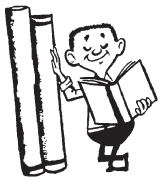
# *Conservation Equations*



| Reading | Problems |
|---|---|
| 4.1 → 4.3, 4.8, 5.1, 5.8, 5.9 | 5.61, 5.134 |
| 6.1, 6.2, 7.2, 7.3 | 6.138, |
| 8.9 → 8.12, 9.1, 9.2 | 8.171, 8.178, 9.159, 9.160 |

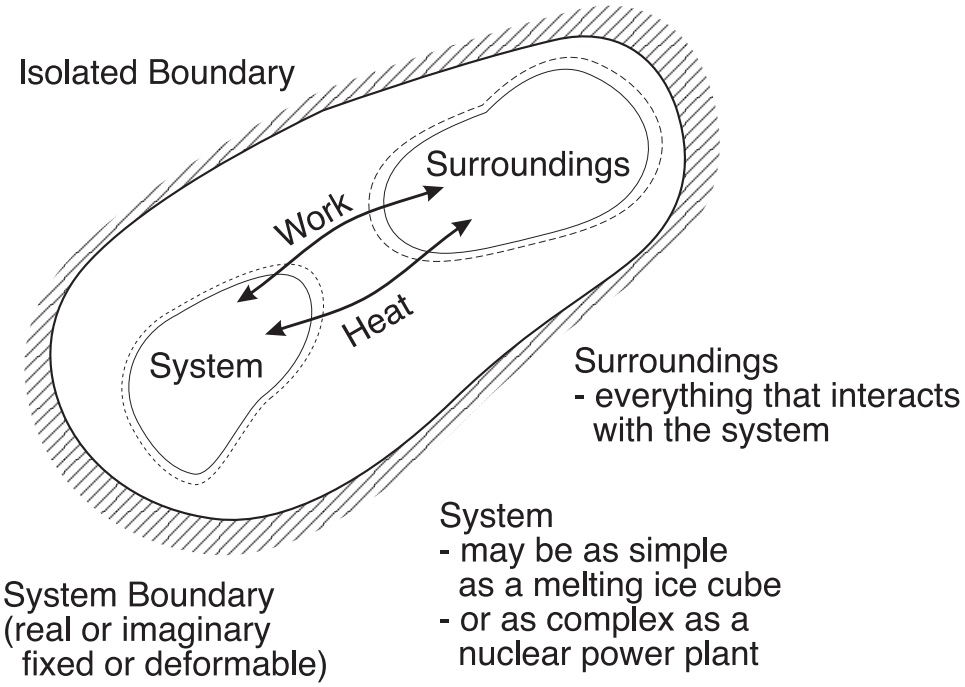
## Definitions

***SYSTEM:***

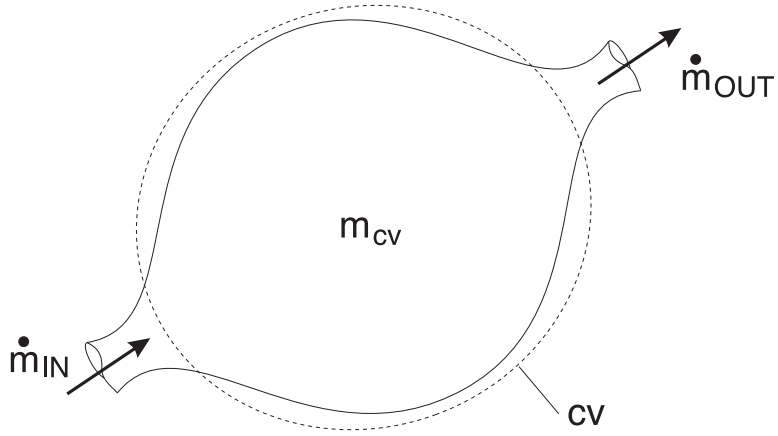
- any specified collection of matter under study.

***WORK & HEAT TRANSFER:***

- thermodynamics deals with these properties of matter as a system interacts with its surroundings through work and heat transfer



## CONSERVATION OF MASS




$$\left\{ \begin{array}{c} rate\ of\ increase \\ of\ mass\ within \\ the\ CV \end{array} \right\} = \left\{ \begin{array}{c} net\ rate\ of \\ mass\ flow \\ IN \end{array} \right\} - \left\{ \begin{array}{c} net\ rate\ of \\ mass\ flow \\ OUT \end{array} \right\}$$

$$\frac{d}{dt}(m_{CV}) = \dot{m}_{IN} - \dot{m}_{OUT}$$

where:

$$\begin{aligned} m_{CV} &= \int_V \rho \, dV \\ \dot{m}_{IN} &= (\rho \, v^* \, A)_{IN} \\ \dot{m}_{OUT} &= (\rho \, v^* \, A)_{OUT} \end{aligned}$$

with $v^*$ = average velocity

# First Law of Thermodynamics

**What is the difference between a closed and an open system?**

- is there mass crossing the system boundary? YES: open system NO: closed system

## Control Mass (Closed System)

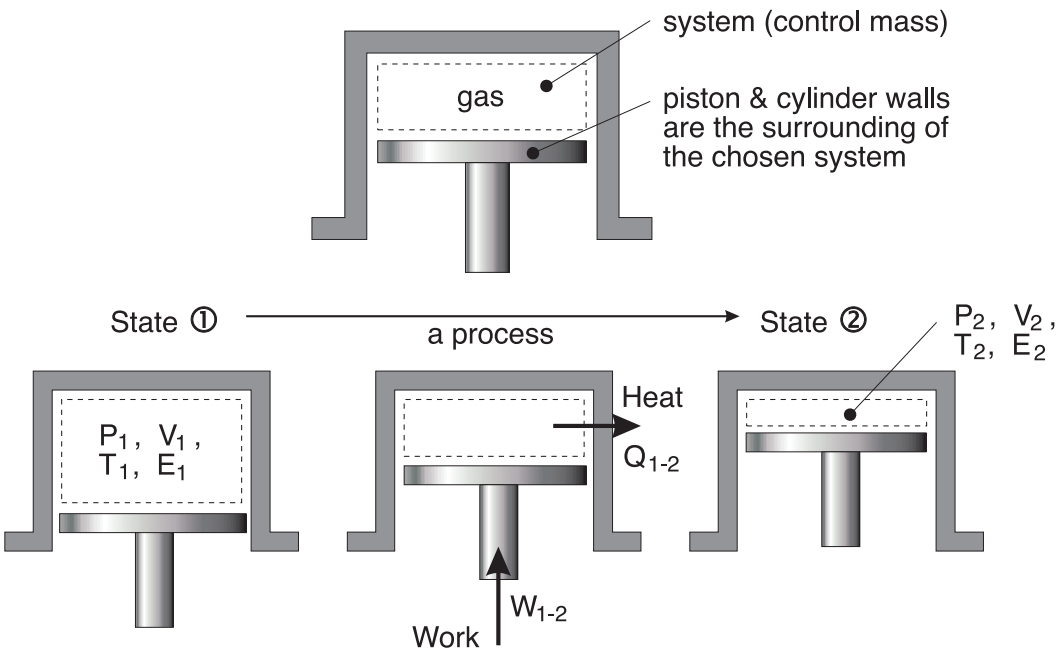
conservation of mass is inherently satisfied

***CONSERVATION OF ENERGY:***

- the energy content of an isolated system is constant

$$\boldsymbol{energy\ entering - energy\ leaving = change\ of\ energy\ within\ the\ system}$$

**Example: A Gas Compressor**

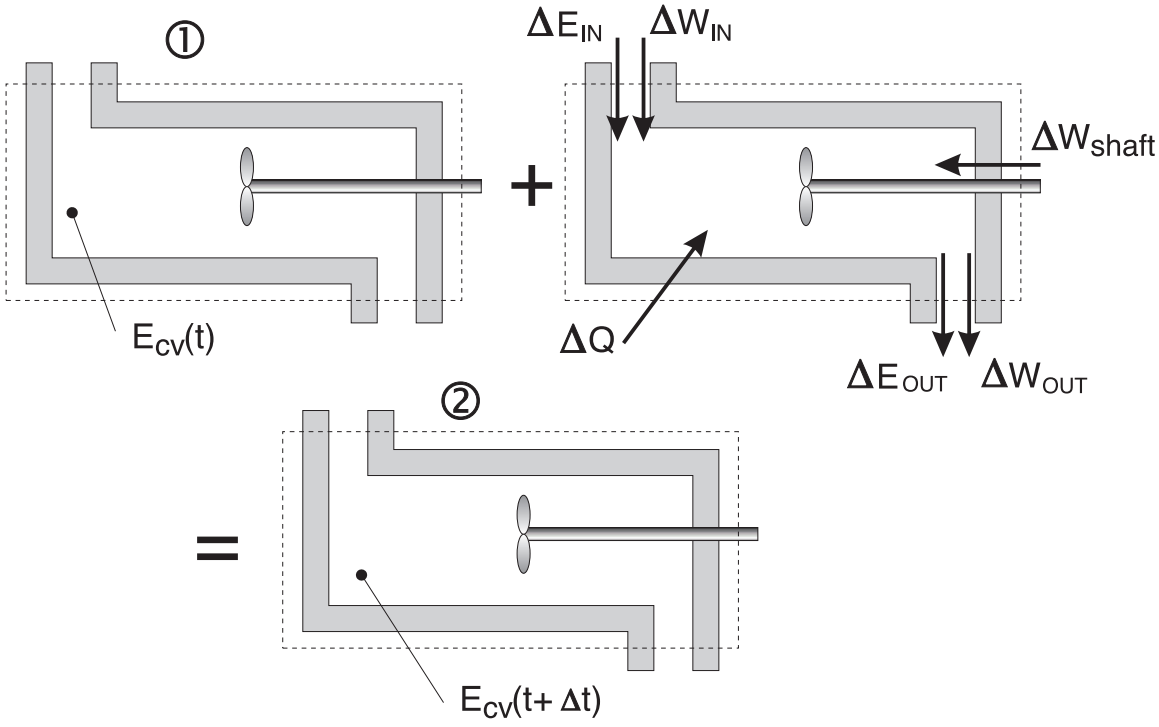



Performing a 1st law energy balance:

$$\left\{ \begin{array}{c} \boldsymbol{Initial} \\ \boldsymbol{Energy} \\ \boldsymbol{E_1} \end{array} \right\} + \left\{ \begin{array}{c} \boldsymbol{Energy\ gain\ W_{1-2}} \\ \boldsymbol{Energy\ loss\ Q_{1-2}} \end{array} \right\} = \left\{ \begin{array}{c} \boldsymbol{Final} \\ \boldsymbol{Energy} \\ \boldsymbol{E_2} \end{array} \right\}$$

## Control Volume Analysis (Open System)

***CONSERVATION OF ENERGY:***




The 1st law states:

$$\boldsymbol{E_{CV}(t) + \Delta Q + \Delta W_{shaft} + (\Delta E_{IN} - \Delta E_{OUT}) + (\Delta W_{IN} - \Delta W_{OUT}) = E_{CV}(t + \Delta t)} \quad (1)$$

where:

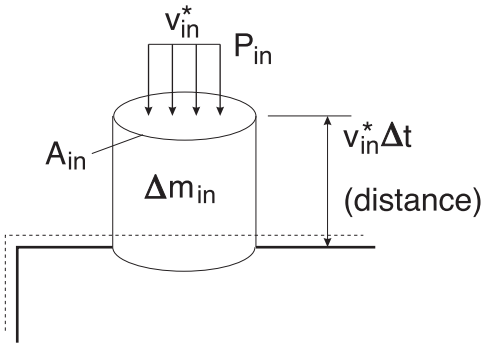
$$\boldsymbol{\Delta E_{IN} = e_{IN} \Delta m_{IN}}$$

$$\boldsymbol{\Delta E_{OUT} = e_{OUT} \Delta m_{OUT}}$$

$$\boldsymbol{\Delta W} = \textbf{flow work}$$

$$\boldsymbol{e = \frac{E}{m} = \underbrace{u}_{internal} + \underbrace{\frac{(v^*)^2}{2}}_{kinetic} + \underbrace{gz}_{potential}}$$

**What is flow work?**




$$\Delta m_{IN} = \rho_{IN} \overbrace{A_{IN}\, v^*_{IN}\, \Delta t}^{volume}$$

$$\Delta W_{IN} = F \cdot distance = \underbrace{P_{IN}\, A_{IN}}_{F} \cdot \underbrace{v^*_{IN}\, \Delta t}_{\Delta s} = \frac{P_{IN}\, \Delta m_{IN}}{\rho_{IN}}$$

with $v = 1/\rho$

$$\Delta W_{IN} = (P\, v\, \Delta m)_{IN} \quad \rightarrow \text{flow work} \tag{2}$$

Similarly

$$\Delta W_{OUT} = (P\, v\, \Delta m)_{OUT} \tag{3}$$

Substituting Eqs. 2 and 3 into Eq. 1 gives the 1st law for a control volume

$$\begin{aligned} E_{CV}(t+\Delta t) - E_{CV}(t) &= \Delta Q + \Delta W_{shaft} + \Delta m_{IN}(e+Pv)_{IN} \\ &\quad - \Delta m_{OUT}(e+Pv)_{OUT} \end{aligned} \tag{4}$$

Equation 4 can also be written as a rate equation $\rightarrow$ divide through by $\Delta t$ and take the limit as $\Delta t \rightarrow 0$

$$\frac{d}{dt} E_{CV} = \dot{Q} + \dot{W}_{shaft} + [\dot{m}(e+Pv)]_{IN} - [\dot{m}(e+Pv)]_{OUT}$$

where:

$$\dot{m} = \rho\, v^*\, A$$

# Second Law of Thermodynamics

## Fundamentals

1. Like mass and energy, every system has entropy.

   *Entropy is a measure of the degree of microscopic disorder and represents our uncertainty about the microscopic state.*

2. Unlike mass and energy, entropy can be produced but it can never be destroyed. That is, the entropy of a system plus its surroundings (i.e. an isolated system) can never decrease (2nd law).

$$\begin{aligned} \mathcal{P}_m &= m_2 - m_1 = 0 \ \text{(conservation of mass)} \\ \mathcal{P}_E &= E_2 - E_1 = 0 \ \text{(conservation of energy)} \rightarrow \text{1st law} \\ \mathcal{P}_S &= S_{gen} = S_2 - S_1 \geq 0 \ \rightarrow \text{2nd law} \end{aligned}$$

   The second law states:

$$(\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$$

   where $\Delta \equiv final - initial$

3. ***Reference:*** In a prefect crystal of a pure substance at $T = 0\ K$, the molecules are completely motionless and are stacked precisely in accordance with the crystal structure. Since entropy is a measure of microscopic disorder, then in this case $S = 0$. That is, there is no uncertainty about the microscopic state.

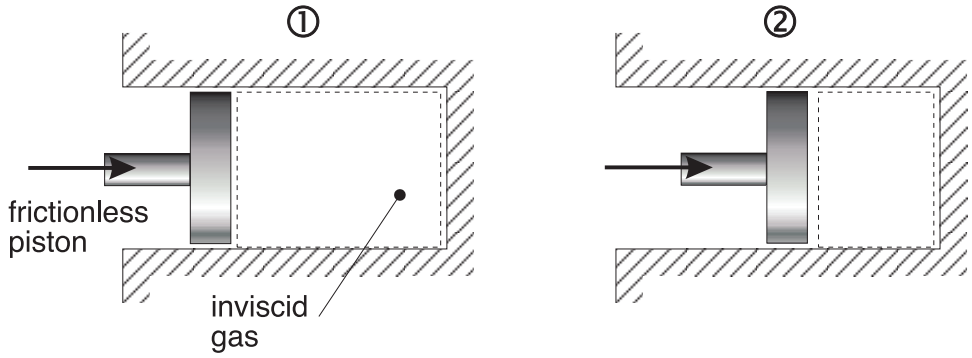
4. ***Relationship to Work:*** For a given system, an increase in the microscopic disorder (that is an increase in entropy) results in a loss of ability to do useful work.

5. ***Heat:*** Energy transfer as heat takes place as work at the microscopic level but in a random, disorganized way. This type of energy transfer carries with it some chaos and thus results in entropy flow in or out of the system.

6. ***Work:*** Energy transfer by work is microscopically organized and therefore entropy-free.

## Definitions

**Example: Slow adiabatic compression of a gas**




Idealization - which makes the process completely reversible

**Reversible Process:** A process $\mathbf{1} \rightarrow \mathbf{2}$ is said to be reversible if the reverse process $\mathbf{2} \rightarrow \mathbf{1}$ restores the system to its original state without leaving any change in either the system or its surroundings.

$\rightarrow$ idealization where $\boldsymbol{S_2 = S_1} \;\; \Rightarrow \boldsymbol{S_{gen} = 0}$

**Adiabatic Process:** A process is adiabatic where there is no energy in the form of heat crossing the boundary.

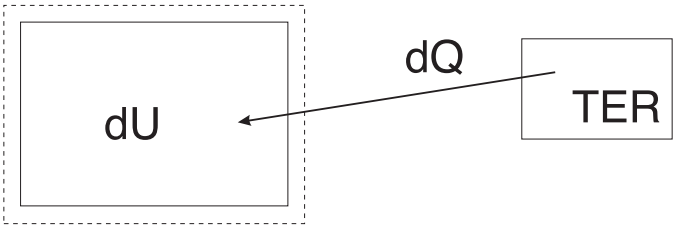
**Isentropic Process:** An isentropic process exhibits no change in the entropy of the state between an initial and final condition. Note: since entropy can be produced as well as transferred, it is important to asses the net result of both mechanisms.

$$\underbrace{\boldsymbol{Reversible}}_{S_{gen}=0} + \underbrace{\boldsymbol{Adiabatic\ Process}}_{Q=0} \Rightarrow \underbrace{\boldsymbol{Isentropic\ Process}}_{S_1=S_2}$$

But does:

$$\boldsymbol{Isentropic\ Process} \Rightarrow \boldsymbol{Reversible} + \boldsymbol{Adiabatic}$$

# Second Law Analysis for a Control Mass




- control mass is uniformly at $\boldsymbol{T_{TER}}$ at all times
- control mass has a fixed size ($\boldsymbol{V = constant}$)

From Gibb's equation

$$\boldsymbol{T_{TER}\, dS = dU + P\, d\cancel{V}^{0}}$$

From the 1st law

$$\boldsymbol{dU = dQ}$$

Therefore for a reversible process

$$\boldsymbol{dS = \frac{dQ}{T_{TER}}}$$

We integrate to give

$$\boldsymbol{S_2 - S_1 = \frac{Q_{1-2}}{T_{TER}}}$$

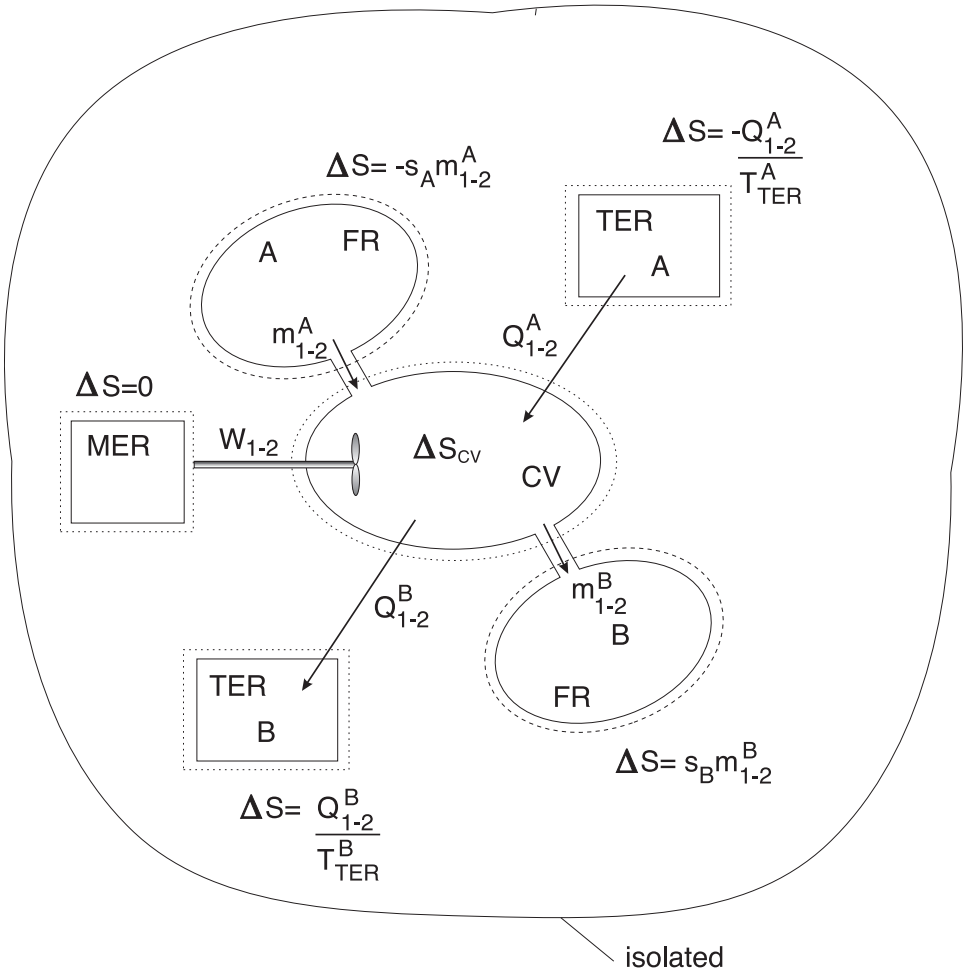
and for a non-reversible process

$$\boldsymbol{dS = \frac{dQ}{T_{TER}} + dS_{gen}}$$

We integrate to give

$$\boldsymbol{S_2 - S_1 = \frac{Q_{1-2}}{T_{TER}} + S_{gen\ 1-2}}$$

# Second Law Analysis for a Control Volume




For the isolated system

$$(\Delta S)_{sys} + (\Delta S)_{sur} = S_{gen} \geq 0$$

$$\Delta S_{CV} - s_A m^A_{1-2} + s_B m^B_{1-2} - \frac{Q^A_{1-2}}{T^A_{TER}} + \frac{Q^B_{1-2}}{T^B_{TER}} = \mathcal{P}_{S_{1-2}} = S_{gen}$$

or as a rate equation

$$\left(\frac{dS}{dt}\right)_{CV} = \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}}\right)_{IN} - \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}}\right)_{OUT} + \dot{S}_{gen}$$

***PROBLEM STATEMENT:***

A bicycle tire has a volume of $\mathbf{1200}\ \boldsymbol{cm^3}$ which is considered to be constant during "inflation". Initially the tire contains air at atmospheric conditions given as $\boldsymbol{P_0 = 100\ kPa}$ and $\boldsymbol{T_0 = 20\ °C}$. A student then hooks up a bicycle pump and begins to force air from the atmosphere into the tire. After pumping stops and a new equilibrium is reached, the tire pressure is $\mathbf{600}\ \boldsymbol{kPa}$ and the air temperature in the tire is $\mathbf{20}\ \boldsymbol{°C}$.

a) Determine the mass $[\boldsymbol{kg}]$ of air added to the tire.
b) Determine the minimum amount of work $[\boldsymbol{kJ}]$ required to reach this end state.
c) If more than the minimum work is actually used, where does this extra energy go? That is what becomes of the extra energy?