309 Final Exam

1. **Fundamentals**
   - properties, property tables
     - interpolation
     - temperature and pressure dependence
   - units, unit conversions
   - conservation equations
     - energy balances
     - conservation of mass: steady flow
   - force balance
   - assumptions

2. **Steady state conduction**

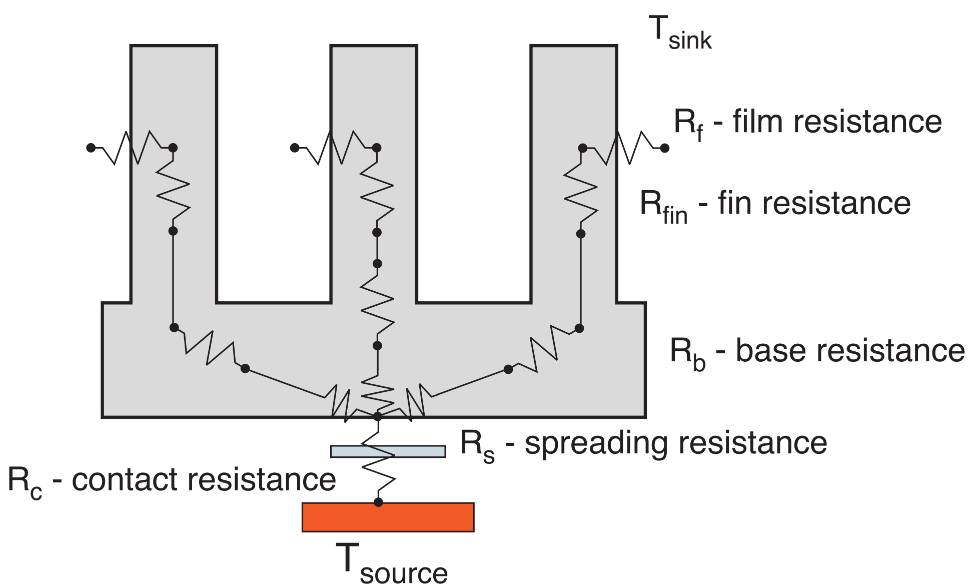
- thermal resistance networks

Conduction: $R_{cond} = \dfrac{L}{kA}$ $\quad R = \dfrac{\ln(r_2/r_1)}{2\pi k \mathcal{L}}$ $\quad R = \dfrac{r_o - r_i}{4\pi k r_i r_o}$

Convection: $R_{conv} = \dfrac{1}{hA}$

Radiation: $R_{rad} = \dfrac{1}{h_{rad}A} \longrightarrow h_{rad} = \epsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr})$

Contact: $R_c = \dfrac{1}{h_c A} \longrightarrow h_c$ see Table 10-2



- series resistance

$$\dot{Q} = \frac{T_{source} - T_{sink}}{R_{total}} \qquad \text{where} \quad R_{total} = \sum_{i=1}^{n} R_i$$

- parallel resistance

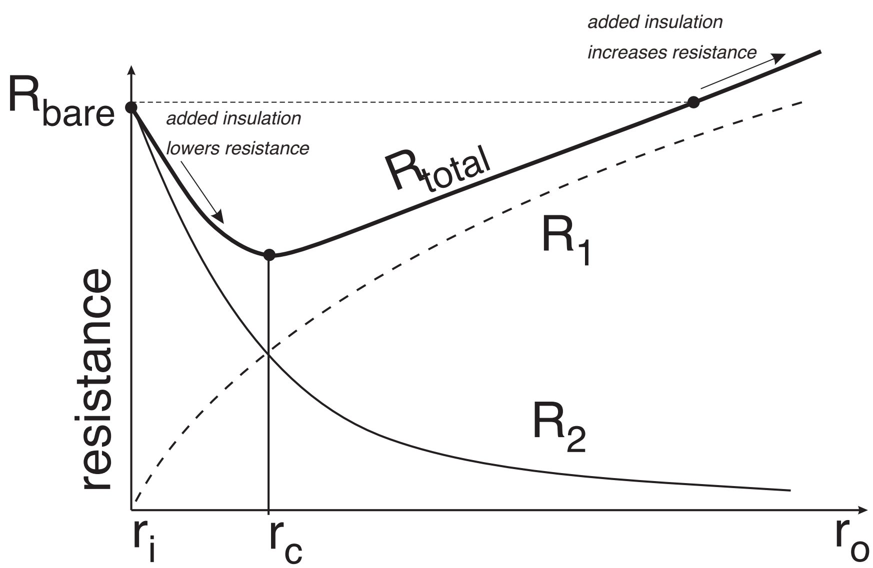
$$\dot{Q} = \sum \dot{Q}_i = (T_{source} - T_{sink})\left(\sum \frac{1}{R_i}\right) \qquad \text{where} \qquad \frac{1}{R_{total}} = \sum_{1}^{n} \frac{1}{R_i}$$

- overall heat transfer coefficient

$$UA = \frac{1}{R_{total}}$$

- critical thickness of insulation

$$r_{cr,cyl} = \frac{k}{h} \quad [m] \qquad\qquad r_{cr,sphere} = \frac{2k}{h} \quad [m]$$



- finned rectangular surfaces
  - temperature profile

$$\frac{\theta(x)}{\theta_b} \Rightarrow \quad \text{for i) prescribe tip temperature, ii) adiabatic tip, and infinitely long fin}$$

  - heat flow rate

$$\dot{Q}_b \Rightarrow \quad \text{for i) prescribe tip temperature, ii) adiabatic tip, and infinitely long fin}$$

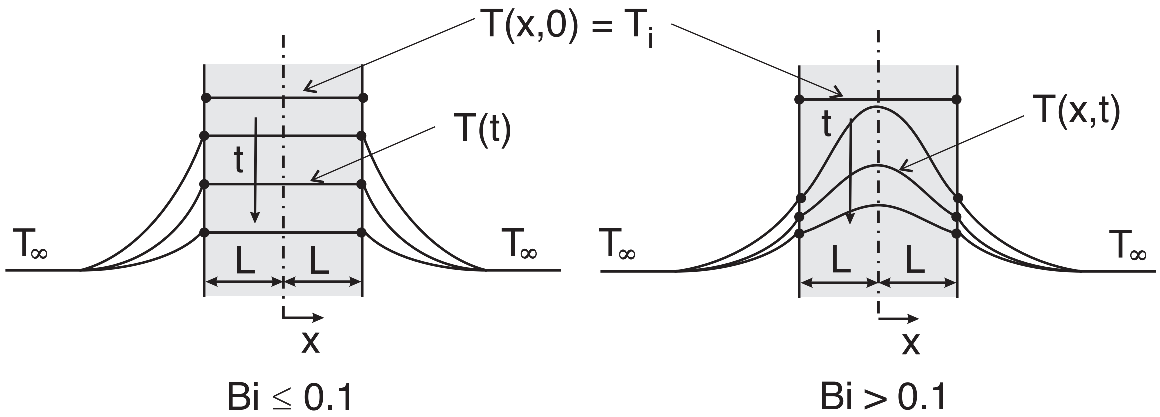
  - efficiency and effectiveness (analytical and graphical)

$$\eta = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate when the entire fin is at } T_b} = \frac{\dot{Q}_b}{hPL\theta_b}$$

$$\epsilon_{fin} = \frac{\text{total fin heat transfer}}{\text{the heat transfer that would have occurred through the base area in the absence of the fin}} = \frac{\dot{Q}_b}{hA_c\theta_b}$$

  - rectangular and non-rectangular cross sections (tables and charts)

3. **Transient conduction**



(a) Lumped system analysis

- $$Bi = \frac{hV}{kA} < 0.1 \quad \Leftarrow \text{slab} - L \quad \text{cylinder} = r/2 \quad \text{sphere} - r/3$$
- $$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-t/(R_{th} \cdot C_{th})} = e^{-t/\tau} = e^{-bt}$$
- $$Q_{total} = mC(T_i - T_\infty)[1 - e^{-t^*/\tau}]$$

(b) Approximate analytical method

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The analytical solution to this equation takes the form of a series solution

$$\frac{T(x,t) - T_\infty}{T_i - T_\infty} = \sum_{n=1,3,5...}^{\infty} A_n e^{\left(-\frac{\lambda_n}{L}\right)^2 \alpha t} \cos\left(\frac{\lambda_n x}{L}\right)$$

- $Bi > 0.1$ and $Fo > 0.2$
- **Plane Wall:** $$\theta_{wall}(x,t) = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 Fo} \cos(\lambda_1 x/L)$$

Cylinder: $$\theta_{cyl}(r,t) = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 Fo} \,\mathrm{J}_0(\lambda_1 r/r_o)$$

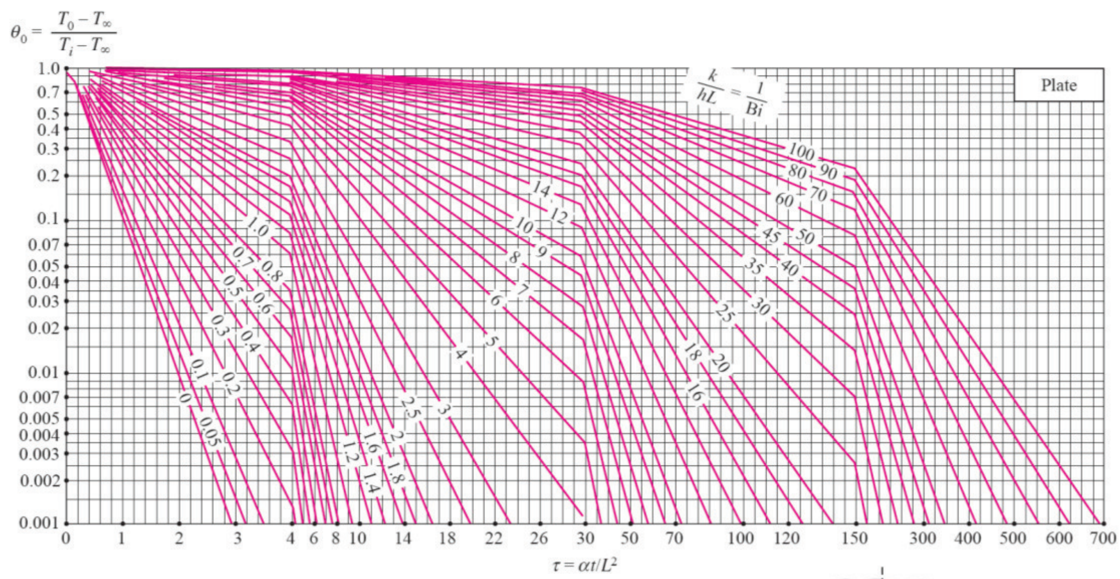
Sphere: $$\theta_{sph}(r,t) = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 Fo} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

- solutions available for $\dfrac{Q}{Q_{max}}$ where

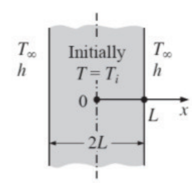
$$Q_{max} = mc_p(T_\infty - T_i) = \rho \mathcal{V} c_p(T_\infty - T_i)$$

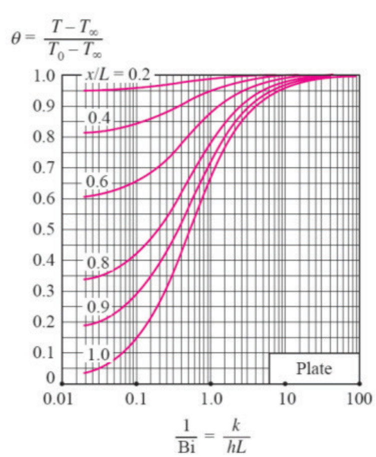
(c) Heisler charts

- find T_0 at the center for a given time (Table 11-15 a, Table 11-16 a or Table 11-17 a)
- find T at other locations at the same time (Table 11-15 b, Table 11-16 b or Table 11-17 b)
- find Q_{tot} up to time t (Table 11-15 c, Table 11-16 c or Table 11-17 c)

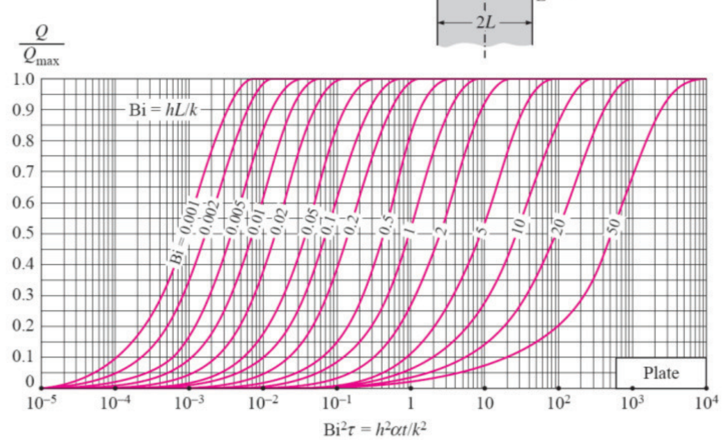


(*a*) Midplane temperature (From M. P. Heisler, Temperat ure Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





(*b*) Temperature distribution (From M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



(*c*) Heat transfer (*From H. Gröber et al.*)

4. **Forced convection**

- empirical correlations

$$Nu = f(Re, Pr) = C_2 \cdot Re^m \cdot Pr^n = \frac{h_x \cdot x}{k_f} \quad \Leftarrow \text{(plate)}$$

(a) external flow

- transition, laminar to turbulent

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

- correlations:
  - local versus average
  - laminar versus turbulent versus blended
  - UWT versus UWF
  - range of Prandtl number
  - other conditions

(b) internal flow

- mean velocity

$$U_m = \frac{1}{A_c}\int_{A_c} u \, dA = \frac{\dot{m}}{\rho_m A_c}$$

$$Re_D = \frac{U_m D}{\nu}$$

- boundary layer thickness
  - hydrodynamic BL

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu}\right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

  - hydrodynamic entry length

$$L_h \approx 0.05 Re_D D \qquad \text{(laminar flow)}$$

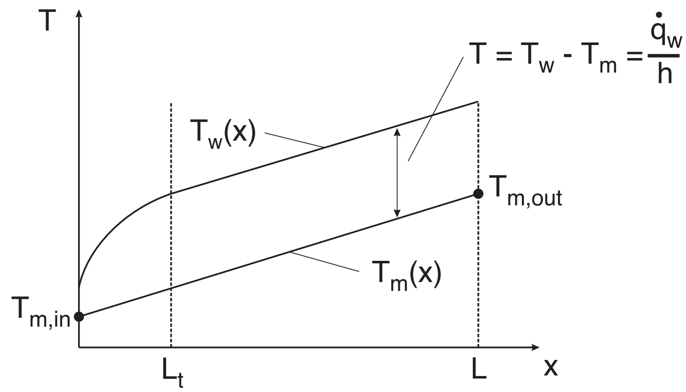
  - thermal entry length

$$L_t \approx 0.05 Re_D Pr D = Pr L_h \qquad \text{(laminar flow)}$$

- Uniform wall heat flux

$$T_{m,x} = T_{m,i} + \frac{\dot{q}_w A}{\dot{m} C_p}$$

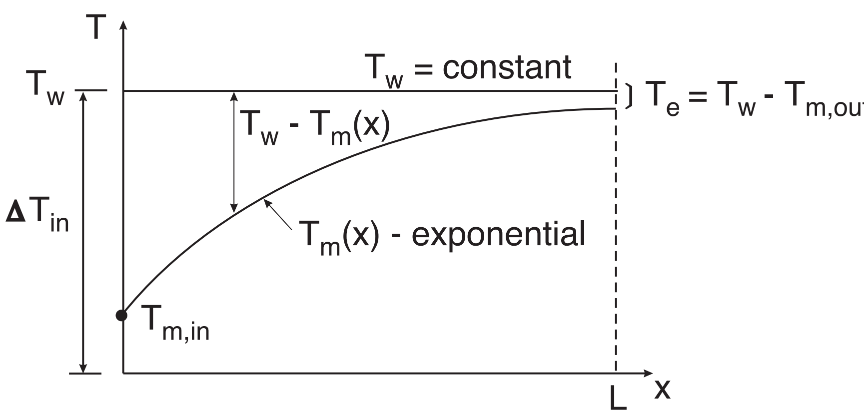
$$T_w = T_m + \frac{\dot{q}_w}{h}$$



- Isothermal wall temperature

$$T_{out} = T_w - (T_w - T_{in}) \exp[-hA/(\dot{m} C_p)]$$

$$\Delta T_{ln} = \frac{T_{out} - T_{in}}{\ln\left(\dfrac{T_w - T_{out}}{T_w - T_{in}}\right)} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})} \Rightarrow \dot{Q} = hA\Delta T_{ln}$$



- correlations

5. **Natural convection**

- correlations

$$Nu = f(Gr, Pr) \equiv C Gr^m Pr^n \qquad \text{where} \quad Ra = Gr \cdot Pr$$

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \underbrace{Gr_{\mathcal{L}}^{1/4} Pr^{1/4}}_{Ra^{1/4}}$$

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)D^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \underbrace{Gr_D^{1/4} Pr^{1/4}}_{Ra_D^{1/4}}$$

6. **Radiation**

(a) Blackbody radiation

- blackbody emissive power

$$E_b = \sigma\, T^4 \quad [W/m^2] \qquad \Leftarrow \text{ Stefan-Boltzmann law}$$

- blackbody radiation function

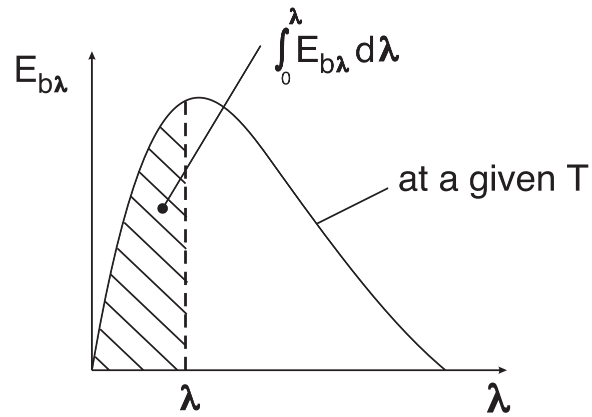
$$f_{0\to\lambda} = \frac{\displaystyle\int_0^t \frac{C_1 T^5 (1/T) dt}{t^5[\exp(C_2/t) - 1]}}{\sigma T^4}$$

$$= \frac{C_1}{\sigma}\int_0^{\lambda T} \frac{dt}{t^5[\exp(C_2/t) - 1]}$$

$$= f(\lambda T)$$

$$f_{\lambda_1\to\lambda_2} = f(\lambda_2 T) - f(\lambda_1 T)$$

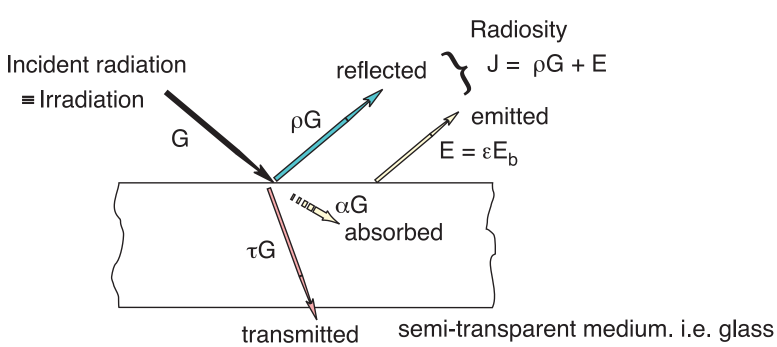
$$f_{\lambda\to\infty} = 1 - f_{0\to\lambda}$$



- emissivity

$$\epsilon(T) = \frac{\text{radiation emitted by surface at temperature } T}{\text{radiation emitted by a black surface at } T}$$

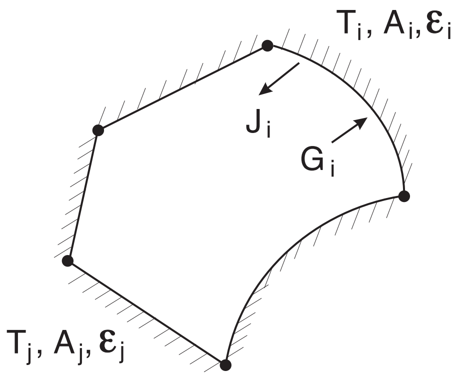
$$= \frac{\displaystyle\int_0^\infty E_\lambda(T)\, d\lambda}{\displaystyle\int_0^\infty E_{b\lambda}(T)\, d\lambda} = \frac{\displaystyle\int_0^\infty \epsilon_\lambda(T) E_{b\lambda}(T)\, d\lambda}{E_b(T)} = \frac{E(T)}{\sigma T^4}$$

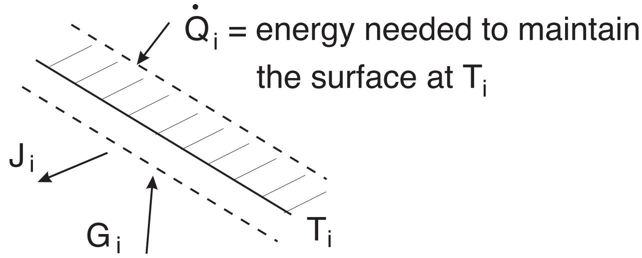


- View Factor

$$F_{i\to j} = \frac{\dot{Q}_{i\to j}}{A_i J_i} = \frac{\text{radiation reaching } j}{\text{radiation leaving } i}$$

- radiation exchange between surfaces





  - assumptions: enclosure, diffuse-gray, opaque surfaces, isothermal surfaces, uniform radiosity and irradiation, non-participating medium

  - surface resistance

$$Q_i = \frac{E_{b,i} - J_i}{\left(\frac{1-\epsilon_i}{\epsilon_i A_i}\right)} \equiv \frac{\text{potential difference}}{\text{surface resistance}}$$

  - space resistance

$$\dot{Q}_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(\frac{1}{A_i F_{i\rightarrow j}}\right)} \equiv \frac{\text{potential difference}}{\text{space resistance}}$$

  - simplifying assumptions

    * black surfaces

$$\epsilon_i = \alpha_i = 1$$

      and

$$J_i = E_{b,i} = \sigma T_i^4 \quad \Leftarrow$$

    * re-radiating, fully insulated surfaces

      heat flow into the node equal heat flow out of the node

$$\dot{Q}_i = 0$$