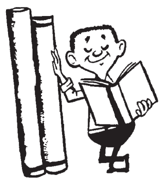
Availability



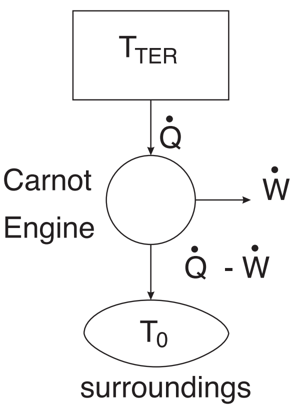
Reading	Problems
10.1 → 10.4	10.59, 10.65, 10.66, 10.67
	10.69, 10.75, 10.81, 10.88

Second Law Analysis of Systems

AVAILABILITY:

- the theoretical maximum amount of reversible work that can be obtained from a system at a given state P_1 and T_1 when interacting with a reference atmosphere at the constant pressure and temperature P_0 and T_0.

What do we mean by work potential of a system?



Notice that:

$$\eta = \frac{\dot{W}}{\dot{Q}} = 1 - \frac{T_0}{T_{TER}}$$

Therefore

$$\dot{W} = \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)$$

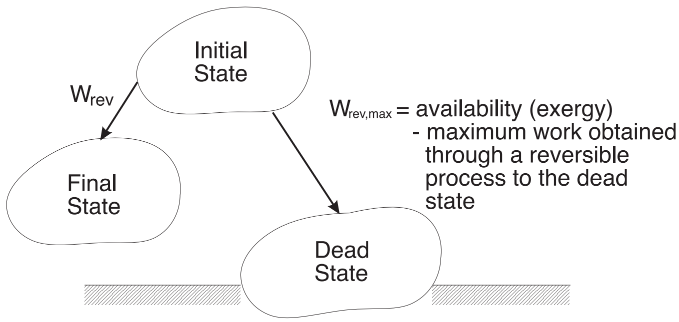
This term represents the work potential (availability) of a given TER with respect to the surroundings (dead state) at T_0.

The following observations can be made about availability:

1. Availability is a **property** - since any quantity that is fixed when the state is fixed is a property.
2. Availability is a **composite property** - since its value depends upon an external datum
3. Availability of a system is 0 at its **dead state** when $T = T_0$ and $P = P_0$.
4. Unless otherwise stated, assume the dead state to be:

$$P_0 = 1\ atm$$

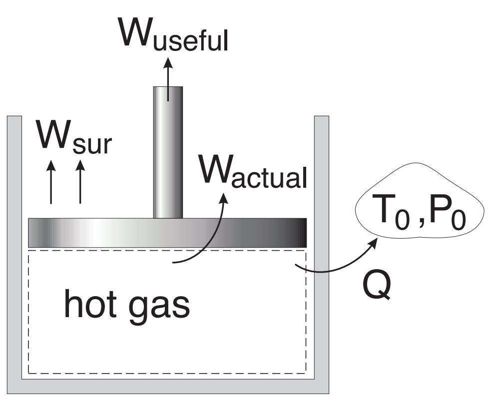
$$T_0 = 25^\circ C$$



5. The maximum work is obtained through a reversible process to the dead state.

$$\underbrace{REVERSIBLE\ WORK}_{W_{rev}} = \underbrace{USEFUL\ WORK}_{W_{useful}} + \underbrace{IRREVERSIBILITY}_{I}$$

Control Mass Analysis



- we know

$$W_{rev} = W_{useful} \ + \ I$$

but as shown in the figure, the actual work of the process is divided into two components

$$W_{actual} = W_{useful} + W_{sur}$$

- where W_{sur} is the part of the work done against the surroundings to displace the ambient air

$$W_{sur} = P_0(V_2 - V_1) = -P_0(V_1 - V_2)$$

To find W_{actual}, from the 1st law

$$E_1 - Q - W_{actual} = E_2 \quad \rightarrow \quad Q = E_1 - E_2 - W_{actual}$$

From the 2nd law

$$\begin{aligned} S_{gen} &= \Delta S_{system} + \Delta S_{sur} \geq 0 \\ &= S_2 - S_1 + \frac{Q}{T_0} \end{aligned}$$

But from the 1st law balance we know

$$\frac{Q}{T_0} = \frac{E_1 - E_2 - W_{actual}}{T_0}$$

and when we combine this with the 2nd law

$$S_{gen} = S_2 - S_1 + \frac{E_1 - E_2 - W_{actual}}{T_0}$$

which leads to

$$W_{actual} = (E_1 - E_2) + T_0(S_2 - S_1) - T_0 S_{gen}$$

or by reversing the order of S_2 and S_1

$$W_{actual} = (E_1 - E_2) - T_0(S_1 - S_2) - T_0 S_{gen}$$

But we also know that

$$W_{useful} = W_{actual} - W_{sur}$$

therefore

$$W_{useful} = (E_1 - E_2) - T_0(S_1 - S_2) + \underbrace{P_0(V_1 - V_2)}_{-W_{sur}} - T_0 S_{gen}$$

and

$$\begin{aligned} W_{rev} &= W_{useful} + I \\ &= W_{actual} - W_{sur} + I \end{aligned}$$

where

$$I = T_0 S_{gen}$$

Therefore

$$W_{rev} = (E_1 - E_2) - T_0(S_1 - S_2) + P_0(V_1 - V_2)$$

Define

$$\begin{aligned} \Phi &= CONTROL\ MASS\ AVAILABILITY \\ &= W_{rev} \text{ (in going to the dead state)} \\ &= (E - E_0) - T_0(S - S_0) + P_0(V - V_0) \end{aligned}$$

where the specific availability is defined as

$$\phi = \frac{\Phi}{m}$$

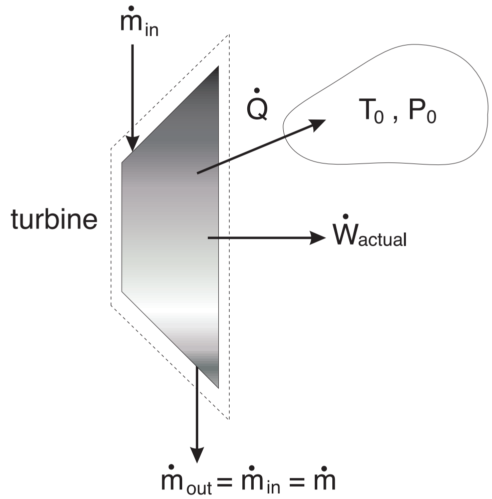
The availability destroyed is

$$I = W_{rev} - W_{useful} = T_0 S_{gen} = T_0\ S_{gen}$$

This can be referred to as: irreversibilities, availability destruction or loss of availability.

Control Volume Analysis

Consider a steady state, steady flow (SS-SF) process



From the 1st law

$$\frac{dE_{cv}}{dt}\nearrow^{0} = -\dot{W}_{actual} - \dot{Q} + \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{in} - \left[\dot{m}(h + \frac{(v^*)^2}{2} + gz)\right]_{out} \qquad (1)$$

From the 2nd law

$$\frac{dS_{cv}}{dt}\nearrow^{0} = \left(\dot{m}s + \frac{\dot{Q}\nearrow^{0}}{T_{TER}}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}}{T_0}\right)_{out} + \dot{S}_{gen} \qquad (2)$$

Combining (1) and (2) through the $\dot{Q}$ term, leads to the actual work output of the turbine, given as

$$\begin{aligned} \dot{W}_{actual} &= \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{in} - \left[\dot{m}\left(h + \frac{(v^*)^2}{2} + gz - T_0 s\right)\right]_{out} - T_0\dot{S}_{gen} \\ &= \dot{m}\left[-T_0\Delta s + \Delta h + \Delta KE + \Delta PE\right] - (T_0\dot{S}_{gen}) \qquad (3) \end{aligned}$$

$\dot{W}_{actual}$ is the actual work output of the turbine.

The specific flow availability, ψ, is given as

$$\psi = -T_0(s - s_0) + (h - h_0) + \left(\frac{(v^*)^2}{2} - \frac{(v_0^*\nearrow^{0})^2}{2}\right) + g(z - z_0\nearrow^{0}) \qquad (4)$$

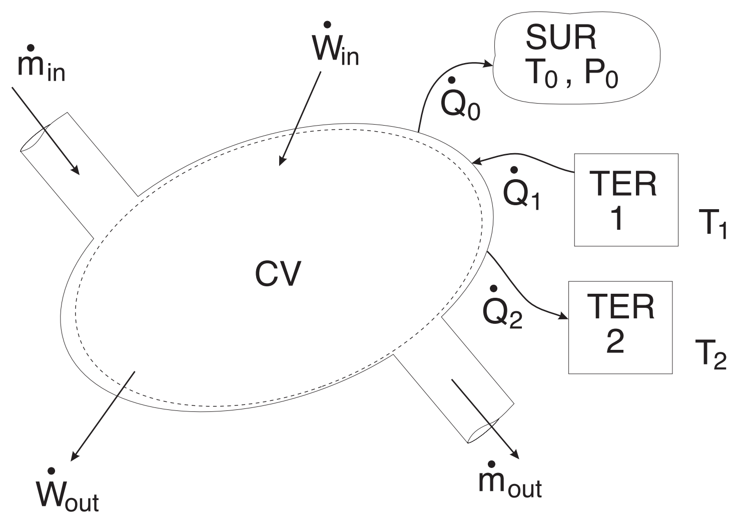
For a steady state, steady flow process where we assume KE=PE=0

$$\dot{W}_{rev} = (\dot{m}\psi)_{in} - (\dot{m}\psi)_{out} \qquad (5)$$

$$\dot{I} = \dot{W}_{rev} - \dot{W}_{actual} = T_0\dot{S}_{gen} = T_0\,\dot{S}_{gen} \qquad (6)$$

$$\psi = (h - h_0) - T_0(s - s_0) \qquad (7)$$

The Exergy Balance Equation



From the 1st law

$$\frac{dE_{cv}}{dt} = \dot{W}_{in} - \dot{W}_{out} - \dot{Q}_0 + \dot{Q}_1 - \dot{Q}_2 + [\dot{m}(e + Pv)]_{in} - [\dot{m}(e + Pv)]_{out} \qquad (1)$$

From the 2nd law

$$\frac{dS_{cv}}{dt} = \left(\dot{m}s - \frac{\dot{Q}_0}{T_0} + \frac{\dot{Q}_1}{T_1}\right)_{in} - \left(\dot{m}s + \frac{\dot{Q}_2}{T_2}\right)_{out} + \dot{S}_{gen} \qquad (2)$$

Multiply (2) by T_0 and subtract from (1) to eliminate Q_0, which leads to the generalized exergy equation

$$\frac{d}{dt}(E - T_0S)_{CV} = \dot{W}_{in} - \dot{W}_{out} + [\dot{m}(e + Pv - T_0s)]_{in}$$

$$
\begin{aligned}
&[\dot{m}(e + Pv - T_0 s)]_{out} + \left(\dot{Q}_1 - \frac{T_0 \dot{Q}_1}{T_1}\right)_{in} \\
&- \left(\dot{Q}_2 - \frac{T_0 \dot{Q}_2}{T_2}\right)_{out} - T_0 \dot{S}_{gen} \qquad (3)
\end{aligned}
$$

We can rewrite Eq. (3) in a generalized form by introducing the definitions of X and ψ.

$$
\begin{aligned}
\frac{d\Phi}{dt} &= P_0 \frac{dV_{CV}}{dt} + \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{in} \\
&\quad - \left[\dot{W} + \dot{m}\psi + \dot{Q}\left(1 - \frac{T_0}{T_{TER}}\right)\right]_{out} - \dot{I}
\end{aligned}
$$

where

$$
\begin{aligned}
\dot{I} &= T_0 \dot{S}_{gen} \\
&= \text{exergy destruction rate} \\
\Phi &= [(E - E_0) + P_0(V - V_0) - T_0(S - S_0)] \\
&= \text{non-flow exergy} \\
\psi &= (h - h_0) - T_0(s - s_0) + \frac{1}{2}\left[(v^*)^2 - (v_0^*)^2\right] + g(z - z_0) \\
&= \text{flow exergy} \\
\dot{W}_{useful} &= (\underbrace{\dot{W}_{in} - \dot{W}_{out}}_{\dot{W}_{actual}}) - \underbrace{\left(P_0 \frac{dV_{CV}}{dt}\right)}_{W_{sur}}
\end{aligned}
$$

Efficiency and Effectiveness

1. **First law efficiency (thermal efficiency)**

$$\eta = \frac{net\ work\ output}{gross\ heat\ input} = \frac{W_{net}}{Q_{in}}$$

Carnot cycle

$$\eta = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

2. **Second Law Efficiency (effectiveness)**

$$\eta_{2nd} = \frac{net\ work\ output}{maximum\ reversible\ work} = \frac{net\ work\ output}{availability}$$

Turbine → $\eta_{2nd} = \dfrac{\dot{W}/\dot{m}}{\psi_e - \psi_i}$

Compressor → $\eta_{2nd} = \dfrac{\psi_e - \psi_i}{\dot{W}/\dot{m}}$

Heat Source → $\eta_{2nd} = \dfrac{\dot{W}/\dot{m}}{\dot{Q}/\dot{m}\left[1 - \dfrac{T_0}{T_{TER}}\right]}$

3. **Isentropic efficiency (process efficiency)**

   (a) adiabatic turbine efficiency

$$\eta_T = \frac{work\ of\ actual\ adiabatic\ expansion}{work\ of\ reversible\ adiabatic\ expansion} = \frac{W_{act}}{W_S}$$

   (b) adiabatic compressor efficiency

$$\eta_C = \frac{work\ of\ reversible\ adiabatic\ compression}{work\ of\ actual\ adiabatic\ compression} = \frac{W_S}{W_{act}}$$

PROBLEM STATEMENT:

$\boldsymbol{2\ kg}$ of air in a piston-cylinder device is expanded reversibly and isothermally from $\boldsymbol{700\ kPa}$ and $\boldsymbol{250\ ^\circ C}$ to a pressure of $\boldsymbol{125\ kPa}$. During the process, heat is added from a thermal energy reservoir at $\boldsymbol{250\ ^\circ C}$. Assume the dead state conditions to be $\boldsymbol{T_0 = 25\ ^\circ C}$ and $\boldsymbol{P_0 = 101.325\ kPa}$.

a) Determine the amount of work transfer [$\boldsymbol{kJ}$] to the piston and the amount of heat transfer [$\boldsymbol{kJ}$] from the source.
b) Determine the availability transfer [$\boldsymbol{kJ}$] due to work and heat.
c) Determine the increase in availability [$\boldsymbol{kJ}$] of the air in the cylinder.
d) Physically, how do you explain that, although the internal energy of the air did not change, its availability did?