THE ROLE OF SPREADING RESISTANCE ON EFFECTIVE CONDUCTIVITY IN LAMINATED SUBSTRATES

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ABSTRACT

A comparative study is presented that demonstrates the importance of including both material resistance and spreading resistance in the calculation of effective conductivity for printed circuit board applications. Results show that models based exclusively on cross-plane and in-plane resistive networks are not adequate for predicting effective conductivity in multilayer, laminated printed circuit boards. The mixed boundary conditions found in most microelectronic applications accentuate the importance of spreading resistance between heat sources and the convective boundaries. A dynamic internet-based tool for calculating effective conductivity in laminated substrates is presented that provides a convenient approach for calculating the combined effects of both the bulk material and spreading resistances.

NOMENCLATURE

a	=	source half length, m
A	=	cross sectional flow area, m^2
b	=	source half width, m
c	=	substrate half length, m
d	=	substrate half width, m
h	=	heat transfer coefficient, $W / m^2 K$
k	=	thermal conductivity, W / mK
L	=	flow length, m
i, m, n	=	counters
N	=	total number of layers
t	=	layer thickness, m
R	=	thermal resistance, $^{o}C / W$

Greek Symbols

β	=	composite eigenvalue in length and width
δ	=	eigenvalue associated with the length variable
λ	=	eigenvalue associated with the width variable
ε	=	relative source area
κ	=	conductivity ratio, $= k_2 / k_1$
ϕ	=	functional value used in spreading resistance

Subscripts

1,2	=	top and bottom layers, respectively
e	=	effective
f	=	fluid
p	=	parallel
s	=	series
T	=	total

INTRODUCTION

Thermal modeling of heat conduction in multilayered printed circuit boards is sometimes simplified through the use of an effective conductivity, a parameter that combines the influence of individual layer conductivities into a unique property value that can be applied to the analysis as a single, homogeneous value. Several schemes have been proposed for calculating effective conductivity, including what are generally considered to be the lower and upper bounds of effective conductivity, the series or “cross-plane conductivity” and the parallel or “in-plane conductivity”.

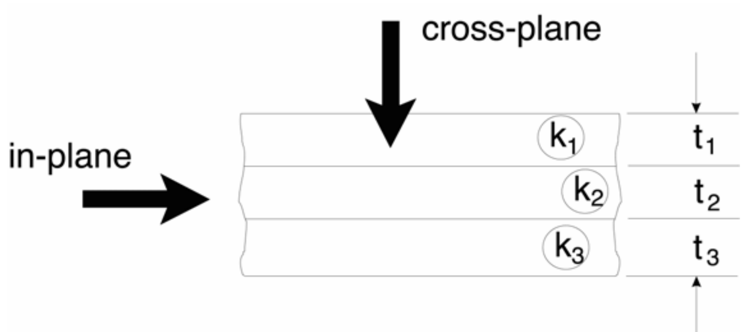

Figure 1: Simple Laminated Structure

Given a simple laminated structure shown in Fig. 1, the cross-plane and in-plane conductivities can be calculated as follows:

- cross-plane conductivity $$k_s = \sum_{i=1}^{N} t_i \Big/ \sum_{i=1}^{N} t_i / k_i \qquad (1)$$
- in-plane conductivity $$k_p = \sum_{i=1}^{N} k_i \cdot t_i \Big/ \sum_{i=1}^{N} t_i \qquad (2)$$

Simple averaging procedures using estimates of thermal conductivity based on the directional dependence of heat flow, as given in Eqs. 1 and 2, can provide an accurate measure of effective conductivity when the difference between the bounding solutions is small. This is generally not the case in PCBs where the ratio between the effective conductivity, based on the parallel and series paths, is typically of order 25:1 or greater. The arithmetic mean, $\equiv (k_s + k_p)/2$, tends to weight the in-plane estimate more heavily, thereby providing the highest prediction of average effective conductivity. The harmonic mean, $\equiv 2k_s \cdot k_p \big/ \left(k_s + k_p\right)$, weights the cross-plane conductivity more heavily, which results in the lowest prediction of effective conductivity. The geometric mean, $\equiv \sqrt{k_s \cdot k_p}$, provides a more neutral estimate of the two limiting values of effective conductivity and is generally taken as the preferred averaging scheme for multidirectional heat flow in laminated substrates.

Methods used to calculate effective conductivity that are based solely on these series/parallel resistive networks must be restricted to applications where boundary conditions are uniformly applied and all intrinsic resistances including the spreading resistance can be neglected. However, the discrete heat sources common to printed circuit board applications lead to mixed boundary value problems that do not lend themselves to these simplified analyses. The spreading resistance, often of similar magnitude to the bulk resistance, must be accounted for in the calculation of effective conductivity, and any estimates of conductivity based exclusively on series and parallel resistor networks can lead to significant miscalculations of board temperatures.

SPREADING RESISTANCE

The calculation of effective conductivity in a composite structure is based on determining a single value of thermal conductivity that can be used in a homogeneous substrate where overall thickness and surface area are preserved, such that

$$k_e = \frac{t_T}{R_T \; A}, \qquad t_T = \sum_{i=1}^{N} t_i \tag{3}$$

To accurately calculate a single value of k_e that captures all resistance components between the heat source and sink, the resistance associated with the bulk material properties and thermal spreading are combined by superposition:

$$R_T = R_{bulk} + R_{spread} \tag{4}$$

While the in-plane and cross-plane models presented in Eqs. 1 and 2 can be used to calculate the effective thermal conductivity of the bulk materials, they cannot be expected to capture the effects of the spreading resistance. The spreading resistance is influenced by many factors in addition to layer thickness and thermal conductivity, such as heat source dimensions and location, the relative position of layers in relation to the source, and the convective boundary conditions.

Yovanovich et al. (1998) have derived a detailed analytical expression for spreading resistance in a two-layer, rectangular substrate with a concentric, rectangular heat source, as shown in Fig. 2.

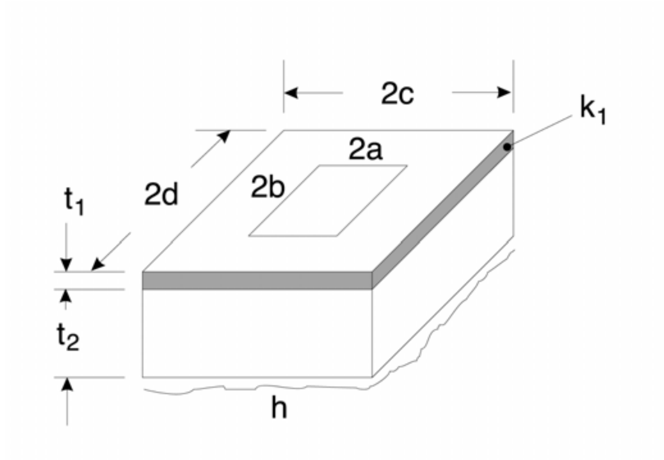

Figure 2: Test Coupon for Spreading Resistance Calculations

The spreading resistance is calculated based on the sum of two strip source solutions and a rectangular source solution:

$$R_{spread} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{\sin(b\lambda)^2 \sin(a\delta)^2}{a^2\lambda^2 b^2 \delta^2 \beta\, c\, d\, k_1} \cdot \phi_{m,n} + \sum_{n=1}^{\infty} \frac{1}{2} \frac{\sin(a\delta)^2}{a^2\delta^2 c\, d\, k_1} \cdot \phi_n + \sum_{m=1}^{\infty} \frac{1}{2} \frac{\sin(b\lambda)^2}{\lambda^2 b^2 c\, d\, k_1} \cdot \phi_m \tag{5}$$

The functional dependence of Eq. 5 clearly shows that the resistance between the source and the sink cannot be based exclusively on in-plane or cross-plane board resistances or a combination of the two resistances. The calculation of spreading resistance requires that conditions imposed at the boundaries, such as source size, source location, and convective conditions, be incorporated into the calculation of an overall resistance and the corresponding effective conductivity.

DISCUSSION

The influence of spreading resistance on effective conductivity is examined through a selective parametric analysis of important design parameters. The base case used in this study consists of a 50 *mm* x 50 *mm* x 1.62 *mm* substrate with two signal planes and three dielectric planes as described in Table 1. A 25 *mm* x 25 *mm* heat source is concentrically located on the top surface, with 2.5 *W* of heat input directed into the substrate. A uniform convective boundary condition of 5 W / m^2K is imposed on the upper and lower exposed surfaces (not on the heat source). Since the coupon is designed to represent an isolated section of a larger PCB, the side walls of the coupon are treated as adiabatic, representing a plane of symmetry within the PCB. In each of these test cases, the overall dimensions of the test coupon, the total heat input to the source, and the volume fractions of copper and FR4 are preserved.

Modeling

Using the geometric and thermophysical properties of the test coupon given in Table 1, estimates of bulk conductivity can be determined using Eqs. 1 and 2, as shown in Table 2.

Table 1: Default Configuration for Test Coupon

Layer	**Material**	**Thickness** *(mm)*	**Conductivity** *(W / mK)*
1	FR4	0.5263	0.4
2	Copper	0.0356	400
3	FR4	0.5263	0.4
4	Copper	0.0356	400
5	FR4	0.5263	0.4

Table 2: Calculated Values of Bulk Conductivity

Method	**Bulk Conductivity** *(W / mK)*
in-plane	17.62
cross-plane	0.42
arithmetic mean	9.02
geometric mean	2.71
harmonic mean	0.82

All simulations performed in this study were conducted using an analytical model based on a general three-dimensional Fourier series solution applied to laminated substrates with arbitrarily specified boundary conditions, as detailed in Culham and Yovanovich (1997). The solution procedure provides a convenient means of calculating the total resistance to heat flow between a heat source on a multilayer stack and the fluid/solid interface over which the convective boundary conditions are applied. Once the total solid-body resistance between the heat source and the convection cooled surfaces is calculated, an iterative procedure is used to find the total solid resistance and equivalent effective conductivity for a single layer substrate with the same total thickness and boundary conditions.

The following study will examine the relationship between effective conductivity and three design variables that must be considered in the thermal analysis of printed circuit boards:

- relative location of high and low conductivity layers in relation to the heat source
- heat source size
- heat source location

Layer Placement

The five layer test coupon described in Table 1 is used as the base case for parametric study. The number and thicknesses of the copper and FR4 layers are preserved but the position of the layers in the stack are varied for the five test cases, as shown in Table 3 and Fig. 3.

Case 1 is a limiting case where both copper layers are placed directly under the heat source. In this configuration, the spreading resistance is minimized and the calculated value of effective conductivity approaches the in-plane value of conductivity. However, since the side walls are insulated, the flow of heat in the substrate turns from the in-plane direction towards the convective boundaries on the upper and lower surfaces. The added resistance associated with this turning assures that the calculated value of effective conductivity will always be less than the upper limit set by the true in-plane conductivity.

Table 3: Material and Thickness for Layer Position Study (all dimensions in *mm*)

Layer	**Case**				
	1	**2**	**3**	**4**	**5**
1	Copper 0.0356	Copper 0.0356	FR4 0.5263	FR4 0.5263	FR4 0.5263
2	Copper 0.0356	FR4 0.5263	Copper 0.0356	Copper 0.0356	FR4 0.5263
3	FR4 0.5263	FR4 0.5263	FR4 0.5263	Copper 0.0356	FR4 0.5263
4	FR4 0.5263	FR4 0.5263	Copper 0.0356	FR4 0.5263	Copper 0.0356
5	FR4 0.5263	Copper 0.0356	FR4 0.5263	–	Copper 0.0356

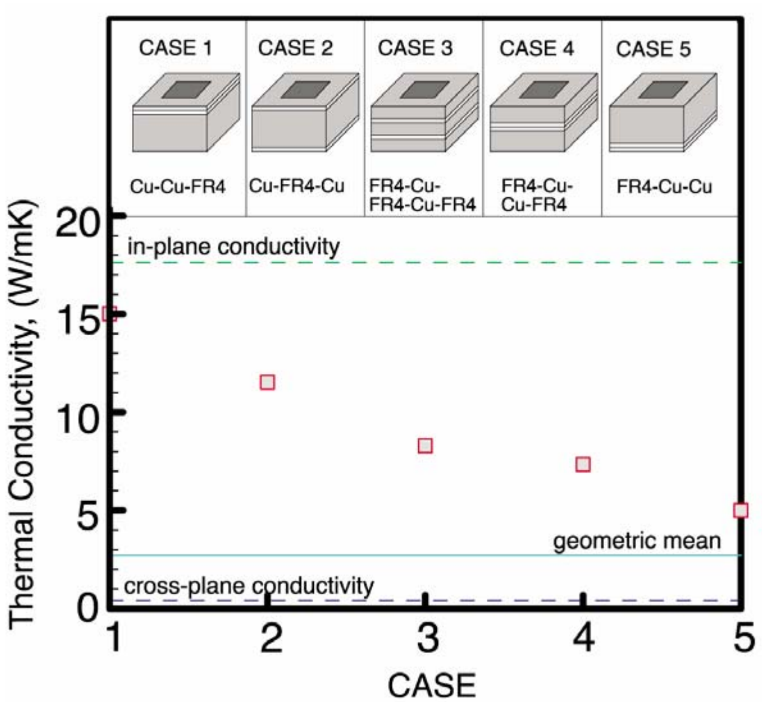

Figure 3: Effect of Layer Location on Effective Conductivity

Cases 2, 3 and 4 demonstrate configurations in which the spreading resistance plays a proportionately increasing role. The calculated values of effective conductivity lie midway between the limiting cases set by the in-plane and cross-plane conductivities. In each case the constriction to heat flow under the heat source is affected by the laminate structure in the PCB. The final example given in Case 5 with both copper layers on the bottom results in the largest spreading resistance and the lowest value of effective conductivity.

As shown in Fig. 3, the effective conductivity can vary significantly as a result of the placement of the various layers in the PCB. Neither the limiting conditions nor the averaging schemes provide a means to account for the effect of spreading resistance. For the five cases shown, the effective conductivity varies between 15.02 and 5.00 *W / mK*, more than 80% higher than the estimated effective conductivity obtained using the geometric mean. The arithmetic mean provides the most neutral estimate of effective thermal conductivity in this case but errors of up to 80% are obtained.

Source Size

The relative source area, ε = source area/board area, is varied between 0.01 and 1while preserving the laminate structure and the boundary conditions as shown in Fig.4. This study is analogous to varying the packaging density on a populated PCB to determine the effect on heat flow in the board.

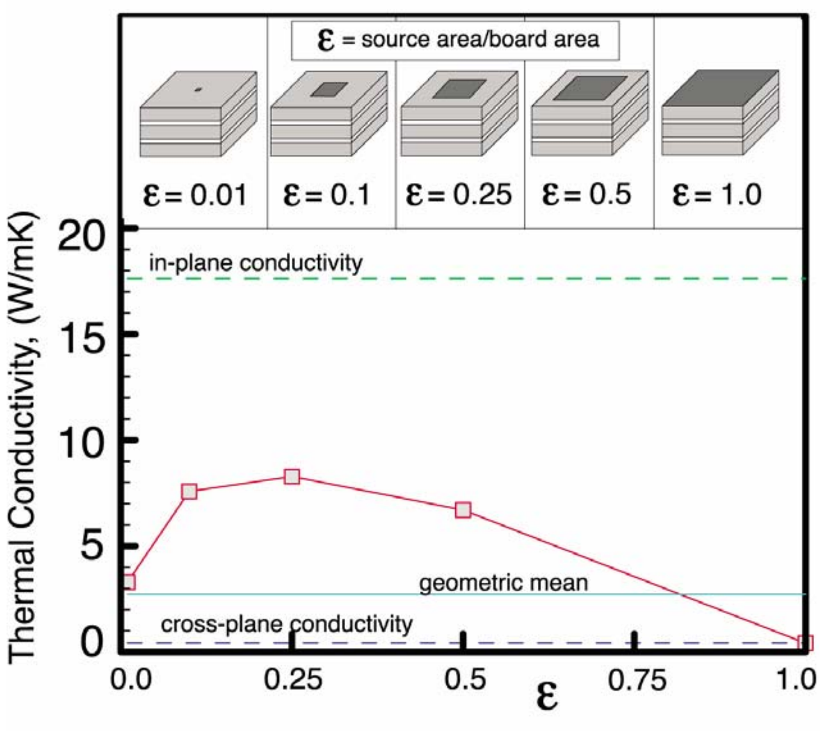

Figure 4: Effect of Heat Source Size on Effective Conductivity

For the fully populated board, $\varepsilon = 1$, there is no spreading resistance and the heat flows uniformly through each layer between the heat source and the sink. The bulk resistance of the laminate structure is based on a series path resistor network and the effective conductivity is equivalent to the cross-plane conductivity. As the relative size of the source is reduced, $\varepsilon < 1$, the exposed surface for convection cooling increases and the overall resistance tends to decrease, resulting in a higher effective conductivity. However, as the relative source area shrinks below $\varepsilon = 0.1$, spreading resistance increases significantly and the resulting effective conductivity decreases.

The variation in the effective conductivity associated with changes in relative source area is a clear demonstration of the interaction of the bulk and spreading resistances. As the source area changes, the spreading resistance is affected by changes in both the convectively cooled area and the constriction resistance near the source. The interaction of these effects leads to a specific value of ε where the effective conductivity takes on a maximum value. The use of the geometric mean of the in-plane and cross-plane conductivities in this case could result in errors of up 70% for certain values of ε.

Source Location

The previous studies all pertained to concentrically located heat sources on a test coupon. However, in some instances the heat source may not be centrally located on a substrate, thus influencing the heat flow path established between the source and the sink.

The parametric study of source location effects examined three cases, as shown in Figure 5. Case 1 is the base case example with a centrally located heat source, where the effective conductivity is calculated as 8.28 W/mK. Case 2 demonstrates a translation of the source along one axis while Case 3 is a result of a translation along two axes. The effect on effective conductivity is significant in both instances, with a 42% and 67% increase, respectively.

Preliminary inspection of the Case 2 and 3 configurations would indicate that the spreading resistance should be significantly higher than with the concentrically located heat source used in Case 1. This in itself would result in a decrease in the calculated effective conductivity for Case 2

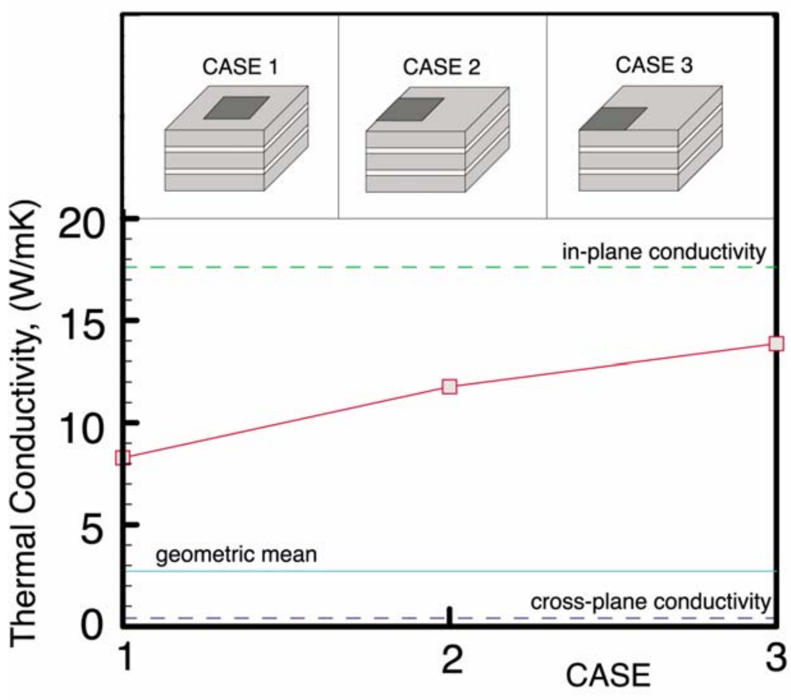

Figure 5: Effect of Heat Source Location on Effective Conductivity

and 3; however, the effective conductivity is based on the combined effect of the resistance of the solid and a ratio of the cross sectional flow area, *A*, to the flow length, *L*, as shown in Eq. 6:

$$k_e = 1 \Big/ \left(R_{solid} \cdot \frac{A}{L} \right) \tag{6}$$

When the heat flow is one-dimensional, the cross sectional flow area and the flow length are clearly defined but for a two-dimensional flow pattern these values become more difficult to quantify. Even without having the ability to quantify the length of the flow path or the cross sectional flow area, we can see that both Case 2 and 3 will provide an increase in the heat flow path between the source and the sink, which leads to a higher effective conductivity.

WEB TOOL

While it is readily apparent that the calculation of effective conductivity of a laminated circuit board must include both material bulk resistance and spreading resistance, the procedure required can be extremely complex. An analytical spreading resistance solution, as shown in Eq. 5, can be coupled with a bulk resistance solution based on in-plane and cross-plane conductivities to provide an accurate measure of the total resistance and the effective conductivity of a laminated structure. The formulations for calculating spreading resistance become progressively more complex for laminated structures with more than 2 layers. An alternate approach for the calculation of effective conductivity, as used in the calculations presented in this study, uses a three-dimensional Fourier series solution that provides a mixed boundary condition on the planar surfaces. While this approach is well documented, the programming of the solution procedure can be complex and time consuming.

These difficulties are overcome through the implementation of a dynamic, internet-based tool for calculating effective conductivity. An interactive, real-time solution tool is available for use by the general public at the Microelectronics Heat Transfer Laboratory website, *http://www.mhtl.uwaterloo.ca.* The “Effective Conductivity” calculator maintained under the ***Online Tools*** section of the Web page provides a convenient procedure for calculating effective conductivity in laminated structures. Figure 6 shows an example of the interface, detailing the calculation of effective conductivity for the default test coupon used in this study. The user is prompted for the overall dimensions and thermophysical properties of the laminated substrate. A surface mounted heat source is prescribed on one of the planar surfaces of the test coupon along with a uniformly prescribed convective coefficient along the top and bottom surfaces.

SUMMARY

The comparative study to ascertain the role of spreading resistance in the calculation of effective conductivity has revealed that both the material resistance and the spreading resistance must be accounted for if a representative, effective conductivity is to be determined for multilayer printed circuit boards with mixed boundary conditions. The in-plane or parallel path conductivity provides an upper bound on effective conductivity while the cross-plane or series path conductivity provides the lower bound. Conventional averaging schemes do not provide a good estimate for the range of design conditions examined.

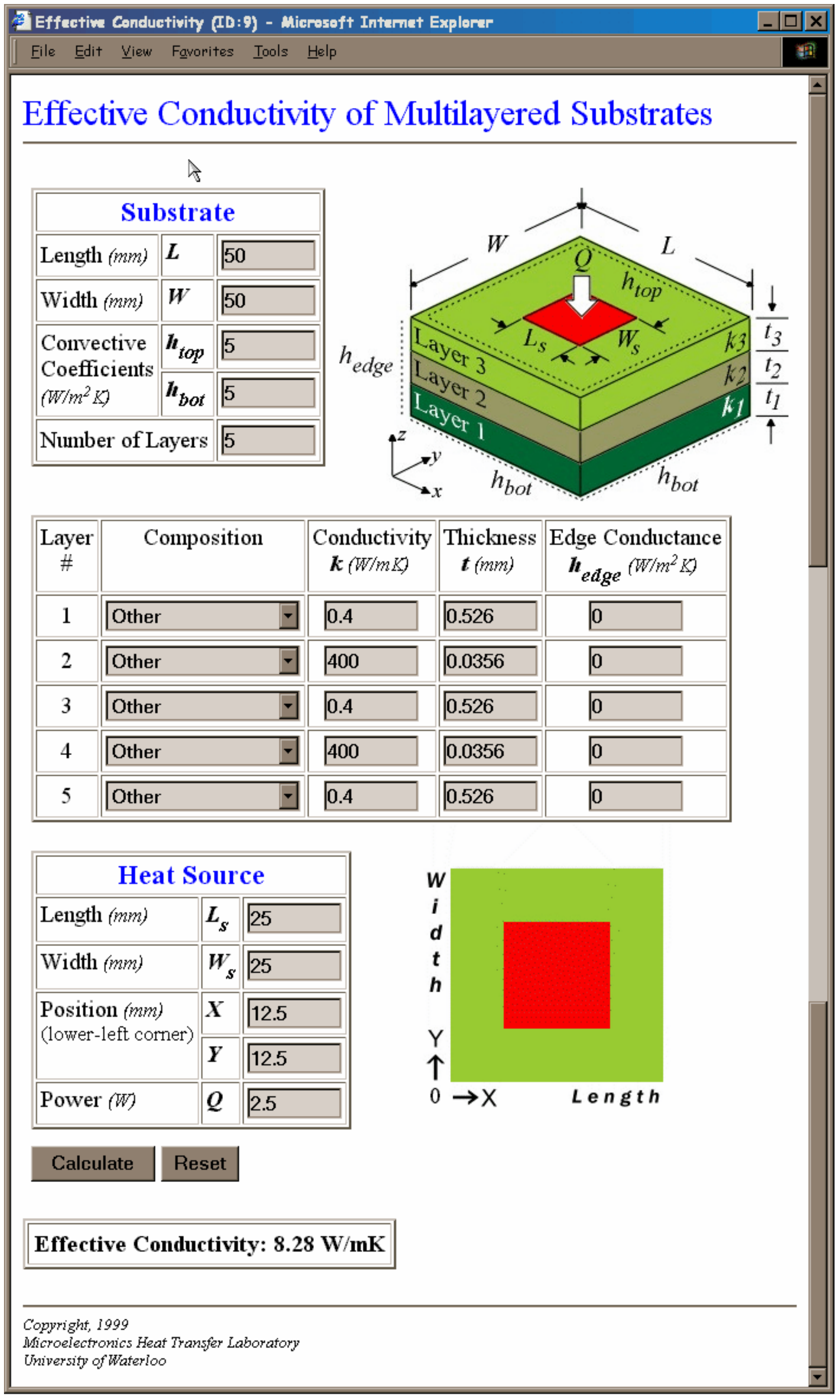

Figure 6: Internet-based Effective Conductivity Calculation Tool

Although series and parallel resistance paths are commonly used to calculate effective conductivity, these methods do not provide any mechanisms for including the effects of layer location in a multilayer stack, packaging density or heat source placement. Effective conductivity can vary by as much as 300% when the conductive signal planes are relocated from adjacent to the source of heat to the opposite boundary. The effective conductivity can range from the lower bound, i.e. the cross-plane conductivity, for a full populated board to a maximum value at roughly 25% density for the cases examined in this study. The placement of heat sources near adiabatic boundaries can have a major influence on the effective conductivity but the effect on the heat flow path length probably has a more significant impact in the calculation.

Any procedure for calculating effective conductivity that does not include both the bulk material and spreading resistance should be avoided when calculating temperature or heat flux distribution in multilayer printed circuit boards.

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BIOGRAPHIES

J. Richard Culham:

Professor Culham is an Associate Professor of Mechanical Engineering at the University of Waterloo. He is the director and a founding member of the Microelectronics Heat Transfer Laboratory. He received his doctoral degree from the University of Waterloo, with a research focus in the areas of analytical and experimental modeling of conjugate heat transfer for microelectronics applications.

Current research interests include modeling and characterization of contacting interfaces, development of analytical and empirical models at micro- and nano-scales, optimization of electronics systems using entropy generation minimization and the characterization of thermophysical properties in electronics materials.

Professor Culham has over 50 publications in refereed journals and conferences in addition to numerous technical reports related to microelectronics cooling. He is a member of ASME, IEEE and the Professional Engineers Association of Ontario. He is also a member of the ASME K16 Committee on Heat Transfer in Electronic Equipment and the Thermal Program Chair for ITHERM 2000.

Peter Teertstra:

Peter Teertstra is a research engineer who joined the Microelectronics Heat Transfer Laboratory in 1993 after receiving a Masters degree for his numerical study of steady-state and transient heat conduction for oblate spheroids with mixed boundary conditions. Peter has participated in the development of analytical heat transfer models for microelectronics applications and the programming of many of these models into web-based online tools. He has also aided in the development of heat transfer analysis software, and is responsible for numerical simulations using commercial software packages, including FLOTHERM. His research interests include analytical modeling of convection and conduction heat transfer, and numerical CFD modeling.

Peter has participated in 5 journal publications, 9 conference papers and 8 technical reports. He and his co-authors received the 1994 AIAA Thermophysics Best Paper Award for the paper "Modeling Transient Conduction from Isothermal Convex Bodies of Arbitrary Shape".

Peter is currently enrolled in a part-time Doctoral program in the Department of Mechanical Engineering at the University of Waterloo, where he is studying natural convection heat transfer in enclosures for microelectronics applications.

M. Michael Yovanovich:

Dr. Yovanovich is a Professor of Mechanical Engineering and Electrical Engineering at the University of Waterloo. He is also a founder and Director of the Microelectronics Heat Transfer Laboratory in the Department of Mechanical Engineering. His research in the field of thermal modeling includes analysis of complex heat conduction problems, natural, forced, and mixed convection heat transfer from complex geometries, and contact resistance theory and applications.

Dr. Yovanovich has published more than 250 journal and conference papers and numerous technical reports. He has been a consultant to several North American nuclear, aerospace and microelectronics industries and national laboratories.

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