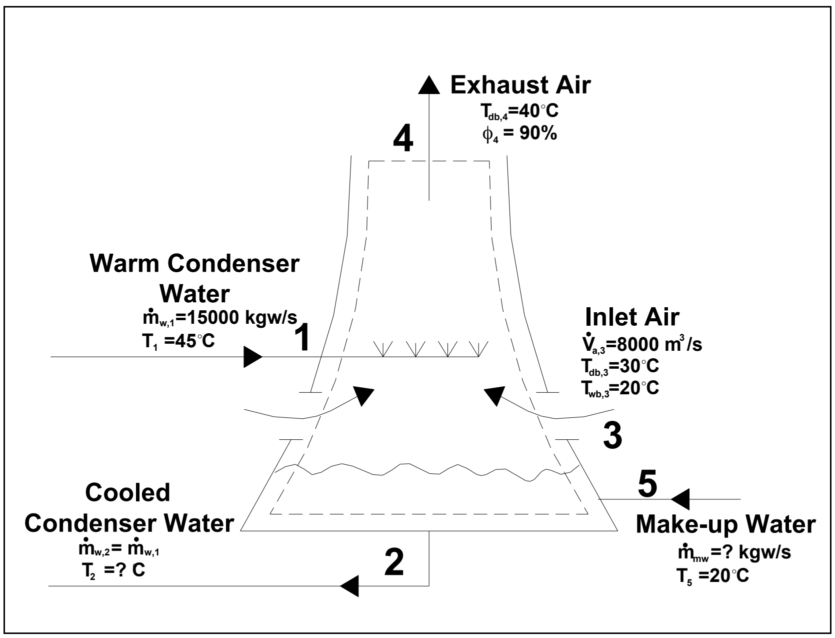
**ME 354 Tutorial, Week#11**
**Non-Reacting Mixtures – Psychrometrics Applied to a Cooling Tower**

Water exiting the condenser of a power plant at 45°C enters a cooling tower with a mass flow rate of 15000 kg/s. A stream of cooled water is returned to the condenser from the cooling tower with the same flow rate. Make-up water is added in a separate stream at 20°C. Atmospheric air enters the cooling tower at 30°C and a wet bulb temperature of 20°C. The volumetric flow rate of air into the cooling tower is 8000 $m^3/s$. Moist air exits the tower at 40°C and 90% relative humidity. Assume an atmospheric pressure of 101.3 kPa.

Determine:

a) the mass flow rate of dry air,
b) the mass flow rate of make-up water, and
c) the temperature of the cooled liquid water exiting the cooling tower.

**Step 1: Draw a diagram to represent the system**




**Step 2: Prepare a property table**

| $H_20$ | T (°C) | m (kg/s) | h (kJ/kg) |
|---|---|---|---|
| 1 (sat. liq) | 45 | 15000 | |
| 2 (sat. liq) | | 15000 | |
| 5 (sat. liq) | 20 | | |

| Air | $T_{db}$ (°C) | $T_{wb}$ (°C) | φ (%) | w ($kg_v/kg_a$) | v ($m^3/kg_a$) | h ($kJ/kg_a$) |
|---|---|---|---|---|---|---|
| 3 | 30 | 20 | | | | |
| 4 | 40 | | 90 | | | |

**Step 3: State your assumptions**

Assumptions:

1) The cooling tower operates under steady conditions
2) $\Delta KE, \Delta PE \approx 0$
3) Cooling tower is rigid and adiabatic $\therefore W_{cv} = 0$ & $Q_{cv} = 0$ .
4) Assume all liquid water is saturated
5) The pressure is constant throughout the cooling tower at 101.3 kPa.

**Step 4: Calculations**

**Part a)**

The mass flow rate of dry air can be determined using the volumetric flow rate of air into the cooling tower (given in the problem as 8000 $m^3/s$) and the specific volume of this air as shown in Eq1.

$$m_{a,3} = \frac{\dot{V}_{a,3}}{v_{a,3}} \qquad \text{(Eq1)}$$

We can determine the specific volume of the air entering the cooling tower by determining the state point of location 3 on the psychrometric chart using $T_{db,3}$ = 30°C and $T_{wb,3}$ = 20°C.

From the psychrometric chart,

$\rightarrow v_{a,3} = 0.873\ m^3/kg_a$

Substituting this value and the given volumetric flow rate into Eq1 we can determine the mass flow rate of dry air.

$$\rightarrow m_{a,3} = \frac{\dot{V}_{a,3}}{v_{a,3}} = \frac{8000\left[\frac{m^3}{s}\right]}{0.873\left[\frac{m^3}{kg_a}\right]} = \mathbf{9163.8\ kg_a/s} \quad \textbf{Answer a)}$$

**Part b)**

To determine the mass flow rate of the make-up water, denoted as $\dot{m}_{mw}$, we can perform a mass balance on the water entering/exiting our cooling tower control volume. At location 1 we have the stream of water entering the cooling tower from the condenser, which we will denote as $\dot{m}_{w,1}$. At location 2 we have the stream of water exiting the cooling tower to be returned to the condenser, which we will denote as $\dot{m}_{w,2}$. We are told in the problem statement that $\dot{m}_{w,1}$ = $\dot{m}_{w,2}$ = $\dot{m}_{w}$. At location 3, we have the moisture entering the cooling tower control volume carried in by the incoming air. We will denote this as $\dot{m}_{v,3}$. At location 4, we have the moisture leaving the cooling tower control volume carried out by the exiting air. We will denote this as $\dot{m}_{v,4}$. The mass balance on the water is performed in Eq2.

$$\dot{m}_{w} - \dot{m}_{w} + \dot{m}_{v,3} - \dot{m}_{v,4} + \dot{m}_{mw} = 0 \rightarrow \dot{m}_{mw} = \dot{m}_{v,4} - \dot{m}_{v,3} \qquad \text{(Eq2)}$$

Note: We could have developed Eq2 immediately by reasoning that the amount of water that needs to be “made-up” for will be equal to the amount of moisture that is picked up in the cooling tower by the air and exhausted.

We can express $\dot{m}_{v,3}$ & $\dot{m}_{v,4}$ in terms of the corresponding mass flow rates of dry air at location 3 & 4 and their respective humidity ratios $w_3$ & $w_4$ as shown in Eq3 and Eq4.

$$\dot{m}_{v,3} = w_3\, \dot{m}_{a,3} \qquad \text{(Eq3)}$$

$$\dot{m}_{v,4} = w_4\, \dot{m}_{a,4} \qquad \text{(Eq4)}$$

Substituting Eq3 and Eq4 into Eq2 we obtain Eq5.

$$\dot{m}_{mw} = \dot{m}_{v,4} - \dot{m}_{v,3} = w_4\, \dot{m}_{a,4} - w_3\, \dot{m}_{a,3} \qquad \text{(Eq5)}$$

From our assumption that the cooling tower operates in a steady manner, the mass flow rate of air will be constant i.e. $\dot{m}_{a,3}$ = $\dot{m}_{a,4}$ = $\dot{m}_{a}$. Eq5 can be rewritten as Eq6.

$$\dot{m}_{mw} = \dot{m}_{a}\left(w_4 - w_3\right) \qquad \text{(Eq6)}$$

Recall that we determined the mass flow rate of dry air in part a). We can determine the humidity ratio of the air entering the cooling tower by using state point 3 on the psychrometric chart.

From the psychrometric chart,

$\rightarrow w_3 = 10.6\ g_v/kg_a = 0.0106\ kg_v/kg_a$

Unfortunately, state point 4 ($T_{db,4}$ = 40°C & $\phi$=90%) is off the psychrometric chart so we will have to calculate the value of $w_4$ using equation 13-11b from Cengel and Boles as shown below.

From Table A-4 @ T= 40°C, $P_g$ = 7.384 kPa.

$$\rightarrow w_4 = \frac{0.622\phi_4 P_g}{P - \phi_4 P_g} = \frac{0.622(0.9)(7.384)}{101.3 - 0.9(7.384)} = 0.0437\left[\frac{kg_v}{kg_a}\right]$$

Substituting these values into Eq6, we can determine the mass flow rate of the make-up water.

$$\rightarrow \dot{m}_{mw} = \dot{m}_a(w_4 - w_3) = 9163.8\left[\frac{kg_a}{s}\right](0.0437 - 0.0106)\left[\frac{kg_v}{kg_a}\right]$$

**= 303.3 $kg_v$/s      Answer b)**

**Part c)**

We can determine the temperature of the cooled liquid water exiting the cooling tower by first determining its enthalpy. Since we are assuming it is a saturated liquid we can use the enthalpy to interpolate in Table A-4 to determine its temperature. To find the enthalpy of the water exiting the cooling tower we must perform an energy balance on the cooling tower control volume. At location 1, the rate of energy entering the control volume carried in by the stream of water coming from the condenser is $\dot{m}_w h_{w,1}$, where h is used to denote enthalpy. At location 2 we have the stream of water leaving the cooling tower carrying away energy at a rate of $\dot{m}_w h_{w,2}$. At location 3, the moist air carries energy at a rate of $\dot{m}_a h_3$ into the control volume. At location 4, we have moist air leaving the cooling tower control volume carrying energy out at a rate of $\dot{m}_a h_4$. The make-up water carries energy into the control volume at a rate of $\dot{m}_{mw} h_{mw}$. Combining all of these statements into one expression we obtain Eq7. Note: We have made use of our list of assumptions in developing Eq7.

$$\dot{m}_w h_{w,1} - \dot{m}_w h_{w,2} + \dot{m}_a h_3 - \dot{m}_a h_4 + \dot{m}_{mw} h_{mw} = 0 \qquad \text{(Eq7)}$$

We can isolate the enthalpy of the water at location 2 by rearranging Eq7 as shown in Eq8.

$$h_{w,2} = h_{w,1} + \frac{\dot{m}_a (h_3 - h_4) + \dot{m}_{mw} h_{mw}}{\dot{m}_w} \qquad \text{(Eq8)}$$

We have previously determined $\dot{m}_a$ and $\dot{m}_{mw}$, and we are given $\dot{m}_w$ in the problem statement.

**$h_{w,1}$**

Since we have assumed saturated liquid water at location 1 we can determine $h_{w,1}$ from Table A-4 using $T_1$ = 45°C.

$$\rightarrow h_{w,1} = 188.45\left[\frac{kJ}{kg_w}\right]$$

**$h_3$**

Using state point 3 on the psychrometric chart we can determine $h_3$.

$$\rightarrow h_3 = 58\left[\frac{kJ}{kg_a}\right]$$

**$h_4$**

As stated previously, state point 4 is off the psychrometric chart so we must calculate $h_4$. Using equation 13-1a from Cengel and Boles, we can calculate the enthalpy of DRY AIR alone.

$$\rightarrow h_{a,4} = \left(1.005\left[\frac{kJ}{kg_a \bullet {}^\circ C}\right]\right)(40^\circ C) = 40.2\left[\frac{kJ}{kg_a}\right]$$

We can calculate the enthalpy of the MOISTURE in the air using Table A-4 for $h_g$ @ T = 40°C.

$$\rightarrow h_{g,4} = 2574.3\left[\frac{kJ}{kg_v}\right]$$

To combine the dry air and moisture enthalpies at location 4 into one term, $h_4$, we need to convert the enthalpy of the moisture to be on a "per kg of dry air" basis by multiplying it by the humidity ratio, $w_4$.

$$\rightarrow h_4 = h_{a,4} + w_4 h_{g,4} = 40.2\left[\frac{kJ}{kg_a}\right] + \left(0.0437\left[\frac{kg_v}{kg_a}\right]\right)\left(2574.3\left[\frac{kJ}{kg_v}\right]\right) = 152.7\left[\frac{kJ}{kg_a}\right]$$

**$h_{mw}$**

Since we have assumed saturated liquid water at location 5 we can determine $h_{mw}$ from Table A-4 using $T_5$=20°C.

$$\rightarrow h_{mw} = 83.96\left[\frac{kJ}{kg_w}\right]$$

Substituting these values into Eq8, we can determine the enthalpy of the water at location 2.

$$h_{w,2} = 188.45\left[\frac{kJ}{kg_w}\right] + \frac{\left(9163.8\left[\frac{kg_a}{s}\right]\right)\left((58-152.7)\left[\frac{kJ}{kg_a}\right]\right) + \left((303.3)\left[\frac{kg_w}{s}\right](83.96)\left[\frac{kJ}{kg_w}\right]\right)}{15000\left[\frac{kg_w}{s}\right]}$$

$$\rightarrow h_{w,2} = 188.45\left[\frac{kJ}{kg_w}\right] - 56.16\left[\frac{kJ}{kg_w}\right] = 132.3\left[\frac{kJ}{kg_w}\right]$$

As stated previously, since we have assumed saturated liquid water at location 2 we can use $h_{w,2}$ to find the corresponding temperature in Table A-4. From Table A-4 we find that $h_{w,2}$ lies in between the enthalpies corresponding to temperatures of 30°C and 35°C. We can interpolate in between them to find the temperature of the water exiting the cooling tower.

$$\frac{132.3-125.79}{146.68-125.79} = \frac{T_2 - 30^\circ C}{35^\circ C - 30^\circ C}$$

**$\rightarrow T_2$ = 31.6°C      Answer c)**

**Step 5: Summary**

a) the mass flow rate of dry air is 9163.8 kga/s
b) the mass flow rate of make-up water is 303.3 kgw/s, and
c) the temperature of the cooled liquid water exiting the cooling tower is 31.6°C.