**Availability**

**Reading**

10.1 → 10.4

**Problems**

10.59, 10.65, 10.66, 10.67
10.69, 10.75, 10.81, 10.88

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**Second Law Analysis of Systems**

*Availability:*

- the theoretical maximum amount of reversible work that can be obtained from a system at a given state $P_1$ and $T_1$ when interacting with a reference atmosphere at the constant pressure and temperature $P_0$ and $T_0$.

What do we mean by work potential of a system?

Notice that:

$$\eta = \frac{\dot{W}}{\dot{Q}} = 1 - \frac{T_0}{T_{TER}}$$

Therefore

$$\dot{W} = \dot{Q} \left(1 - \frac{T_0}{T_{TER}}\right)$$

This term represents the work potential (availability) of a given TER with respect to the surroundings (dead state) at $T_0$.

The following observations can be made about availability:

1. Availability is a **property** - since any quantity that is fixed when the state is fixed is a property.

2. Availability is a **composite property** - since its value depends upon an external datum

3. Availability of a system is 0 at its **dead state** when $T = T_0$ and $P = P_0$.

4. Unless otherwise stated, assume the dead state to be:

   $$P_0 = 1 \text{ atm}$$
   $$T_0 = 25^\circ C$$
5. The maximum work is obtained through a reversible process to the dead state.

$$W_{\text{rev max}} = \text{available (exergy)}$$
- maximum work obtained through a reversible process to the dead state

$$W_{\text{rev}} = W_{\text{useful}} + I$$

**Control Mass Analysis**

- we know

$$W_{\text{rev}} = W_{\text{useful}} + I$$

but as shown in the figure, the actual work of the process is divided into two components

$$W_{\text{actual}} = W_{\text{useful}} + W_{\text{sur}}$$
• where $W_{\text{sur}}$ is the part of the work done against the surroundings to displace the ambient air

$$W_{\text{sur}} = P_0(V_2 - V_1) = -P_0(V_1 - V_2)$$

To find $W_{\text{actual}}$, from the 1st law

$$E_1 - Q - W_{\text{actual}} = E_2 \quad \rightarrow \quad Q = E_1 - E_2 - W_{\text{actual}}$$

From the 2nd law

$$S_{\text{gen}} = \Delta S_{\text{system}} + \Delta S_{\text{sur}} \geq 0$$

$$= S_2 - S_1 + \frac{Q}{T_0}$$

But from the 1st law balance we know

$$\frac{Q}{T_0} = \frac{E_1 - E_2 - W_{\text{actual}}}{T_0}$$

and when we combine this with the 2nd law

$$S_{\text{gen}} = S_2 - S_1 + \frac{E_1 - E_2 - W_{\text{actual}}}{T_0}$$

which leads to

$$W_{\text{actual}} = (E_1 - E_2) + T_0(S_2 - S_1) - T_0S_{\text{gen}}$$

or by reversing the order of $S_2$ and $S_1$

$$W_{\text{actual}} = (E_1 - E_2) - T_0(S_1 - S_2) - T_0S_{\text{gen}}$$

But we also know that

$$W_{\text{useful}} = W_{\text{actual}} - W_{\text{sur}}$$
therefore

\[ W_{useful} = (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) - T_0S_{gen} \]

and

\[ W_{rev} = W_{useful} + I = W_{actual} - W_{sur} + I \]

where

\[ I = T_0S_{gen} \]

Therefore

\[ W_{rev} = (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2) \]

Define

\[ \Phi = CONTROL\ MASS\ AVAILABILITY \]

\[ = W_{rev} \text{ (in going to the dead state)} \]

\[ = (E - E_0) - T_0(S - S_0) + P_0(V - V_0) \]

where the specific availability is defined as

\[ \phi = \frac{\Phi}{m} \]

The availability destroyed is

\[ I = W_{rev} - W_{useful} = T_0S_{gen} = T_0 S_{gen} \]

This can be referred to as: irreversibilities, availability destruction or loss of availability.
Control Volume Analysis

Consider a steady state, steady flow (SS-SF) process

From the 1st law

\[
\frac{dE_{cv,0}}{dt} = -\dot{W}_{actual} - \dot{Q} + \left[ \dot{m} (h + \frac{(v^*)^2}{2} + gz) \right]_{in} - \left[ \dot{m} (h + \frac{(v^*)^2}{2} + gz) \right]_{out} \tag{1}
\]

From the 2nd law

\[
\frac{dS_{cv,0}}{dt} = \left( \dot{m} s + \frac{\dot{Q}^0}{T_{TER}} \right)_{in} - \left( \dot{m} s + \frac{\dot{Q}}{T_0} \right)_{out} + \dot{S}_{gen} \tag{2}
\]

Combining (1) and (2) through the $\dot{Q}$ term, leads to the actual work output of the turbine, given as

\[
\dot{W}_{actual} = \left[ \dot{m} \left( h + \frac{(v^*)^2}{2} + gz - T_0 s \right) \right]_{in} - \left[ \dot{m} \left( h + \frac{(v^*)^2}{2} + gz - T_0 s \right) \right]_{out} - T_0 \dot{S}_{gen}
\]

\[
= \dot{m} \left[ -T_0 \Delta s + \Delta h + \Delta KE + \Delta PE \right] - (T_0 \dot{S}_{gen}) \tag{3}
\]

$\dot{W}_{actual}$ is the actual work output of the turbine.

The specific flow availability, $\psi$, is given as

\[
\psi = -T_0 (s - s_0) + (h - h_0) + \left( \frac{(v^*)^2}{2} - \frac{(v_0^*)^2}{2} \right) + g(z - z_0) \tag{4}
\]
For a steady state, steady flow process where we assume KE=PE=0

\[ \dot{W}_{rev} = (\dot{m}\psi)_{in} - (\dot{m}\psi)_{out} \quad (5) \]

\[ i = \dot{W}_{rev} - \dot{W}_{actual} = T_0\dot{S}_{gen} = T_0\dot{S}_{gen} \quad (6) \]

\[ \psi = (h - h_0) - T_0(s - s_0) \quad (7) \]

The Exergy Balance Equation

\[ \frac{dE_{cv}}{dt} = \dot{W}_{in} - \dot{W}_{out} + \dot{Q}_0 + \dot{Q}_1 - \dot{Q}_2 + [\dot{m}(e + Pv)]_{in} - [\dot{m}(e + Pv)]_{out} \quad (1) \]

From the 2nd law

\[ \frac{dS_{cv}}{dt} = \left( \dot{m}s - \frac{\dot{Q}_0}{T_0} + \frac{\dot{Q}_1}{T_1} \right)_{in} - \left( \dot{m}s + \frac{\dot{Q}_2}{T_2} \right)_{out} + \dot{S}_{gen} \quad (2) \]

Multiply (2) by \( T_0 \) and subtract from (1) to eliminate \( Q_0 \), which leads to the generalized exergy equation

\[ \frac{d}{dt}(E - T_0S)_{CV} = \dot{W}_{in} - \dot{W}_{out} + [\dot{m}(e + Pv - T_0s)]_{in} \]
\[ [\dot{m}(e + P\nu - T_0s)]_{out} + \left( \dot{Q}_1 - \frac{T_0\dot{Q}_1}{T_1} \right)_{in} \]

\[ - \left( \dot{Q}_2 - \frac{T_0\dot{Q}_2}{T_2} \right)_{out} - T_0\dot{S}_{gen} \quad (3) \]

We can rewrite Eq. (3) in a generalized form by introducing the definitions of \( X \) and \( \psi \).

\[
\frac{d\Phi}{dt} = P_0 \frac{dV_{CV}}{dt} + \left[ \dot{W} + \dot{m}\psi + \dot{Q} \left( 1 - \frac{T_0}{T_{TER}} \right) \right]_{in} \\
- \left[ \dot{W} + \dot{m}\psi + \dot{Q} \left( 1 - \frac{T_0}{T_{TER}} \right) \right]_{out} - \dot{i}
\]

where

\[ \dot{i} = T_0\dot{S}_{gen} \]

= exergy destruction rate

\[ \Phi = [(E - E_0) + P_0(V - V_0) - T_0(S - S_0)] \]

= non-flow exergy

\[ \psi = (h - h_0) - T_0(s - s_0) + \frac{1}{2} \left[ (v^*)^2 - (v_0^*)^2 \right] + g(z - z_0) \]

= flow exergy

\[ \dot{W}_{useful} = \left( \dot{W}_{in} - \dot{W}_{out} \right) - \left( P_0 \frac{dV_{CV}}{dt} \right) \]

\[ \dot{W}_{actual} \]

\[ \dot{W}_{sur} \]
Efficiency and Effectiveness

1. First law efficiency (thermal efficiency)

\[ \eta = \frac{\text{net work output}}{\text{gross heat input}} = \frac{W_{\text{net}}}{Q_{\text{in}}} \]

Carnot cycle

\[ \eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{T_L}{T_H} \]

2. Second Law Efficiency (effectiveness)

\[ \eta_{2nd} = \frac{\text{net work output}}{\text{maximum reversible work}} = \frac{\text{net work output}}{\text{availability}} \]

Turbine \( \rightarrow \) \( \eta_{2nd} = \frac{\dot{W}}{\dot{m}} \frac{\psi_e - \psi_i}{\dot{W}} \)

Compressor \( \rightarrow \) \( \eta_{2nd} = \frac{\psi_e - \psi_i}{\dot{W}/\dot{m}} \)

Heat Source \( \rightarrow \) \( \eta_{2nd} = \frac{\dot{W}/\dot{m}}{\dot{Q}/\dot{m}} \left[ 1 - \frac{T_0}{T_{TER}} \right] \)

3. Isentropic efficiency (process efficiency)

(a) adiabatic turbine efficiency

\[ \eta_T = \frac{\text{work of actual adiabatic expansion}}{\text{work of reversible adiabatic expansion}} = \frac{W_{\text{act}}}{W_S} \]

(b) adiabatic compressor efficiency

\[ \eta_C = \frac{\text{work of reversible adiabatic compression}}{\text{work of actual adiabatic compression}} = \frac{W_S}{W_{\text{act}}} \]
2 kg of air in a piston-cylinder device is expanded reversibly and isothermally from 700 kPa and 250 °C to a pressure of 125 kPa. During the process, heat is added from a thermal energy reservoir at 250 °C. Assume the dead state conditions to be \( T_0 = 25 \, ^\circ C \) and \( P_0 = 101.325 \, kPa \).

a) Determine the amount of work transfer \([kJ]\) to the piston and the amount of heat transfer \([kJ]\) from the source.

b) Determine the availability transfer \([kJ]\) due to work and heat.

c) Determine the increase in availability \([kJ]\) of the air in the cylinder.

d) Physically, how do you explain that, although the internal energy of the air did not change, its availability did?

\[
\text{TER} \quad 250 \, ^\circ C
\]

\[
2 \, \text{kg air}
\]

\[
T_0, \, P_0
\]