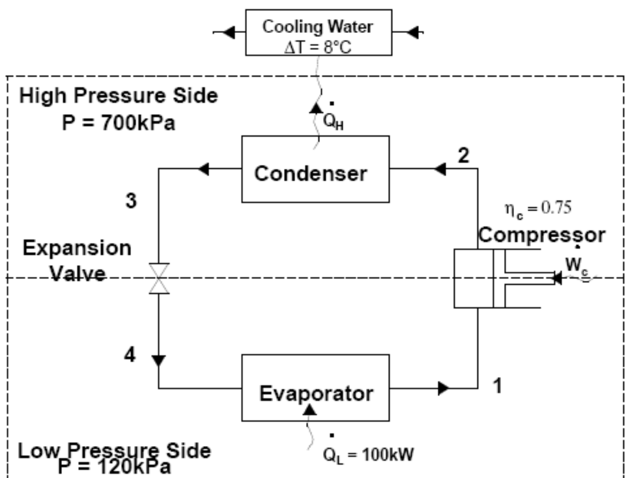
ME 354 – Tutorial, Week#5 –Refrigeration Cycle

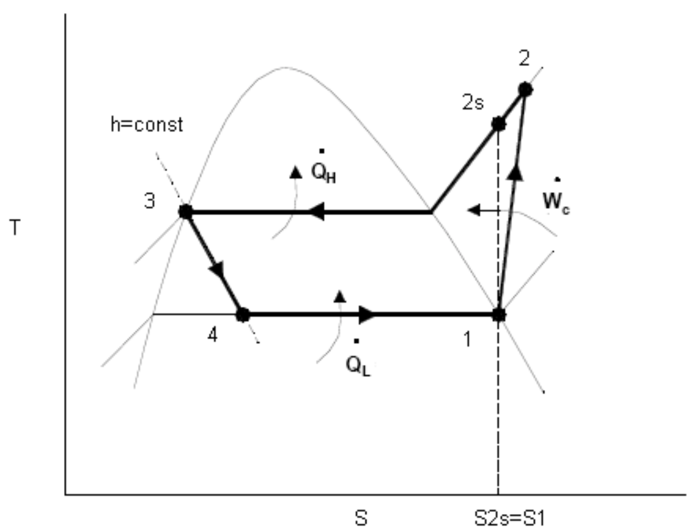
A large refrigeration plant is to be maintained at -15^{o}C, and it requires refrigeration at a rate of 100 kW. The condenser of the plant is to be cooled by liquid water, which experiences a temperature rise of 8^{o}C as it flows over the coils of the condenser. The plant uses refrigerant-134a between the pressure limits of 120 kPa and 700 kPa. Assuming the compressor has an isentropic efficiency of 75%, determine:

a) The mass flow rate of the refrigerant
b) The power input to the compressor
c) The mass flow rate of the cooling water
d) The rate of exergy destruction associated with the compression process (assume T_0=25^{o}C)

Step 1: Draw a diagram to represent the system



To better visualize what is happening during the cycle we can draw a T-s process diagram.



Step 2: Write out what is required to solve for

a) The mass flow rate of the refrigerant
b) The power input to the compressor
c) The mass flow rate of the cooling water
d) The rate of exergy destruction associated with the compression process (assume $T_0=25^oC$)

Step 3: Property table

	T (°C)	P (kPa)	h (kJ/kg)	s (kJ/k*K)
1 (sat vap)		120		
2		700		
2s		700		s_1
3 (sat liq)		700		
4		120	h_3	

Step 4: Assumptions

Assumptions:

1) $\Delta ke, \Delta pe = 0$
2) SSSF
3) Throttling process is adiabatic
4) State 1 is a saturated vapour @ P=120 kPa
5) State 3 is a saturated liquid @ P=700 kPa

Step 5: Solve
Part a)

We can find the mass flow rate of the refrigerant, m_{refrig}, by looking at a device in our cycle that we know the energy transfer (work or heat transfer), in or out. We are given the heat transfer into the evaporator, $\dot{Q}_e$ = 100 kW, so $\dot{m}_{refrig}$ can be determined from an energy balance on the evaporator as shown in Eq1.

$$\dot{m}_{refrig} = \frac{\dot{Q}_e}{(h_1 - h_4)} \quad \text{(Eq1)}$$

Since we know state 1 is a saturated vapor at a pressure of 120kPa, we can determine h_1 using Table A-12.

State 1→Table B-5@P=0.12 MPa

→ $h_1 = h_{g@P=120\ kPa} = 384.46$ kJ/kg
→ $s_1 = s_{g@P=120\ kPa} = 1.74187$ kJ/kgK

Performing an energy balance on the throttling valve using the assumptions $\Delta ke, \Delta pe = 0$ and the process is adiabatic gives $h_4=h_3$. So h_4 will be known if h_3 is known. State 3 is saturated liquid at a pressure of 700 kPa, therefore h_3 can be determined using Table A-12.

State 3→Table A-12@P=0.7 MPa

→ $h_3 = h_4 = hf_{@P=700\ kPa} = 236.90$ kJ/kg

Substituting these values into Eq1 the value of $\dot{m}_{refrig}$ can be determined.

$$\rightarrow \dot{m}_{refrig} = \frac{\dot{Q}_L}{(h_1 - h_4)} = \frac{100\left[\frac{kJ}{s}\right]}{(384.46 - 236.90)\left[\frac{kJ}{kg}\right]} \text{ (corrected)}$$

=0.68 kg/s Answer a)

Part b)

The power input to the compressor, $\dot{W}_c$, can be found by performing an energy balance on the compressor as shown in Eq2.

$$\dot{W}_c = \dot{m}_{refrig}(h_2 - h_1) \tag{Eq2}$$

$\dot{m}_{refrig}$ was calculated in part a) but (h_2-h_1) must still be determined. The problem statement provides an isentropic efficiency for the compressor. Rearranging the definition of the isentropic efficiency, as shown in Eq3, an expression for (h_2-h_1) in terms of known quantities $\dot{m}_{refrig}$, h_1, and h_{2s}, which can be determined, is found.

$$\eta_c = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \rightarrow (h_{2s} - h_1) = \frac{(h_{2s} - h_1)}{\eta_c} \tag{Eq3}$$

For state 2s, $s_2=s_1=0.94779$ kJ/kg*K. Using this information with Table A-12, it is found that state 2s is outside of the vapor dome and into the superheated region i.e. $s_2>s_{g@P=700\ kPa.}$ Looking in Table A-13 @ P=0.7 MPa, it is found that s_2 lies between the entropies with corresponding temperatures of 30°C and 40°C. Using $s_2=0.94779$ kJ/kg*K to interpolate between the entropies at 30°C and 40°C, we can find the enthalpy at the state 2s.

$$\rightarrow \frac{\left(h_{2s} - h_{s=0.9313}\right)}{\left(h_{s=0.9641} - h_{s=0.9313}\right)} = \frac{0.94779 - 0.9313}{0.9641 - 0.9313} \rightarrow h_{2s} = h_{s=0.9313} + +0.5027\left(h_{s=0.9641} - h_{s=0.9313}\right)$$

$$\rightarrow h_{2s} = 268.45 + (0.5027)(278.57 - 268.45) = 273.54\left[\frac{kJ}{kg}\right] \text{ (Not Correct)}$$

$$h_{2s} = 421.97\left[\frac{kJ}{kg}\right]$$

Note: h_{2s} is what the enthalpy at state 2 would be if the compression process from $1 \rightarrow 2$ was isentropic i.e. $s_2 = s_1$. Substituting Eq3 into Eq2, the value of $\dot{W}_c$ can be determined.

$$\rightarrow \dot{W}_c = \dot{m}_{refrig} \frac{\left(h_{2s} - h_1\right)}{\eta_c} = 0.68\left[\frac{kg}{s}\right] \frac{\left(421.975 - 384.46\right)\left[\frac{kJ}{kg}\right]}{0.75} \text{ (corrected)}$$

=34.01 kJ/s   Answer b)

Part c)

Part c) asks for the mass flow rate of the cooling water, $\dot{m}_{cw}$, used in the condenser. The rate of heat transfer out of the condenser must be equal to the rate of heat transfer into the cooling water. The rate of heat transfer out of the condenser, $\dot{Q}_H$, can be determined by performing an energy balance on the condenser as shown in Eq4.

$$\dot{Q}_H = \dot{m}_{refrig}\left(h_2 - h_3\right) \qquad \text{(Eq4)}$$

The rate of heat transfer into the cooling water can be expressed as a function of the temperature rise the cooling water experiences using Eq5.

$$\dot{Q}_H = \dot{m}_{cw} c \Delta T \qquad \text{(Eq5)}$$

Combining Eq4 and Eq5, and rearranging to isolate for the mass flow rate of the cooling water, Eq6 is obtained.

$$\dot{Q}_H = \dot{m}_{refrig}\left(h_2 - h_3\right) = \dot{m}_{cw} c \Delta T \rightarrow \dot{m}_{cw} = \frac{\dot{m}_{refrig}\left(h_2 - h_3\right)}{c \Delta T} \qquad \text{(Eq6)}$$

h_3 and $\dot{m}_{refrig}$ have been previously determined, so only h_2, c, and ΔT need to be found. Rearranging Eq3, the enthalpy at state 2, h_2, can be found.

$$\rightarrow h_2 = \frac{(h_{2s} - h_1)}{\eta_c} + h_1 = \frac{273.54 - 236.97}{0.75} + 236.97 = \rightarrow h_2 = 285.73\left[\frac{kJ}{kg}\right]$$

From Table A-3, c for water is given as 4.18 kJ/kg*°C. The problem statement gives the temperature rise in the cooling water as 8°C i.e. ΔT =8°C. Substituting the values of h_2, c, ΔT, h_3 and $\dot{m}_{refrig}$ into Eq6, we can determine the mass flow rate of the cooling water.

$$\rightarrow \dot{m}_{cw} = \frac{\dot{m}_{refrig}(h_2 - h_3)}{c\Delta T} = \frac{0.68\left[\frac{kg}{s}\right](285.73 - 88.82)\left[\frac{kJ}{kg}\right]}{4.18\left[\frac{kJ}{kg\,^oC}\right](8)\left[^oC\right]}$$

=4.00 kg/s **Answer c)**

Part d)

The rate of exergy destruction can be expressed in terms of the surroundings temperature, T_0, and the rate of entropy generation, $\dot{S}_{GEN}$, as shown in Eq7.

$$\dot{X}_{destroyed} = T_0\dot{S}_{GEN} \qquad \text{(Eq7)}$$

We can determine the entropy generation term, $\dot{S}_{GEN}$, by applying an entropy balance on the compressor as shown in Eq.8

$$\dot{S}_{in} - \dot{S}_{out} + \dot{S}_{GEN} = \Delta\dot{S}_{system} \qquad \text{(Eq8)}$$

Since there is no heat transfer in or out of the compressor, the only mechanism for entropy transfer into and out of the compressor is by the mass flow. Therefore $\dot{S}_{in}$ and $\dot{S}_{out}$ are equal to $\dot{m}_{refrig}s_1$ and $\dot{m}_{refrig}s_2$ respectively. Using the steady state assumption, $\Delta\dot{S}_{system}$ is zero. Applying the above information to Eq8, Eq9 is obtained.

$$\dot{S}_{GEN} = \dot{m}_{refrig}(s_2 - s_1) \qquad \text{(Eq9)}$$

Substituting Eq9 into Eq7, Eq10 is obtained.

$$\dot{X}_{destroyed} = T_0\dot{m}_{refrig}(s_2 - s_1) \qquad \text{(Eq10)}$$

s_2 can be determined from Table A-13 @ P=0.7 MPa using h_2=285.73 kJ/kg to interpolate between the entropies corresponding to temperatures of 40°C and 50°C.

$$\rightarrow \frac{\left(s_2 - s_{h==278.57}\right)}{\left(s_{h=288.53} - s_{h=278.57}\right)} = \frac{285.73 - 278.57}{288.53 - 278.57} \rightarrow s_2 = s_{h=278.57} + +0.719\left(s_{h=288.53} - s_{h=278.57}\right)$$

$$\rightarrow s_2 = 0.9641 + (0.719)(0.9954 - 0.9641) = 0.9866\left[\frac{kJ}{kgK}\right]$$

Substituting the known variables, assuming T_0 = 25°C, into Eq10, the rate of exergy destruction during the compression process can be determined.

$$\rightarrow \dot{X}_{destroyed} = \left(273.15 + 25\right)\left[K\right]\left(0.68\right)\left[\frac{kg}{s}\right]\left(0.9866 - 0.94779\right)\left[\frac{kJ}{kgK}\right]$$

=7.87 kg/s Answer d)

Alternative:

The exergy destroyed can also be found by performing an exergy balance on the compressor.

$$\dot{X}_{in} - \dot{X}_{out} - \dot{X}_{destroyed} = \Delta\dot{X}_{system} \qquad \text{(Eq11)}$$

Since there is no heat transfer in or out of the compressor the only mechanism for exergy transfer into and out of the compressor is by the mass flow and the work coming in the compressor. Using the steady state assumption, $\dot{X}_{destroyed}$ is zero. Applying the above information to Eq11, Eq12 is obtained.

$$\dot{m}_{refrig}\left(\Psi_1 - \Psi_2\right) + \dot{W}_c = \dot{X}_{destroyed} \qquad \text{(Eq12)}$$

Replacing Ψ_1 and Ψ_2 by their value,

$$\dot{m}_{refrig}\left(h_1 - h_2 - T_0\left(s_1 - s_2\right)\right) + \dot{W}_c = \dot{X}_{destroyed} \qquad \text{(Eq13)}$$

From Eq2, $\dot{W}_c = \dot{m}_{refrig}\left(h_2 - h_1\right)$. Substituting this information in Eq12, Eq14 is obtained.

$$\dot{m}_{refrig}\left(h_1 - h_2 - T_0\left(s_1 - s_2\right)\right) + \dot{m}_{refrig}\left(h_2 - h_1\right) = \dot{X}_{destroyed} \qquad \text{(Eq14)}$$

By simplifying the previous equation, an equation identical to Eq10 for $\dot{X}_{destroyed}$ is obtained.

$$\dot{X}_{destroyed} = T_0 \dot{m}_{refrig} \left(s_2 - s_1 \right) \quad \text{(Eq14)}$$

Step 5: Concluding Remarks & Discussion

The mass flow rate of the refrigerant was found to be 0.68 kg/s. The power input to the compressor was found to be 33.16 kJ/s. The mass flow rate of the cooling water was found to be 4.0 kg/s. The rate of exergy destruction associated with the compression process was found to be 7.87 kJ/s assuming a dead state temperature, T_0=25^{o}C.