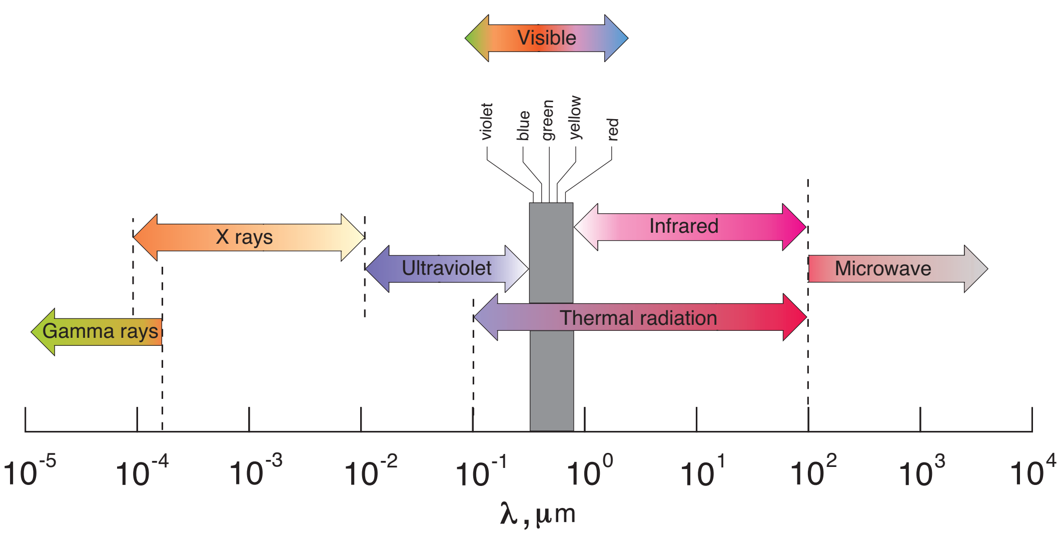
Radiation Heat Transfer

Reading	Problems
15-1 → 15-7	15-27, 15-33, 15-50, 15-57, 15-77, 15-79, 15-96, 15-107, 15-108

Introduction

A narrower band inside the thermal radiation spectrum is denoted as the visible spectrum, that is the thermal radiation that can be seen by the human eye. The visible spectrum occupies roughly $0.4 - 0.7\ \mu m$. Thermal radiation is mostly in the infrared range. As objects heat up, their energy level increases, their frequency, ν, increases and the wavelength of the emitted radiation decreases.



Blackbody Radiation

A blackbody is an ideal radiator that *absorbs* all incident radiation regardless of wavelength and direction.

Definitions

1. **Blackbody emissive power:** the radiation emitted by a blackbody per unit time and per unit surface area

$$E_b = \sigma T^4 \quad [W/m^2] \qquad \Leftarrow \text{ Stefan-Boltzmann law}$$

where Stefan-Boltzmann constant $= 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$ and the temperature T is given in K.

2. **Spectral blackbody emissive power:** the amount of radiation energy emitted by a blackbody per unit surface area and per unit wavelength about the wavelength λ. The following relationship between emissive power, temperature and wavelength is known as *Plank's distribution law*

$$E_{b,\lambda} = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \quad [W/(m^2 \cdot \mu m)]$$

where

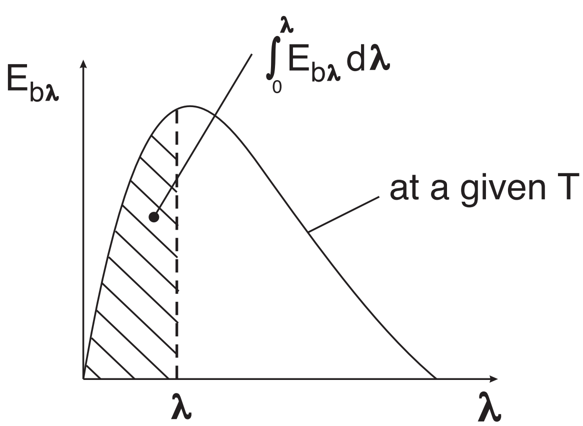
$$\begin{aligned} C_1 &= 2\pi h C_0^2 = 3.74177 \times 10^8 \; [W \cdot \mu m^4/m^2] \\ C_2 &= hC_0/K = 1.43878 \times 10^4 \; [\mu m \cdot K] \end{aligned}$$

The wavelength at which the peak emissive power occurs for a given temperature can be obtained from *Wien's displacement law*

$$(\lambda T)_{max\ power} = 2897.8 \; \mu m \cdot K$$

3. **Blackbody radiation function:** the fraction of radiation emitted from a blackbody at temperature, T in the wavelength band $\lambda = 0 \rightarrow \lambda$

$$f_{0\rightarrow\lambda} = \frac{\int_0^\lambda E_{b,\lambda}(T)\, d\lambda}{\int_0^\infty E_{b,\lambda}(T)\, d\lambda} = \frac{\int_0^\lambda \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} d\lambda}{\sigma T^4}$$



let $t = \lambda T$ and $dt = T\, d\lambda$, then

$$\begin{aligned} f_{0\rightarrow\lambda} &= \frac{\int_0^t \frac{C_1 T^5 (1/T) dt}{t^5[\exp(C_2/t) - 1]}}{\sigma T^4} \\ &= \frac{C_1}{\sigma} \int_0^{\lambda T} \frac{dt}{t^5[\exp(C_2/t) - 1]} \\ &= f(\lambda T) \end{aligned}$$

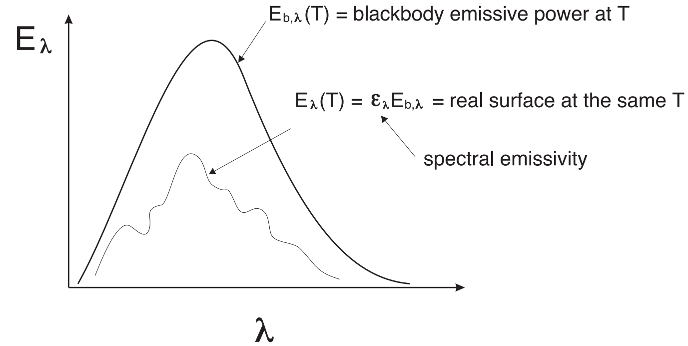
$f_{0\rightarrow\lambda}$ is tabulated as a function λT in Table 15.2

We can easily find the fraction of radiation emitted by a blackbody at temperature T over a discrete wavelength band as

$$\begin{aligned} f_{\lambda_1\rightarrow\lambda_2} &= f(\lambda_2 T) - f(\lambda_1 T) \\ f_{\lambda\rightarrow\infty} &= 1 - f_{0\rightarrow\lambda} \end{aligned}$$

Radiation Properties of Real Surfaces

The thermal radiation emitted by a real surface is a function of surface temperature, T, wavelength, λ, direction and surface properties.



$$E_\lambda = f(T, \lambda, \textbf{direction, surface properties})$$

$\Rightarrow$ spectral emissive power

while for a blackbody, the radiation was only a function of temperature and wavelength

$$E_{b,\lambda} = f(T, \lambda) \quad \rightarrow \quad \text{diffuse emitter} \quad \Rightarrow \text{independent of direction}$$

Definitions

1. **Diffuse surface:** properties are independent of direction.
2. **Gray surface:** properties are independent of wavelength.
3. **Emissivity:** defined as the ratio of radiation emitted by a surface to the radiation emitted by a blackbody at the same surface temperature.

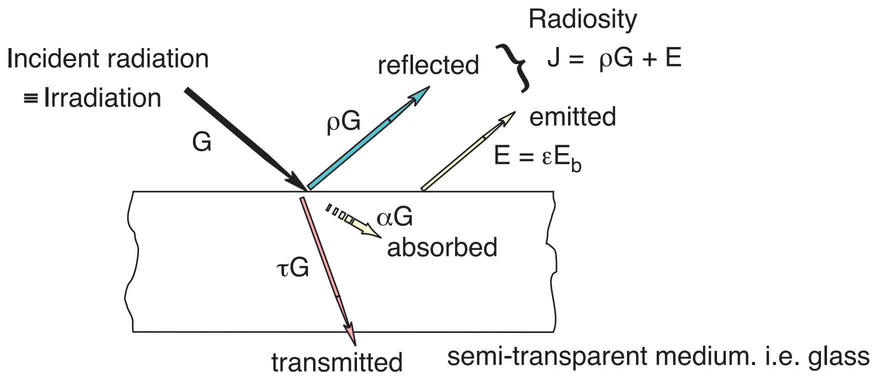
$$\epsilon(T) = \frac{\text{radiation emitted by surface at temperature } T}{\text{radiation emitted by a black surface at } T}$$

$$= \frac{\int_0^\infty E_\lambda(T)\, d\lambda}{\int_0^\infty E_{b\lambda}(T)\, d\lambda} = \frac{\int_0^\infty \epsilon_\lambda(T) E_{b\lambda}(T)\, d\lambda}{E_b(T)} = \frac{E(T)}{\sigma T^4}$$

where ϵ changes rather quickly with surface temperature.

Typical Emissivity Values	
metal (polished)	$\epsilon \approx 0.1$
metal (oxidized)	$\epsilon \approx 0.3 - 0.4$
skin	$\epsilon \approx 0.9$
graphite	$\epsilon \approx 0.95$

4. **Irradiation, G:** the radiation energy incident on a surface per unit area and per unit time



An energy balance based on incident radiation gives

$$G = \rho G + \alpha G + \tau G$$

where

$$\left.\begin{array}{rcl} \rho & = & \text{reflectivity} \\ \alpha & = & \text{absorptivity} \\ \tau & = & \text{transmissivity} \end{array}\right\} \Rightarrow \text{function of } \lambda \ \& \ T \text{ of the incident radiation } G$$

$$\epsilon = \text{emissivity} \qquad \Rightarrow \quad \text{function of } \lambda \ \& \ T \text{ of the emitting surface}$$

If we normalize with respect to the total irradiation

$$\alpha + \rho + \tau = 1$$

In general $\epsilon \neq \alpha$. However, for a diffuse-gray surface (properties are independent of wavelength and direction)

$$\epsilon = \alpha \qquad \text{diffuse-gray surface}$$

5. **Radiosity, J:** the total radiation energy leaving a surface per unit area and per unit time.

For a surface that is gray and opaque, i.e. $\epsilon = \alpha$ and $\alpha + \rho = 1$, the radiosity is given as

$$\begin{aligned} J &= \text{radiation emitted by the surface} + \text{radiation reflected by the surface} \\ &= \epsilon\, E_b + \rho G \\ &= \epsilon \sigma T^4 + \rho G \end{aligned}$$

Since $\rho = 0$ for a blackbody, the radiosity of a blackbody is

$$J = \sigma T^4$$

Diffuse-Gray Surfaces, $\epsilon = \alpha$

Kirchhoff's Law

The absorptivity, $\alpha(\lambda, T, \text{direction})$ *of a non-black surface is always equal to the emissivity,* $\epsilon(\lambda, T, \text{direction})$ *of the same surface when the surface is in thermal equilibrium with the radiation that impinges on it.*

$$\epsilon(\lambda, T, \phi, \theta) = \alpha(\lambda, T, \phi, \theta)$$

To a lesser degree of certainty we can write a more restrictive form of Kirchhoff's law for diffuse-gray surfaces where

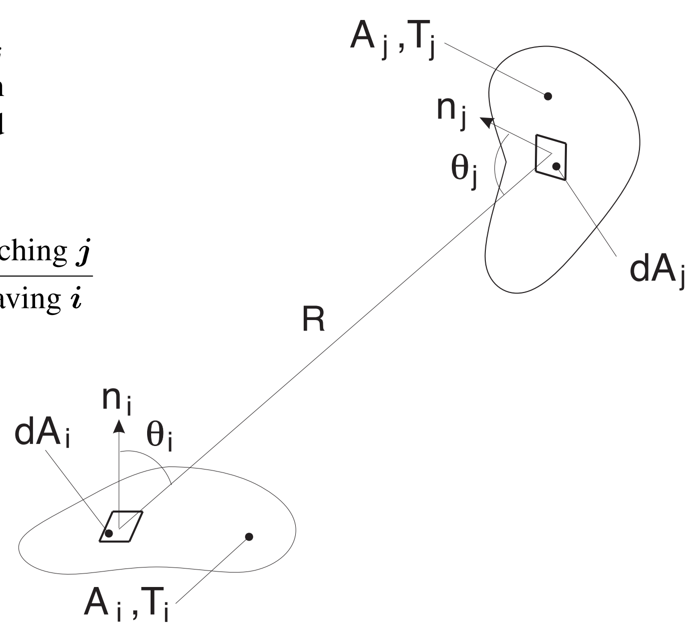
$$\epsilon(T) = \alpha(T)$$

While Kirchhoff's law requires that the radiant source and the surface be in thermal equilibrium, this is seldom the case. The law can still be used but you should proceed with caution when the two temperatures differ by more than $100\ K$.

View Factor (Shape Factor, Configuration Factor)

- **Definition:** The view factor, $F_{i \to j}$ is defined as the fraction of radiation leaving surface i which is intercepted by surface j. Hence

$$F_{i \to j} = \frac{\dot{Q}_{i \to j}}{A_i J_i} = \frac{\text{radiation reaching } j}{\text{radiation leaving } i}$$



$$F_{i \to j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j dA_i$$

$$F_{j \to i} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

View Factor Relations

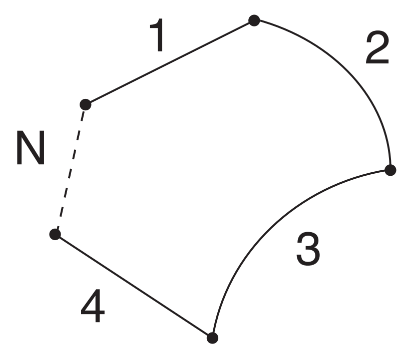
Reciprocity Relation

The last two equations show that

$$A_i F_{i\to j} = A_j F_{j\to i}$$

Summation Relation

$$A_1 J_1 = \dot{Q}_{1\to 1} + \dot{Q}_{1\to 2} + \ldots + \dot{Q}_{1\to N}$$



Therefore

$$1 = \sum_{j=1}^{N} \left(\frac{\dot{Q}_{i\to j}}{A_i J_i} \right) = \sum_{j=1}^{N} F_{i\to j}$$

Hence

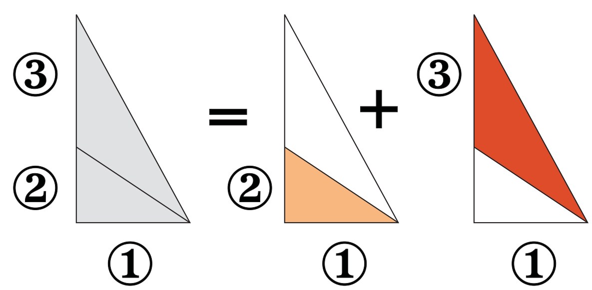
$$\sum_{j=1}^{N} F_{i\to j} = 1 \qquad ; \; i = 1, 2, \ldots, N$$

Note that $F_{i\to i} \neq 0$ for a concave surface. For a plane or convex surface $F_{i\to i} = 0$.

Superposition Relation

If the surface is not available in the tables sometimes it can be treated as the sum of smaller known surfaces to form the full extent of the surface of interest.

$$F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$$

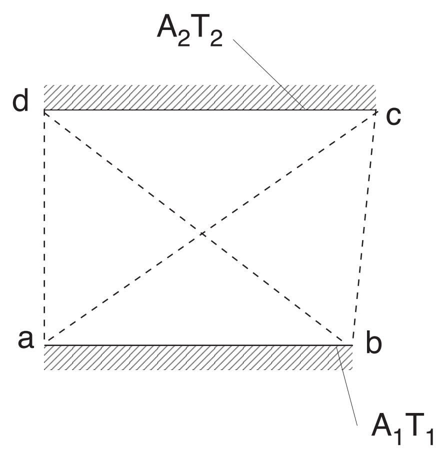


Symmetry Relation

If the problem is symmetric, then the view factors will also be symmetric.

Hottel Crossed String Method

Can be applied to 2D problems where surfaces are any shape, flat, concave or convex. Note for a 2D surface the area, A is given as a length times a unit width.



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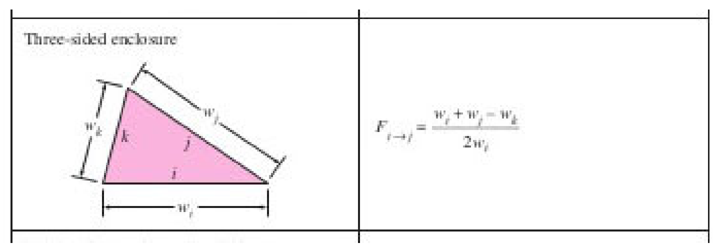


$$A_1F_{12} = A_2F_{21} = \frac{\text{(total crossed)} - \text{(total uncrossed)}}{2}$$

A_1 and A_2 do not have to be parallel

$$A_1F_{12} = A_2F_{21} = \frac{1}{2}[\underbrace{(ac + bd)}_{crossed} - \underbrace{(bc + ad)}_{uncrossed}]$$

Radiation Exchange Between Surfaces

In general, radiation exchange between surfaces should include:

- irradiation of each surface accounting for all energy reflected from other surfaces
- multiple reflections may occur before all energy is absorbed

Diffuse-Gray Surfaces Forming an Enclosure

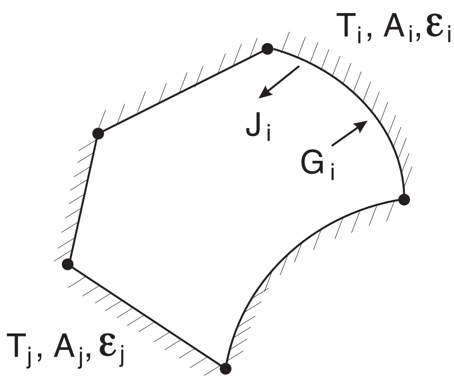
To help simplify radiation analyses in diffuse, gray enclosures we will assume

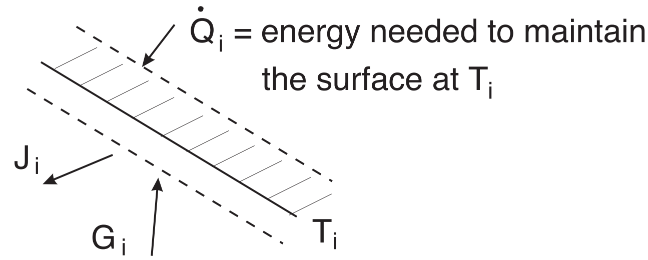
1. each surface of the enclosure is isothermal

2. radiosity, J_i, and irradiation, G_i are uniform over each surface

3. the surfaces are opaque ($\tau_i = 0$) and diffuse-gray ($\alpha_i = \epsilon_i$)

4. the cavity is filled with a fluid which does not participate in the radiative exchange process

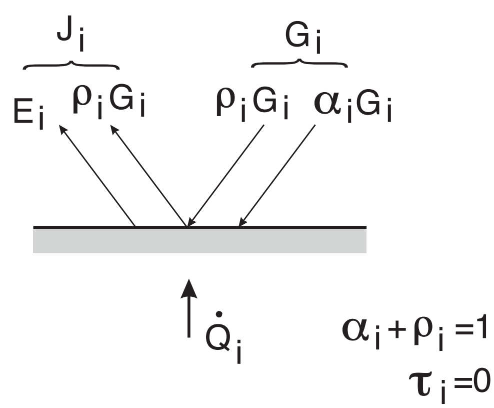




Radiation Heat Transfer To or From a Surface

- an energy balance on the $i'th$ surface gives:

$$\dot{Q}_i = \dot{q}_i A_i = A_i(J_i - G_i)$$



recasting the energy balance:

$$\dot{Q}_i = A_i[\underbrace{(E_i + \rho_i G_i)}_{J_i} - \underbrace{(\rho_i G_i + \alpha_i G_i)}_{G_i}] = A_i(E_i - \alpha_i G_i) \qquad (1)$$

where:

$$\begin{aligned} J_i &= E_i + \rho_i G_i & (2) \quad &\Rightarrow \text{radiosity} \\ E_i &= \epsilon_i E_{b,i} = \epsilon_i \sigma T_i^4 & (3) \quad &\Rightarrow \text{emmisive power} \\ \rho_i &= 1 - \alpha_i = 1 - \epsilon_i & (4) \quad &\Rightarrow \text{since } \alpha_i + \rho_i + \cancelto{0}{\tau_i} = 1 \\ & & &\quad \text{and} \quad \alpha_i = \epsilon_i \end{aligned}$$

Combining Eqs. 2, 3 and 4 gives

$$J_i = \epsilon_i E_{b,i} + (1 - \epsilon_i) G_i \qquad (5)$$

Combining this with Eq. 1 gives the net radiation heat transfer to or from surface "i"

$$Q_i = \frac{E_{b,i} - J_i}{\left(\dfrac{1 - \epsilon_i}{\epsilon_i A_i}\right)} \equiv \frac{\text{potential difference}}{\text{surface resistance}}$$

this surface resistance represents real surface behavior

Note: for a black surface

$$\epsilon_i = \alpha_i = 1$$

and Eq. 5 becomes

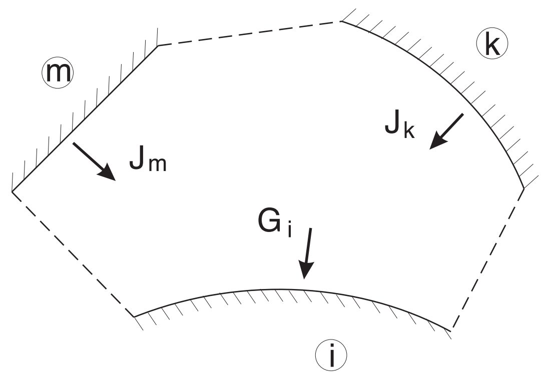
$$J_i = E_{b,i} = \sigma T_i^4 \quad \Leftarrow$$

Radiation Heat Transfer Between Surfaces

- by inspection it is clearly seen that

$$\left\{ \begin{array}{c} \text{irradiation on} \\ \text{surface } i \end{array} \right\} = \left\{ \begin{array}{c} \text{radiation leaving the} \\ \text{remaining surfaces} \end{array} \right\}$$

$$A_i G_i = \sum_{j=1}^{N} F_{j \to i} (A_j J_j) = \sum_{j=1}^{N} A_i F_{i \to j} J_i \qquad \Leftarrow \text{(from reciprocity)}$$



Therefore

$$G_i = \sum_{j=1}^{N} F_{i\to j} J_j$$

Combining this with Eq. 5 gives

$$J_i = \epsilon_i \underbrace{\sigma T_i^4}_{E_{b,i}} + (1-\epsilon_i) \sum_{j=1}^{N} F_{i\to j} J_j$$

In addition by performing an energy balance at surface "i", we can write

$$\begin{aligned} \dot{Q}_i &= \text{energy out} - \text{energy in} \\ &= A_i J_i - \sum_{j=1}^{N} A_i F_{i\to j} J_j \end{aligned}$$

Since the summation rule states $\sum_{j=1}^{N} F_{i\to j} = 1$, the above equation becomes

$$\dot{Q}_i = A_i \left\{ \underbrace{\sum_{j=1}^{N} F_{i\to j}}_{\equiv 1} J_i - \sum_{j=1}^{N} F_{i\to j} J_j \right\}$$

$$\dot{Q}_i = \sum_{j=1}^{N} A_i F_{i\to j} (J_i - J_j)$$

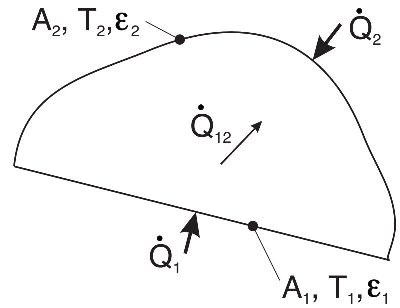
or

$$\dot{Q}_i = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(\dfrac{1}{A_i F_{i\to j}}\right)} \equiv \frac{\text{potential difference}}{\text{space resistance}}$$

- the space resistance can be used for any gray, diffuse and opaque surfaces that form an enclosure

Radiation Exchange in Enclosures

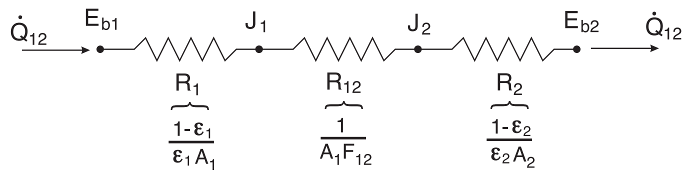
The Two-Surface Enclosure



- radiation <u>from</u> surface 1 must equal radiation <u>to</u> surface 2

$$\dot{Q}_1 = -\dot{Q}_2 = \dot{Q}_{12}$$

- the resistor network will consist of 2 surface resistances and 1 space resistance

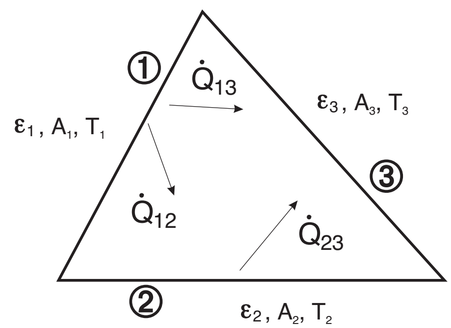


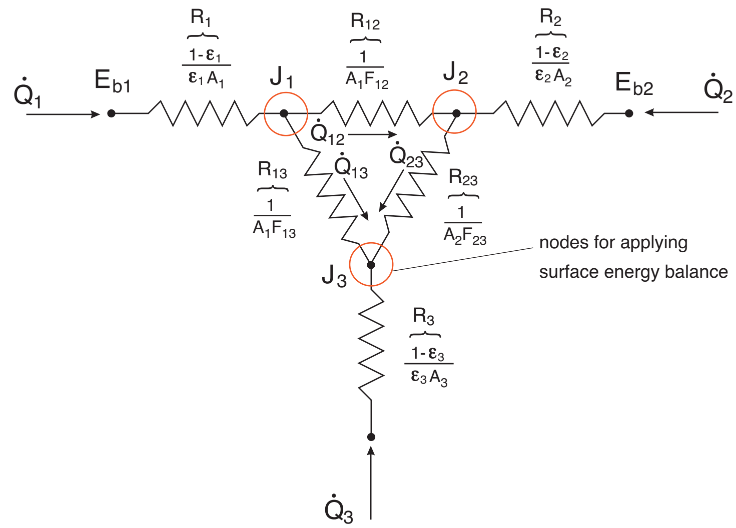
- the net radiation exchange can be determined as follows:

$$\dot{Q}_{12} = \frac{E_{b,1} - E_{b,2}}{R_{total}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\epsilon_2}{\epsilon_2 A_2}}$$

The Three-Surface Enclosure

- radiative heat transfer between all combinations of surfaces must be accounted for
- the resistor network will consist of 3 surface resistances and 3 space resistances
- leads to a system of 3 equations in 3 unknowns
- the algebraic sum of the currents (net radiation transfer) at each node must equal zero. Note: this assumes all heat flow is into the node.
- if the assumed direction of current flow is incorrect, your will get a -ve value of $\dot{Q}$





Performing an energy balance at each node:

at $J_1 \quad \Rightarrow \quad \dot{Q}_1 + \dot{Q}_{12} + \dot{Q}_{13} = 0$

$$\frac{E_{b1} - J_1}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_2 - J_1}{\dfrac{1}{A_1 F_{12}}} + \frac{J_3 - J_1}{\dfrac{1}{A_1 F_{13}}} = 0 \qquad (1)$$

at $J_2 \quad \Rightarrow \quad \dot{Q}_{12} + \dot{Q}_2 + \dot{Q}_{23} = 0$

$$\frac{J_1 - J_2}{\dfrac{1}{A_1 F_{12}}} + \frac{E_{b2} - J_2}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}} + \frac{J_3 - J_2}{\dfrac{1}{A_2 F_{23}}} = 0 \qquad (2)$$

at $J_3 \quad \Rightarrow \quad \dot{Q}_{13} + \dot{Q}_{23} + \dot{Q}_3 = 0$

$$\frac{J_1 - J_3}{\dfrac{1}{A_1 F_{13}}} + \frac{J_2 - J_3}{\dfrac{1}{A_2 F_{23}}} + \frac{E_{b3} - J_3}{\dfrac{1-\epsilon_1}{\epsilon_1 A_1}} = 0 \qquad (3)$$

if surface temperature is known

- given T_i, evaluate $E_{bi} = \sigma T_i^4$
- evaluate all space and surface resistances
- solve for J_1, J_2 and J_3
- determine the heat flow rate as

$$\dot{Q}_i = A_i \sum F_{ij}(J_i - J_j)$$

if surface heat flow rate is known

- replace $\dot{Q}_1$, $\dot{Q}_2$ and/or $\dot{Q}_3$ in Eqs. 1-3
- solve for J_1, J_2 and J_3
- determine the surface temperature as

$$\sigma T_i^4 = J_i + \frac{1 - \epsilon_i}{\epsilon_i} \sum F_{ij}(J_i - J_j)$$

Special Cases

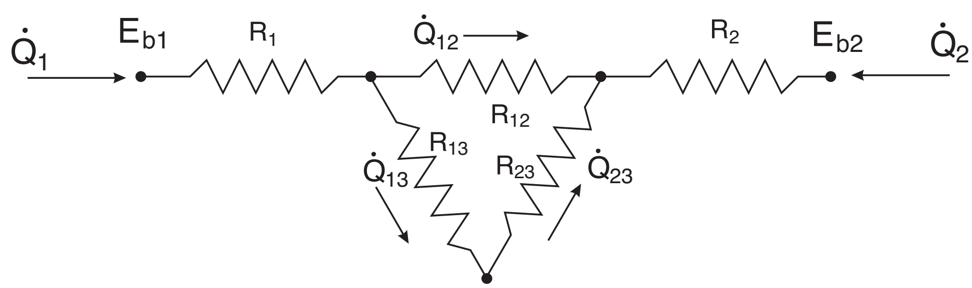
The system of equations for 2 and 3-surface enclosures can simplify further when one or more surfaces are: i) blackbody surfaces or ii) reradiating (fully insulated) surfaces.

blackbody surface: for a blackbody surface $\epsilon = 1$ and the surface resistance goes to zero. As a consequence the radiosity can be calculated directly as a function of surface temperature

$$J_i = E_{bi} = \sigma T_i^4$$

reradiating surface: $Q_i = 0$, therefore the heat flow into the radiosity node equals the heat flow out of the node.

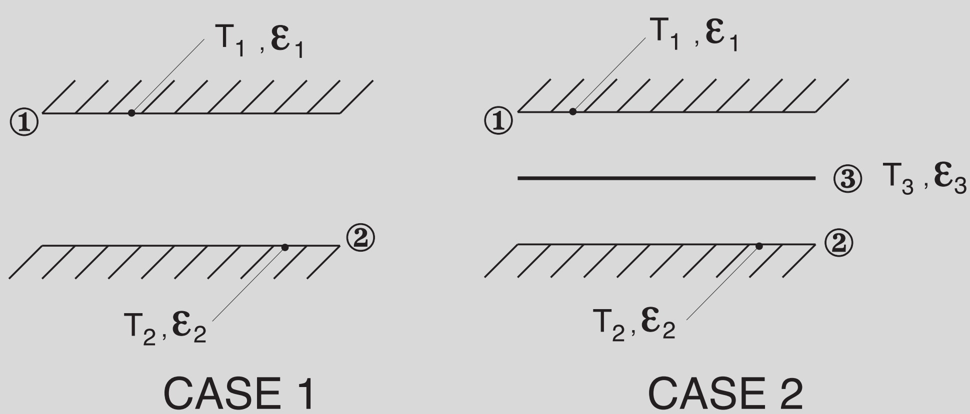
The resistor network simplifies to:



and the system of equations can be easily solved as:

$$\dot{Q}_1 = -\dot{Q}_2 = \frac{E_{b1} - E_{b2}}{R_1 + \underbrace{\left[\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right]^{-1}}_{\text{parallel resistance}} + R_2}$$

Example 7-1: Consider two very large parallel plates with diffuse, gray surfaces, Determine the net rate of radiation heat transfer per unit surface area, $\dot{Q}_{12}/A$, between the two surfaces. For Case 2, also determine T_3, the temperature of a radiation shield, positioned midway between surfaces 1 and 2.

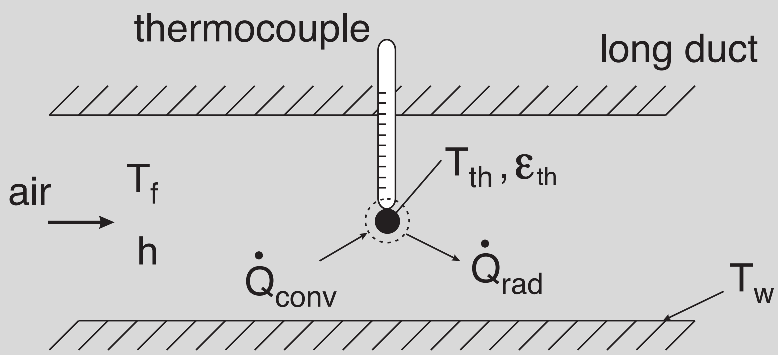


Given:

$$\begin{aligned} \epsilon_1 &= 0.2 & T_1 &= 800\ K \\ \epsilon_2 &= 0.7 & T_2 &= 500\ K \\ \epsilon_3 &= 0.02 & A_1 &= A_2 = A_3 = A \end{aligned}$$

Assume steady state conditions.

Example 7-2: A thermocouple is suspended between two parallel surfaces as shown in the figure below. Find T_f, the temperature of the air stream by performing an energy balance on the thermocouple.



Given:

$$T_w = 400\ K \qquad T_{th} = 650\ K$$
$$\epsilon_{th} = 0.6 \qquad h = 80\ W/(m^2 \cdot K)$$

Assume steady state conditions.

Example 7-3: Consider a room that is $4\ m$ long by $3\ m$ wide with a floor-to-ceiling distance of $2.5\ m$. The four walls of the room are well insulated, while the surface of the floor is maintained at a uniform temperature of $30\ °C$ using an electric resistance heater. Heat loss occurs through the ceiling, which has a surface temperature of $12°C$. All surfaces have an emissivity of 0.9.

a) determine the rate of heat loss, (W), by radiation from the room.
b) determine the temperature, (K), of the walls.