$\mathcal{ME}$ 201 ADVANCED CALCULUS

Assignment 9: *Vector Fields*
March 16, 2018

1. Calculate the following vector field quantities:

   (a) ∇f if $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$

   ANSWER: $\nabla f = -\dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2 + y^2 + z^2)^{3/2}}$

   (b) $\nabla \cdot \vec{F}$ if $\vec{F}(x, y, z) = 2xe^y\hat{i} + 3x^2z\hat{j} - 2x^2yz\hat{k}$

   ANSWER: $\nabla \cdot \vec{F} = 2(e^y - x^2y)$

   (c) $\nabla \cdot \vec{F}$ if $\vec{F}(x, y, z) = \dfrac{x\hat{i} + y\hat{j} + \hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

   ANSWER: $\nabla \cdot \vec{F} = \dfrac{2}{\sqrt{x^2 + y^2 + z^2}}$

   (d) $\nabla \times \vec{F}$ if $\vec{F}(x, y, z) = x^2z\hat{i} + 12xyz\hat{j} + 32y^2z^4\hat{k}$

   ANSWER: $\nabla \times \vec{F} = (64yz^4 - 12xy)\hat{i} + x^2\hat{j} + 12yz\hat{k}$

   (e) $\nabla \times \vec{F}$ at $(2, 0)$ if $\vec{F}(x, y) = y\hat{i} - x\hat{j}$

   ANSWER: $\nabla \times \vec{F} = -2\hat{k}$

2. Given the potential function $\phi(x, y)$ where $\phi = -K \ln \sqrt{x^2 + y^2}$ with $x > 0,\ y > 0$ and $K = constant$:

   (a) Determine the gradient, $\nabla\phi$ and calculate the divergence of the gradient, $\nabla \cdot \nabla\phi$.

   (b) Show that $\nabla^2\phi = 0$

   Hint: $\nabla = \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y}$ for this problem