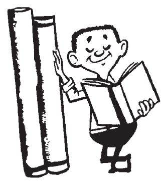
Convection Heat Transfer



Reading	Problems
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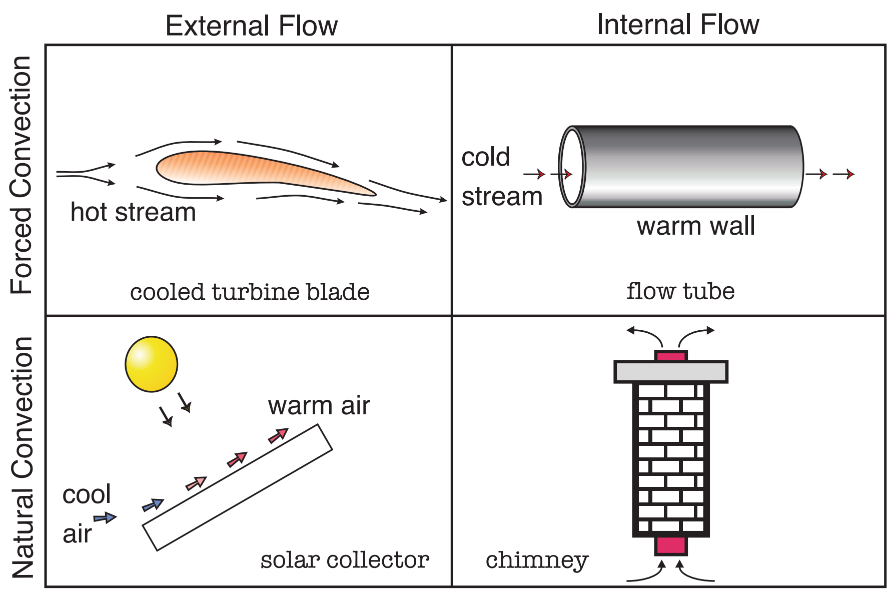
Introduction



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Newton's Law of Cooling

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty)$$

$$\Rightarrow \quad R_{conv} = \frac{1}{hA}$$

Typical Values of h (W/m^2K)

Natural Convection	gases: 3-20 water: 60 - 900
Forced Convection	gases: 30 - 300 oils: 60 - 1800 water: 100 -1500
Boiling	water: 3000 - 10^5
Condensation	steam: 3000 - 10^5

Controlling Factors

Geometry: shape, size, aspect ratio and orientation

Flow Type: forced, natural, laminar, turbulent, internal, external

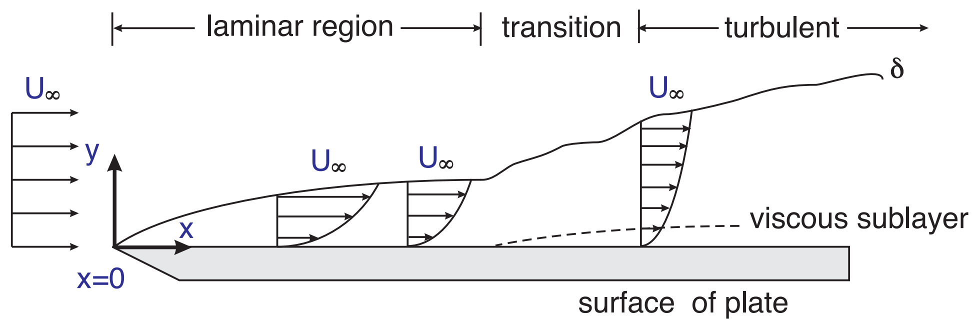
Boundary: isothermal (T_w = constant) or isoflux ($\dot{q}_w$ = constant)

Fluid Type: viscous oil, water, gases or liquid metals

Properties: all properties determined at film temperature
$T_f = (T_w + T_\infty)/2$
Note: ρ and $\nu \propto 1/P_{atm}$ ⇒ see Q12-40
density: ρ $((kg/m^3)$
specific heat: C_p $(J/kg \cdot K)$
dynamic viscosity: μ, $(N \cdot s/m^2)$
kinematic viscosity: $\nu \equiv \mu/\rho$ (m^2/s)
thermal conductivity: k, $(W/m \cdot K)$
thermal diffusivity: α, $\equiv k/(\rho \cdot C_p)$ (m^2/s)
Prandtl number: Pr, $\equiv \nu/\alpha$ $(--)$
volumetric compressibility: β, $(1/K)$

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.

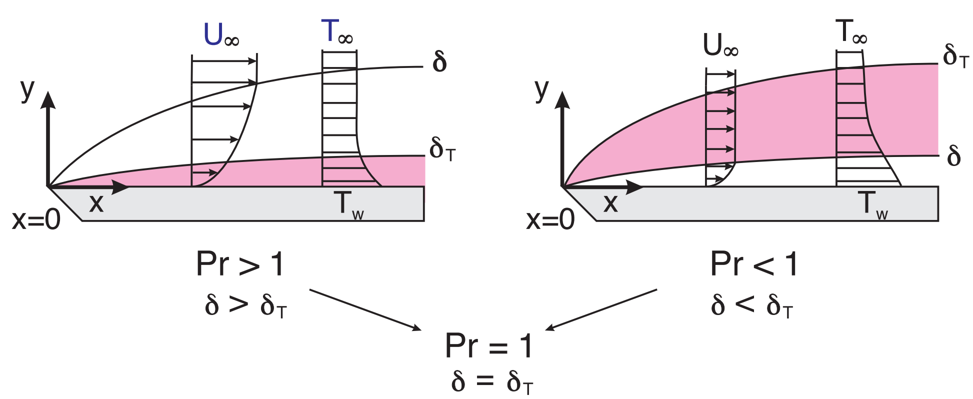


- flow forms thin layers that can slip past one another at different velocities
- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

  - for flow over a flat plate $Re_{cr} \approx 500,000$
  - $x < x_{cr}$ the boundary layer is laminar; $x > x_{cr}$ the boundary layer is turbulent

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity at the surface of the plate, $y = 0$, is set to zero, $U_{@y=0} = 0\ m/s$ because of the *no slip condition* at the wall

- the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99\, U_\infty$ which is denoted as the δ, the *velocity boundary layer thickness*.

- the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at U_∞

Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

- for *low Prandtl number* fluids the velocity boundary layer is fully contained within the thermal boundary layer

- conversely, for *high Prandtl number* fluids the thermal boundary layer is contained within the velocity boundary layer

Flow Over Plates

1. Laminar Boundary Layer Flow, Isothermal (UWT)

All laminar formulations for $Re < 500,000$. The local value of the Nusselt number is given as

$$\boxed{Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}} \quad \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6$$

An average value of the heat transfer coefficient for the full extent of the plate can be obtained by using the mean value theorem.

$$\boxed{Nu_L = \frac{h_L L}{k_f} = 0.664\, \text{Re}_L^{1/2}\, \text{Pr}^{1/3}} \quad \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6$$

For <u>low Prandtl numbers</u>, i.e. liquid metals

$$\boxed{Nu_x = 0.565\, Re_x^{1/2}\, Pr^{1/2}} \quad \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6$$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

All turbulent formulations for $500,000 \leq Re \leq 10^7$. The local Nusselt number is given as

$$\boxed{Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3}} \quad \Rightarrow \begin{array}{l}\text{local, turbulent, UWT,} \\ 0.6 < Pr < 60\end{array}$$

and the average Nusselt number is

$$\boxed{Nu_L = 0.037\ Re_L^{0.8}\ Pr^{1/3}} \Rightarrow \begin{array}{l}\text{average, turbulent, UWT,} \\ 0.6 < Pr < 60\end{array}$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When $(T_w - T_\infty)$ is constant

$$h_L = \frac{1}{L}\int_0^L h dx = \frac{1}{L}\left\{\int_0^{x_{cr}} h_x^{lam} dx + \int_{x_{cr}}^L h_x^{tur} dx\right\}$$

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037\ Re_L^{0.8} - 871)\ Pr^{1/3}} \Rightarrow \begin{array}{l}\text{average, combined, UWT,} \\ 0.6 < Pr < 60, \\ 500,000 \leq Re_L \leq 10^7\end{array}$$

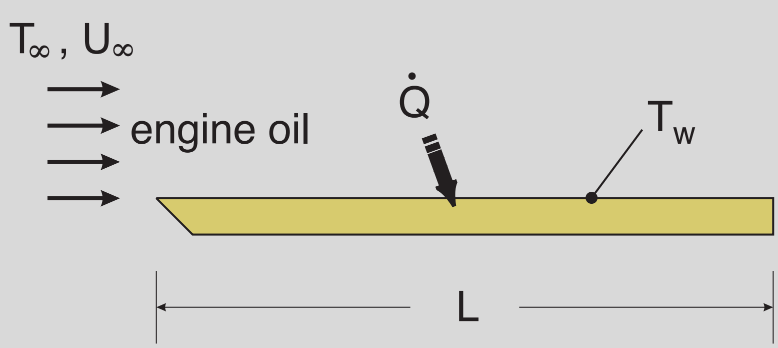
4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453\ Re_x^{1/2}\ Pr^{1/3}} \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308\ Re_x^{4/5}\ Pr^{1/3}} \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6$$

Example 6-1: Hot engine oil with a bulk temperature of $60\ °C$ flows over a horizontal, flat plate $5\ m$ long with a wall temperature of $20\ °C$. If the fluid has a free stream velocity of $2\ m/s$, determine the heat transfer rate from the oil to the plate if the plate is assumed to be of unit width.



Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Bernstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$Nu_D = 0.3 + \frac{0.62\, Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$\Rightarrow$ average, UWT, $Re_D < 10^7$, $0 \leq Pr \leq \infty$, $Re_D \cdot Pr > 0.2$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 12-3 are commonly used, where

$$Nu_D \approx \frac{\overline{h}D}{k} = C\, Re_D^m\, Pr^{1/3}$$

$\Rightarrow$ see Table 12-3 for conditions

3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

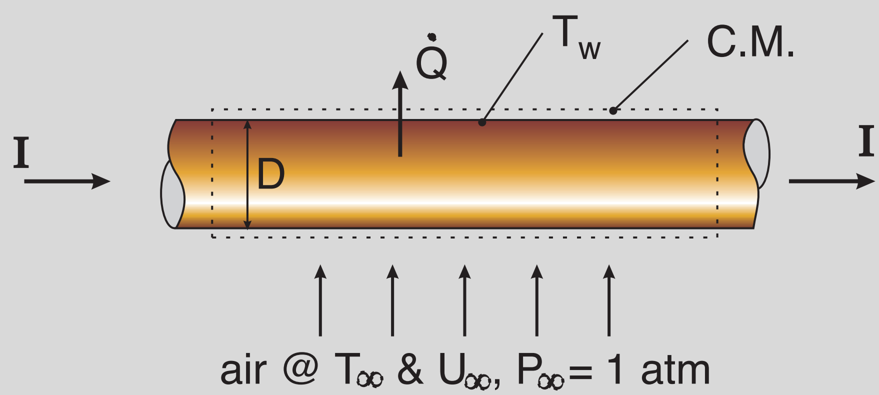
For flow over an isothermal sphere of diameter D, Whitaker recommends

$$Nu_D = 2 + \left[0.4\, Re_D^{1/2} + 0.06\, Re_D^{2/3}\right] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$\Rightarrow$ average, UWT, $0.7 \leq Pr \leq 380$, $3.5 < Re_D < 80,000$

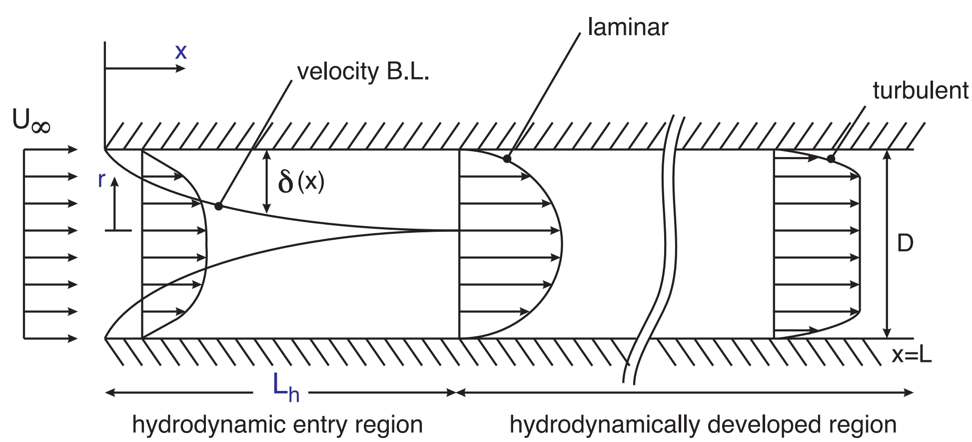
where the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on the free stream temperature, T_∞ and the dynamic viscosity of the fluid at the surface, μ_s, is based on the surface temperature, T_s. All other properties are based on T_∞.

Example 6-2: An electric wire with a $1\ mm$ diameter and a wall temperature of $325\ K$ is cooled by air in cross flow with a free stream temperature of $275\ K$. Determine the air velocity required to maintain a steady heat loss per unit length of $70\ W/m$.



Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below



The Reynolds number is given as: $Re_D = \dfrac{U_m D}{\nu}$. For flow in a tube:

$Re_D < 2300$	laminar flow
$2300 < Re_D < 4000$	transition to turbulent flow
$Re_D > 4000$	turbulent flow

For engineering calculations, we typically assume that $Re_{cr} \approx 2300$, therefore

$$Re_D \begin{cases} < Re_{cr} & \text{laminar} \\ > Re_{cr} & \text{turbulent} \end{cases}$$

Hydrodynamic (Velocity) Boundary Layer

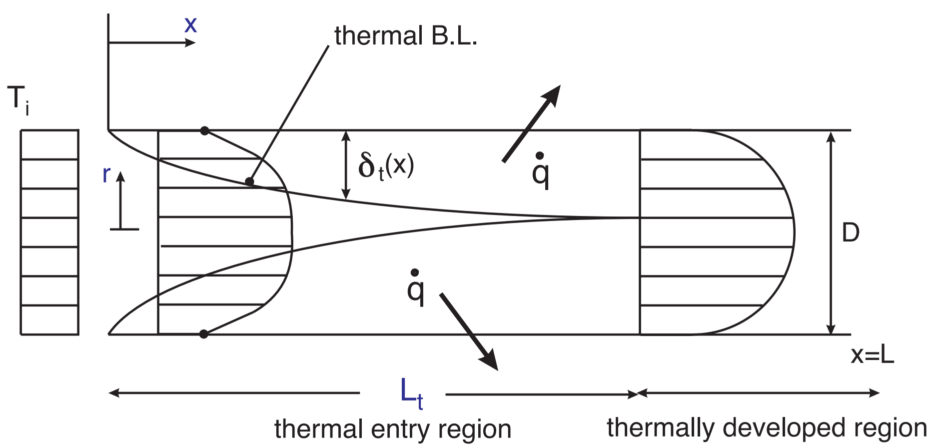
- when the boundary layer grows to the tube radius, r, the boundary layers merge
  - this flow length is called the flow entrance length, L_h
  - $0 \leq x \leq L_h$ is the hydrodynamic entrance region
  - $L_h < x \leq L$ is the fully developed region or hydrodynamically developed region
- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu}\right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

- the hydrodynamic entry length can be approximated as

$$L_h \approx 0.05 Re_D D \qquad \text{(laminar flow)}$$

Thermal Boundary Layer



- a thermal entrance region develops from $0 \leq x \leq L_t$
- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D = Pr L_h \qquad \text{(laminar flow)}$$

- for turbulent flow $L_h \approx L_t \approx 10D$

Wall Boundary Conditions

1. Uniform Wall Heat Flux: The total heat transfer from the wall to the fluid stream can be determined by performing an energy balance over the tube. If we assume steady flow conditions, $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ then the energy balance becomes

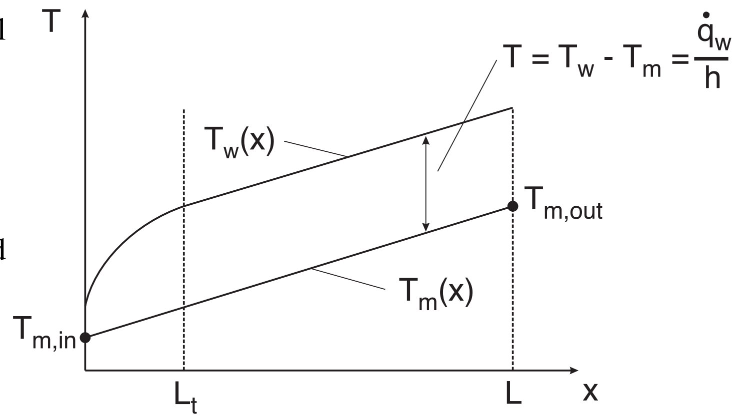
$$\dot{Q} = \dot{q}_w A = \dot{m}(h_{out} - h_{in}) = \dot{m} C_p (T_{out} - T_{in})$$

Since the wall flux $\dot{q}_w$ is uniform, the local mean temperature is linear with x.

$$T_{m,x} = T_{m,i} + \frac{\dot{q}_w A}{\dot{m} C_p}$$

The surface temperature can be determined from

$$T_w = T_m + \frac{\dot{q}_w}{h}$$



2. Isothermal Wall: Using Newton's law of cooling we can determine the average rate of heat transfer to or from a fluid flowing in a tube

$$\dot{Q} = hA \underbrace{(T_w - T_m)}_{average\ \Delta T}$$

From an energy balance over a control volume in the fluid, we can determine

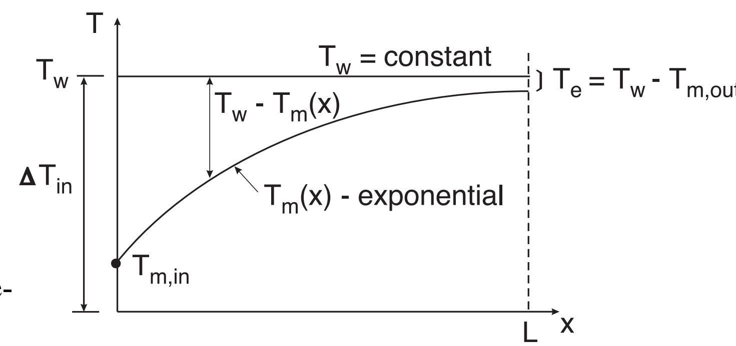
$$\dot{Q} = \dot{m} C_p dT_m$$

Equating the two equations above we find

$$\dot{m} C_p dT_m = hA \underbrace{(T_w - T_m)}_{average\ \Delta T}$$

By isolating the temperature terms and integrating we obtain

$$\ln\left(\frac{T_w - T_{out}}{T_w - T_{in}}\right) = -\frac{hA}{\dot{m} C_p}$$



Because of the exponential temperature decay within the tube, it is common to present the mean temperature from inlet to outlet as a log mean temperature difference where

$$\Delta T_{ln} = \frac{T_{out} - T_{in}}{\ln\left(\frac{T_w - T_{out}}{T_w - T_{in}}\right)} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})} \Rightarrow \dot{Q} = hA\Delta T_{ln}$$

1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$\boxed{Nu_D = 3.66}$ $\Rightarrow$ fully developed, laminar, UWT, $L > L_t$ & L_h

$\boxed{Nu_D = 4.36}$ $\Rightarrow$ fully developed, laminar, UWF, $L > L_t$ & L_h

$$\boxed{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L}\right)^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}} \quad \Rightarrow$$

developing laminar flow, UWT,
$Pr > 0.5$
$L < L_h$ or $L < L_t$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction with Table 13-1 to determine the Reynolds number and in turn the Nusselt number.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Boelter equation (Eq. 13-68) can be used

$$\boxed{Nu_D = 0.023\, Re_D^{0.8}\, Pr^n} \quad \Rightarrow$$

turbulent flow, UWT or UWF,
$0.7 \leq Pr \leq 160$
$Re_D > 2,300$
$n = 0.4$ heating
$n = 0.3$ cooling

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

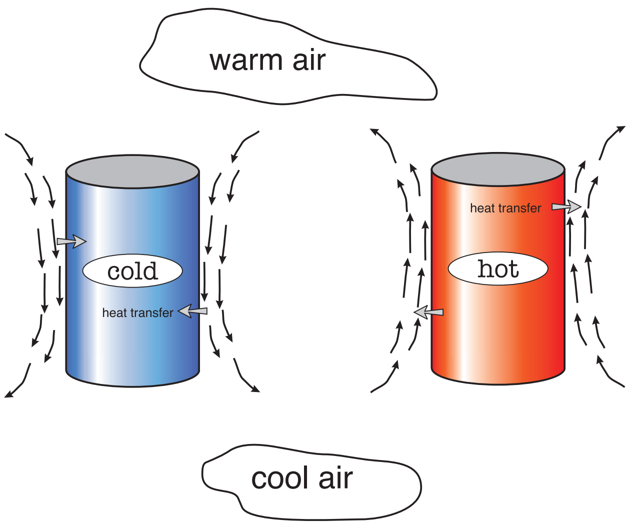
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2}\left(T_{m,in} + T_{m,out}\right)$$

except for μ_s which is evaluated at the wall temperature, T_s.

Natural Convection

What Drives Natural Convection?



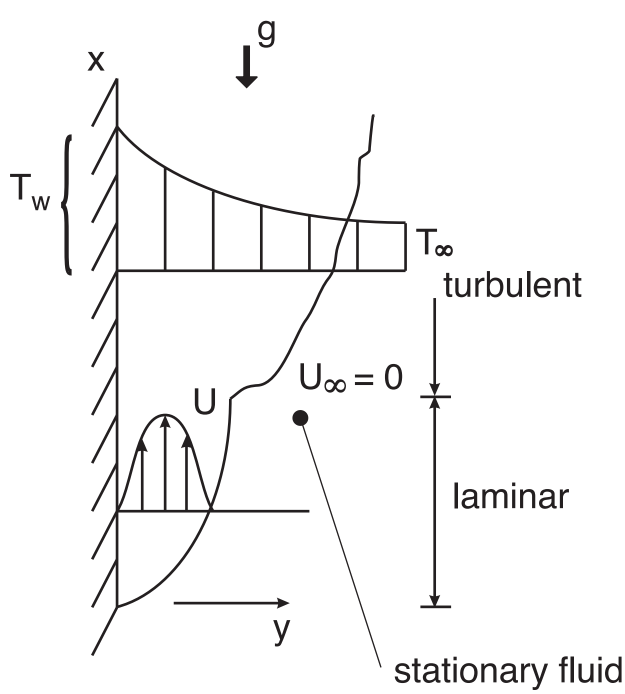
- fluids tend to expand when heated and contract when cooled at constant pressure
- therefore a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface
- a lighter fluid will flow upward and a cooler fluid will flow downward
- as the fluid sweeps the wall, heat transfer will occur in a similar manner to boundary layer flow however in this case the bulk fluid is stationary as opposed to moving at a constant velocity in the case of forced convection

In natural convection, the *Grashof number* is analogous to the Reynolds number.

$$Gr = \frac{\text{buouancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}$$

Natural Convection Over Surfaces

- natural convection heat transfer depends on geometry and orientation
- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero
- the velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:



Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$Nu = f(Gr, Pr) \equiv C Gr^m Pr^n \qquad \text{where} \quad Ra = Gr \cdot Pr$$

- C depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- m depends on type of flow (laminar or turbulent)
- n depends on the type of fluid and type of flow
- Table 14-1 should be used to find Nusselt number for various combinations of geometry and boundary conditions
  - for ideal gases $\beta = 1/T_f, \ (1/K)$
  - all fluid properties are evaluated at the film temperature, $T_f = (T_w + T_\infty)/2$.

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \ \underbrace{Gr_{\mathcal{L}}^{1/4} \ Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\underbrace{\frac{g\beta(T_w - T_\infty)D^3}{\nu^2}}_{\equiv Gr} \right)^{1/4} \left(\underbrace{\frac{\nu}{\alpha}}_{\equiv Pr} \right)^{1/4} = C \ \underbrace{Gr_D^{1/4} \ Pr^{1/4}}_{Ra_D^{1/4}}$$

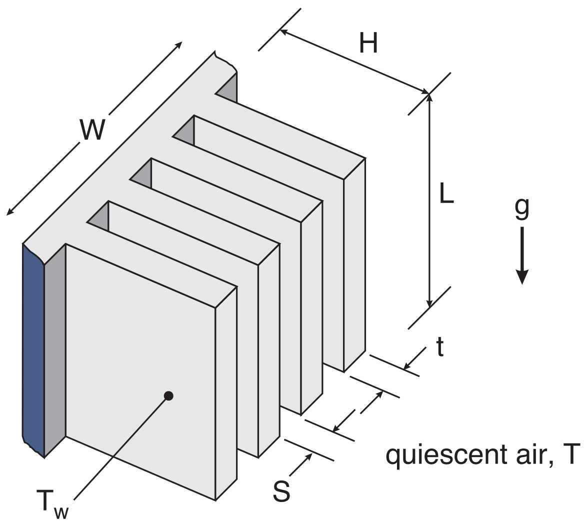
where

$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

Natural Convection From Plate Fin Heat Sinks

The average Nusselt number for an isothermal plate fin heat sink with natural convection can be determined using

$$Nu_S = \frac{hS}{k_f} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}}\right]^{-0.5}$$



Two factors must be considered in the selection of the number of fins

- more fins results in added surface area and reduced boundary layer resistance,

$$R \downarrow = \frac{1}{hA \uparrow}$$

- more fins leads to a decrease fin spacing and a decrease in the heat transfer coefficient

$$R \uparrow = \frac{1}{h \downarrow A}$$

A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714\left(\frac{L}{Ra_L^{1/4}}\right)$$

with

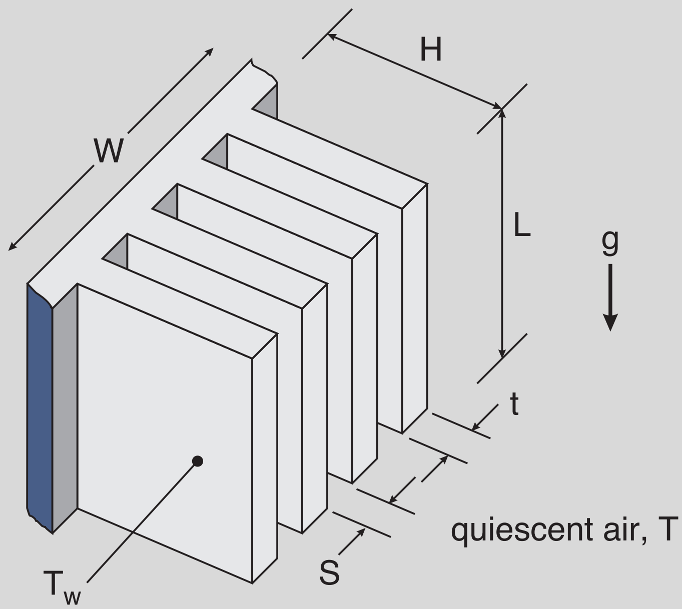
$$Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} Pr$$

The corresponding value of the heat transfer coefficient is

$$h = 1.307 k_f / S_{opt}$$

All fluid properties are evaluated at the film temperature.

Example 6-3: Find the optimum fin spacing, S_{opt} and the rate of heat transfer, $\dot{Q}$ for the following plate fin heat sink cooled by natural convection.



Given:

$$
\begin{aligned}
W &= 120\ mm & H &= 24\ mm \\
L &= 18\ mm & t &= 1\ mm \\
T_w &= 80\ ^\circ C & T_\infty &= 25\ ^\circ C \\
P_\infty &= 1\ atm & \text{fluid} &= \text{air}
\end{aligned}
$$

Find: S_{opt} and the corresponding heat transfer, $\dot{Q}$