**Convection Heat Transfer**

<table>
<thead>
<tr>
<th>Reading</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-1 → 12-8</td>
<td>12-40, 12-49, 12-68, 12-70, 12-87, 12-98</td>
</tr>
<tr>
<td>14-1 → 14-4</td>
<td>14-18, 14-24, 14-45, 14-82</td>
</tr>
</tbody>
</table>

**Introduction**

**Newton’s Law of Cooling**

\[ \dot{Q}_{\text{conv}} = \frac{\Delta T}{R_{\text{conv}}} = hA(T_w - T_\infty) \]

\[ \Rightarrow R_{\text{conv}} = \frac{1}{hA} \]

**Typical Values of \( h \) (\( W/m^2K \))**

<table>
<thead>
<tr>
<th>Type</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Convection</td>
<td>gases: 3-20</td>
</tr>
<tr>
<td></td>
<td>water: 60 - 900</td>
</tr>
<tr>
<td>Forced Convection</td>
<td>gases: 30 - 300</td>
</tr>
<tr>
<td></td>
<td>oils: 60 - 1800</td>
</tr>
<tr>
<td></td>
<td>water: 100 - 1500</td>
</tr>
<tr>
<td>Boiling</td>
<td>water: 3000 - 10^5</td>
</tr>
<tr>
<td>Condensation</td>
<td>steam: 3000 - 10^9</td>
</tr>
</tbody>
</table>

**Controlling Factors**

- **Geometry**: shape, size, aspect ratio and orientation
- **Flow Type**: forced, natural, laminar, turbulent, internal, external
- **Boundary**: isothermal \( (T_w = \text{constant}) \) or isoflux \( (\dot{q}_w = \text{constant}) \)
- **Fluid Type**: viscous oil, water, gases or liquid metals
- **Properties**: all properties determined at film temperature \( T_f = (T_w + T_\infty)/2 \)
  - density: \( \rho \)
  - specific heat: \( C_p \)
  - dynamic viscosity: \( \mu \)
  - kinematic viscosity: \( \nu = \frac{\mu}{\rho} \)
  - thermal conductivity: \( k \)
  - thermal diffusivity: \( \alpha = \frac{k}{\rho C_p} \)
  - Prandtl number: \( Pr = \frac{\nu}{\alpha} \)
  - volumetric compressibility: \( \beta = \frac{1}{K} \)
Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.

- flow forms thin layers that can slip past one another at different velocities
- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
  - transition from laminar to turbulent is called the critical Reynolds number, $Re_{cr}$

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

- for flow over a flat plate $Re_{cr} \approx 500,000$
- $x < x_{cr}$ the boundary layer is laminar; $x > x_{cr}$ the boundary layer is turbulent

Boundary Layers

- the region of fluid flow over the plate where viscous effects dominate is called the velocity or hydrodynamic boundary layer
- the velocity at the surface of the plate, $y = 0$, is set to zero, $U_{@y=0} = 0 \text{ m/s}$ because of the no slip condition at the wall
• the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99 \, U_\infty$ which is denoted as the $\delta$, the *velocity boundary layer thickness*.

• the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at $U_\infty$.

**Thermal Boundary Layer**

• the thermal boundary layer is arbitrarily selected as the locus of points where

\[
\frac{T - T_w}{T_\infty - T_w} = 0.99
\]

• for *low Prandtl number* fluids the velocity boundary layer is fully contained within the thermal boundary layer

• conversely, for *high Prandtl number* fluids the thermal boundary layer is contained within the velocity boundary layer

**Flow Over Plates**

1. Laminar Boundary Layer Flow, Isothermal (UWT)

All laminar formulations for $Re < 500,000$. The local value of the Nusselt number is given as

\[
Nu_x = 0.332 \, Re_x^{1/2} \, Pr^{1/3} \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6
\]

An average value of the heat transfer coefficient for the full extent of the plate can be obtained by using the mean value theorem.

\[
Nu_L = \frac{h_L L}{k_f} = 0.664 \, Re_L^{1/2} \, Pr^{1/3} \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6
\]

For *low Prandtl numbers*, i.e. liquid metals

\[
Nu_x = 0.565 \, Re_x^{1/2} \, Pr^{1/2} \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6
\]

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

All turbulent formulations for $500,000 \leq Re \leq 10^7$. The local Nusselt number is given as

\[
Nu_x = 0.0296 \, Re_x^{0.8} \, Pr^{1/3} \Rightarrow 0.6 < Pr < 60
\]
and the average Nusselt number is

\[ Nu_L = 0.037 \frac{Re_L^{0.8}}{Pr^{1/3}} \Rightarrow 0.6 < Pr < 60 \]

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When \((T_w - T_\infty)\) is constant

\[ h_L = \frac{1}{L} \int_0^L h \, dx = \frac{1}{L} \left\{ \int_0^{x_{cr}} h_{x_{lam}} \, dx + \int_{x_{cr}}^L h_{x_{tur}} \, dx \right\} \]

\[ Nu_L = \frac{h_L L}{k} = (0.037 \frac{Re_L^{0.8}}{Pr^{1/3}} - 871) Pr^{1/3} \Rightarrow 0.6 < Pr < 60, \quad 500,000 \leq Re_L \leq 10^7 \]

4. Laminar Boundary Layer Flow, Isoflux (UWF)

\[ Nu_x = 0.453 \frac{Re_x^{1/2}}{Pr^{1/3}} \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6 \]

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

\[ Nu_x = 0.0308 \frac{Re_x^{4/5}}{Pr^{1/3}} \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6 \]

**Example 6-1:** Hot engine oil with a bulk temperature of 60 °C flows over a horizontal, flat plate 5 m long with a wall temperature of 20 °C. If the fluid has a free stream velocity of 2 m/s, determine the heat transfer rate from the oil to the plate if the plate is assumed to be of unit width.
**Flow Over Cylinders and Spheres**

1. **Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)**

The Churchill-Bernstein (1977) correlation for the average Nusselt number for long \((L/D > 100)\) cylinders is

\[
Nu_D = 0.3 + \frac{0.62 \, Re_D^{1/2} \, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[ 1 + \left( \frac{Re_D}{282,000} \right)^{5/8} \right]^{4/5}
\]

⇒ average, UWT, \(Re_D < 10^7\), \(0 \leq Pr \leq \infty\), \(Re_D \cdot Pr > 0.2\)

All fluid properties are evaluated at \(T_f = (T_w + T_\infty)/2\).

2. **Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)**

The empirical formulations of Zhukauskas and Jakob given in Table 12-3 are commonly used, where

\[
Nu_D \approx \frac{hD}{k} = C \, Re_D^m \, Pr^{1/3}
\]

⇒ see Table 12-3 for conditions

3. **Boundary Layer Flow Over a Sphere, Isothermal (UWT)**

For flow over an isothermal sphere of diameter \(D\), Whitaker recommends

\[
Nu_D = 2 + [0.4 \, Re_D^{1/2} + 0.06 \, Re_D^{2/3}] \, Pr^{0.4} \left( \frac{\mu_\infty}{\mu_s} \right)^{1/4}
\]

⇒ average, UWT, 
\(0.7 \leq Pr \leq 380\)
\(3.5 < Re_D < 80,000\)

where the dynamic viscosity of the fluid in the bulk flow, \(\mu_\infty\) is based on the free stream temperature, \(T_\infty\) and the dynamic viscosity of the fluid at the surface, \(\mu_s\), is based on the surface temperature, \(T_s\). All other properties are based on \(T_\infty\).
Example 6-2: An electric wire with a 1 mm diameter and a wall temperature of 325 K is cooled by air in cross flow with a free stream temperature of 275 K. Determine the air velocity required to maintain a steady heat loss per unit length of 70 W/m.

Internal Flow

Let's consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter $D$ as shown below.

The Reynolds number is given as: $Re_D = \frac{U_m D}{\nu}$. For flow in a tube:

$Re_D < 2300$ \hspace{1cm} \text{laminar flow}$

$2300 < Re_D < 4000$ \hspace{1cm} \text{transition to turbulent flow}$

$Re_D > 4000$ \hspace{1cm} \text{turbulent flow}$
For engineering calculations, we typically assume that $\text{Re}_{cr} \approx 2300$, therefore

$$Re_D \begin{cases} < \text{Re}_{cr} \text{ laminar} \\ > \text{Re}_{cr} \text{ turbulent} \end{cases}$$

**Hydrodynamic (Velocity) Boundary Layer**

- when the boundary layer grows to the tube radius, $r$, the boundary layers merge
  - this flow length is called the flow entrance length, $L_h$
  - $0 \leq x \leq L_h$ is the hydrodynamic entrance region
  - $L_h < x \leq L$ is the fully developed region or hydrodynamically developed region

- the hydrodynamic boundary layer thickness can be approximated as
  $$\delta(x) \approx 5x \left( \frac{U_m x}{\nu} \right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

- the hydrodynamic entry length can be approximated as
  $$L_h \approx 0.05Re_D D \quad \text{(laminar flow)}$$

**Thermal Boundary Layer**

- a thermal entrance region develops from $0 \leq x \leq L_t$
- the thermal entry length can be approximated as
  $$L_t \approx 0.05Re_D Pr D = Pr L_h \quad \text{(laminar flow)}$$

- for turbulent flow $L_h \approx L_t \approx 10D$
Wall Boundary Conditions

1. Uniform Wall Heat Flux: The total heat transfer from the wall to the fluid stream can be determined by performing an energy balance over the tube. If we assume steady flow conditions, \( \dot{m}_{in} = \dot{m}_{out} = \dot{m} \), then the energy balance becomes

\[
\dot{Q} = \dot{q}_w A = \dot{m}(h_{out} - h_{in}) = \dot{m}C_p(T_{out} - T_{in})
\]

Since the wall flux \( \dot{q}_w \) is uniform, the local mean temperature is linear with \( x \).

\[
T_{m,x} = T_{m,i} + \frac{\dot{q}_w A}{\dot{m}C_p}
\]

The surface temperature can be determined from

\[
T_w = T_m + \frac{\dot{q}_w}{h}
\]

2. Isothermal Wall: Using Newton’s law of cooling we can determine the average rate of heat transfer to or from a fluid flowing in a tube

\[
\dot{Q} = hA \left( \frac{T_w - T_m}{\text{average } \Delta T} \right)
\]

From an energy balance over a control volume in the fluid, we can determine

\[
\dot{Q} = \dot{m}C_p dT_m
\]

Equating the two equations above we find

\[
\dot{m}C_p dT_m = hA \left( \frac{T_w - T_m}{\text{average } \Delta T} \right)
\]

By isolating the temperature terms and integrating we obtain

\[
\ln \left( \frac{T_w - T_{out}}{T_w - T_{in}} \right) = -\frac{hA}{\dot{m}C_p}
\]

Because of the exponential temperature decay within the tube, it is common to present the mean temperature from inlet to outlet as a log mean temperature difference where

\[
\Delta T_{in} = \frac{T_{out} - T_{in}}{\ln \left( \frac{T_w - T_{out}}{T_w - T_{in}} \right)} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})} \Rightarrow \dot{Q} = hA\Delta T_{in}
\]
1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where \( Re_D \leq 2300 \)

\[
Nu_D = 3.66 \Rightarrow \text{fully developed, laminar, UWT, } L > L_t \& L_h
\]

\[
Nu_D = 4.36 \Rightarrow \text{fully developed, laminar, UWF, } L > L_t \& L_h
\]

\[
Nu_D = 1.86 \left( \frac{Re_D Pr D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_s} \right)^{0.14} \Rightarrow \text{developing laminar flow, UWT, } Pr > 0.5 \quad P_r > 0.5 \quad L < L_h \text{ or } L < L_t
\]

For non-circular tubes the hydraulic diameter, \( D_h = 4A_c/P \) can be used in conjunction with Table 13-1 to determine the Reynolds number and in turn the Nusselt number.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where \( Re_D \geq 2300 \) the Dittus-Boelter equation (Eq. 13-68) can be used

\[
Nu_D = 0.023 Re^{0.8} Pr^n \Rightarrow n = 0.4 \text{ heating, } \quad n = 0.3 \text{ cooling}
\]

For non-circular tubes, again we can use the hydraulic diameter, \( D_h = 4A_c/P \) to determine both the Reynolds and the Nusselt numbers.

In all cases the fluid properties are evaluated at the mean fluid temperature given as

\[
T_{mean} = \frac{1}{2} (T_{m,in} + T_{m,out})
\]

except for \( \mu_s \) which is evaluated at the wall temperature, \( T_s \).
Natural Convection

What Drives Natural Convection?

- Fluids tend to expand when heated and contract when cooled at constant pressure.
- Therefore, a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface.
- A lighter fluid will flow upward and a cooler fluid will flow downward.
- As the fluid sweeps the wall, heat transfer will occur in a similar manner to boundary layer flow; however, in this case, the bulk fluid is stationary as opposed to moving at a constant velocity in the case of forced convection.

In natural convection, the Grashof number is analogous to the Reynolds number.

\[ Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g \beta (T_w - T_\infty) L^3}{\nu^2} \]

Natural Convection Over Surfaces

- Natural convection heat transfer depends on geometry and orientation.
- Note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero.
- The velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:

10
**Natural Convection Heat Transfer Correlations**

The general form of the Nusselt number for natural convection is as follows:

$$ Nu = f(Gr, Pr) \equiv CGr^m Pr^n $$

where $$ Ra = Gr \cdot Pr $$

- $C$ depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- $m$ depends on type of flow (laminar or turbulent)
- $n$ depends on the type of fluid and type of flow
- Table 14-1 should be used to find Nusselt number for various combinations of geometry and boundary conditions
  - for ideal gases $\beta = 1/T_f$, $(1/K)$
  - all fluid properties are evaluated at the film temperature, $T_f = (T_w + T_\infty)/2$.

1. **Laminar Flow Over a Vertical Plate, Isothermal (UWT)**

The general form of the Nusselt number is given as

$$ Nu_L = \frac{hL}{k_f} = C \left( \frac{g\beta(T_w - T_\infty)L^3}{\nu^2 \equiv Gr} \right)^{1/4} \left( \frac{\nu}{\alpha \equiv Pr} \right)^{1/4} = C \frac{Gr_L^{1/4} Pr^{1/4}}{Ra_L^{1/4}} $$

where

$$ Ra_L = Gr_L Pr = \frac{g\beta(T_w - T_\infty)L^3}{\alpha \nu} $$

2. **Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)**

The general boundary layer correlation is

$$ Nu_D = \frac{hD}{k_f} = C \left( \frac{g\beta(T_w - T_\infty)D^3}{\nu^2 \equiv Gr} \right)^{1/4} \left( \frac{\nu}{\alpha \equiv Pr} \right)^{1/4} = C \frac{Gr_D^{1/4} Pr^{1/4}}{Ra_D^{1/4}} $$

where

$$ Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)L^3}{\alpha \nu} $$
**Natural Convection From Plate Fin Heat Sinks**

The average Nusselt number for an isothermal plate fin heat sink with natural convection can be determined using

\[
Nu_s = \frac{hS}{k_f} = \left[ \frac{576}{(Ra_sS/L)^2} + \frac{2.873}{(Ra_sS/L)^{0.5}} \right]^{-0.5}
\]

Two factors must be considered in the selection of the number of fins:

- more fins results in added surface area and reduced boundary layer resistance,
- more fins leads to a decrease in fin spacing and a decrease in the heat transfer coefficient

\[
R \downarrow = \frac{1}{hA} \uparrow
\]

A basic optimization of the fin spacing can be obtained as follows:

\[
\dot{Q} = hA(T_w - T_\infty)
\]

where the fins are assumed to be isothermal and the surface area is \(2nHL\), with the area of the fin edges ignored.

For isothermal fins with \(t < S\)

\[
S_{opt} = 2.714 \left( \frac{L}{Ra_L^{1/4}} \right)
\]

with

\[
Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2 Pr}
\]

The corresponding value of the heat transfer coefficient is

\[
h = 1.307k_f/S_{opt}
\]

All fluid properties are evaluated at the film temperature.
Example 6-3: Find the optimum fin spacing, $S_{opt}$ and the rate of heat transfer, $\dot{Q}$ for the following plate fin heat sink cooled by natural convection.

Given:

\[
\begin{align*}
W &= 120 \text{ mm} \\
L &= 18 \text{ mm} \\
T_w &= 80 \degree C \\
P_\infty &= 1 \text{ atm}
\end{align*}
\]

\[
\begin{align*}
H &= 24 \text{ mm} \\
t &= 1 \text{ mm} \\
T_\infty &= 25 \degree C \\
\text{fluid} &= \text{air}
\end{align*}
\]

Find: $S_{opt}$ and the corresponding heat transfer, $\dot{Q}$