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Modeling the *f* and *j* Characteristics of the Offset Strip Fin Array

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New models for predicting the thermal-hydraulic characteristics of offset strip fin arrays are developed. These models are developed by combining the asymptotic behaviour for the laminar and turbulent wake regions. Models in these two regions are developed by considering the offset strip fin as an array of short ducts or channels. The proposed models are compared with published experimental data for nineteen configurations of the rectangular offset strip fin. Model predictions are within ± 20 percent for 96 percent of friction factor data and 82 percent for Colburn j factor data. Extension of the new models for offset strip fins having non-rectangular subchannels is also discussed.

Keywords: Extended surface; Compact heat exchangers; Friction factor; Heat transfer coefficient

INTRODUCTION

Many compact heat exchangers take advantage of the offset strip fin (OSF) to provide heat transfer enhancement. This device provides enhancement by means of increased surface area along with larger heat transfer coefficients due to the short interrupted channels. In many applications the designer must optimize heat transfer for a given pumping power or mass flow rate. Optimization of a new design incorporating these devices is best achieved with analytic or empirical models.

A review of the literature reveals that the only analytic model available was developed by Joshi and Webb (1987). However, this model is very complex and requires several parameters which are presented graphically. Although the model represents the correct behavior for very small and very large values of the Reynolds number, the model is unable to predict the data which fall in the transition region. A number of empirical correlations have also been developed by Wieting (1975), Joshi and Webb (1987), and Manglik and Bergles (1990, 1995). The primary disadvantage of the multiple regression correlations is that they are based upon experimental data for 21 OSF configurations found in Kays and London (1984). Since the regression models are based on actual

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experimental data, they are only valid in the Reynolds number range typical of the experiments $200 < Re < 10000$.

With the exception of the correlations developed by Manglik and Bergles (1990, 1995), none of the models or corrleations are valid in the transition region. Later, it will be shown that the correlations of Manglik and Bergles (1990, 1995) underpredict the data for $Re < 200$ and $Re > 10000$. If a new configuration does not fall within the range of parameters for which these correlations were developed, considerable uncertainty in the results must be expected. Finally, all of the correlations developed in the literature assume that the subchannels formed by the interrupted fin are rectangular. However, in many applications the shape is not necessarily rectangular. If an offset fin array is developed from other shapes such as sinusoidal and trapezoidal cross-sections, then these correlations are generally not applicable.

A new model has been developed which overcomes the limitations of the present analytic and empirical models. This new model has been developed by considering the OSF as an array of short channels or straight ducts. Simple analytic models have been developed for the laminar wake and turbulent wake regions which accurately predict the f and j characteristics of the array. The asymptotic models are then combined using the Churchill and Usagi (1972) correlation method to provide a model which is valid in the transition region. This new model is also applicable to OSF arrays which contain non-rectangular subchannels. The new model is able to predict most OSF data in Kays and London (1984) within $+/-20$ percent for 96 percent of f data and 82 percent of j data.

REVIEW OF PUBLISHED MODELS

Manglik and Bergles (1990, 1995) provide an extensive summary of the past experimental, numerical, and analytical work involving offset strip fins. Only a few models for the offset strip fin arrangement are analytically based. The simplest of these models was first proposed by Kays (Kays and Crawford, 1993). This model treats the fin as a flat plate and does not consider the effect of the channel walls.

Kays (Kays and Crawford, 1993) proposed the following simple model based upon forced convection over a flat plate:

$$f = \frac{1.328}{\sqrt{Re_{L_f}}} + \frac{t\,C_D}{2L_f} \tag{1}$$

and

$$j = \frac{0.664}{\sqrt{Re_{L_f}}} \tag{2}$$

where $C_D \approx 0.88$ is the drag coefficient for a flat plate based upon the potential flow solution (Milne-Thomson, 1968) and L_f is the fin length. The potential flow solution for the drag coefficient is also in good agreement with experimental data for rectangular plates having small t/L_f ratios (Blevins, 1984). These models only consider heat transfer and friction from the fin surfaces and do not consider the contributions from the channel walls. Equations (1) and (2) are only valid in the laminar region and were proposed for comparative purposes only.

Wieting (1975) developed multiple regression correlations based upon data from 21 offset strip fin configurations. Wieting (1975) presented correlations for both the laminar and turbulent regions and also developed correlations for the critical Reynolds number. The correlations for the laminar and turbulent regions are presented below:

Laminar $Re_{d_h} \le 1000$

$$f = 7.661(L_f/d_h)^{-0.384}(s/H)^{-0.092}Re_{d_h}^{-0.712} \tag{3}$$

$$j = 0.483(L_f/d_h)^{-0.162}(s/H)^{-0.184}Re_{d_h}^{-0.536} \tag{4}$$

Turbulent $Re_{d_h} \ge 2000$

$$f = 1.136(L_f/d_h)^{-0.781}(t/d_h)^{0.534}Re_{d_h}^{-0.198} \tag{5}$$

$$j = 0.242(L_f/d_h)^{-0.322}(t/d_h)^{0.089}Re_{d_h}^{-0.368} \tag{6}$$

The critical Reynolds numbers for the intersection of the laminar and turbulent asymptotes for the f and j factors are:

$$Re^*_{d_h f} = 41(L_f/d_h)^{0.772}(s/H)^{-0.179}(t/d_h)^{-1.04} \quad (7)$$

$$Re^*_{d_h j} = 61.9(L_f/d_h)^{0.952}(s/H)^{-1.1}(t/d_h)^{-0.53} \quad (8)$$

In addition to developing an analytic model, Joshi and Webb (1987) also developed multiple regression correlations for the laminar and turbulent regions. The correlations for the laminar and turbulent regions are:

Laminar $Re_{d_h} \leq Re^*_{d_h}$

$$f = 8.12(L_f/d_h)^{-0.41}(s/H)^{-0.02}Re_{d_h}^{-0.74} \quad (9)$$

$$j = 0.53(L_f/d_h)^{-0.15}(s/H)^{-0.14}Re_{d_h}^{-0.5} \quad (10)$$

Turbulent $Re_{d_h} \geq Re^*_{d_h} + 1000$

$$f = 1.12(L_f/d_h)^{-0.65}(t/d_h)^{0.17}Re_{d_h}^{-0.36} \quad (11)$$

$$j = 0.21(L_f/d_h)^{-0.24}(t/d_h)^{0.02}Re_{d_h}^{-0.40} \quad (12)$$

The value of the critical Reynolds number $Re^*_{d_h}$ is determined from the following expression

$$Re^*_{d_h} = 257\left(\frac{L_f}{s}\right)^{1.23}\left(\frac{t}{L_f}\right)^{0.58}\left(\frac{d_h}{t + (1.328L_f/Re_{L_f}^{1/2})}\right) \quad (13)$$

Finally, Manglik and Bergles (1990, 1995) also developed correlations based upon multiple regression analysis of available data. Their correlations differ from those of Wieting (1975) and Joshi and Webb (1987) in the definition of the hydraulic diameter as well as providing better geometric correlation. Correlations for the laminar and turbulent regions are:

Laminar $Re_{d_h} \leq Re^*_{d_h}$

$$f = 9.624(s/H)^{-0.186}(t/L_f)^{0.305}(t/s)^{-0.266}Re_{d_h}^{-0.742} \quad (14)$$

$$j = 0.652(s/H)^{-0.154}(t/L_f)^{0.150}(t/s)^{-0.068}Re_{d_h}^{-0.540} \quad (15)$$

Turbulent $Re_{d_h} \geq Re^*_{d_h} + 1000$

$$f = 1.870(s/H)^{-0.094}(t/L_f)^{0.682}(t/s)^{-0.242}Re_{d_h}^{-0.299} \quad (16)$$

$$j = 0.244(s/H)^{-0.104}(t/L_f)^{0.196}(t/s)^{-0.173}Re_{d_h}^{-0.406} \quad (17)$$

The value of the critical Reynolds number $Re^*_{d_h}$ is defined by the expression developed by Joshi and Webb (1987), given earlier. In addition, Manglik and Bergles (1990, 1995) also combined the laminar and turbulent correlations using the method proposed by Churchill and Usagi (1972) to provide a correlation which is valid in the transition region. The resulting correlations which include the transition region are:

$$f = 9.624(s/H)^{-0.186}(t/L_f)^{0.305}(t/s)^{-0.266}Re_{d_h}^{-0.742}[1 + 7.669 \times 10^{-8}(s/H)^{0.92}(t/L_f)^{3.77}(t/s)^{0.236}Re_{d_h}^{4.43}]^{0.1} \quad (18)$$

$$j = 0.652(s/H)^{-0.154}(t/L_f)^{0.150}(t/s)^{-0.068}Re_{d_h}^{-0.540}[1 + 5.63 \times 10^{-5}(s/H)^{0.51}(t/L_f)^{0.46}(t/s)^{-0.106}Re_{d_h}^{1.34}]^{0.1} \quad (19)$$

Equations (18) and (19) are now widely accepted for modelling the characteristics of the offset strip fin array. Since these correlations are the only ones capable of predicting data in the transition region, they will be compared with the new models developed in this work. In all of the above

TABLE I Definitions of hydraulic diameter for offset strip fin models

Model	Definition of d_h	
Wieting (1975)	$(2sH/(s+H))$	(i)
Joshi and Webb (1987)	$(2(s-t)HL_f/(sL_f+HL_f+tH))$	(ii)
Manglik and Bergles (1990)	$(4sHL_f/(2(sL_f+HL_f+tH)+ts))$	(iii)

correlations a different definition for d_h has been chosen. A summary of these definitions is given in Table I. The present work adopts the definition of the hydraulic diameter proposed by Manglik and Bergles (1990), since it is consistent with the definition for complex channels and ducts, *i.e.*, $d_h = 4V_{\text{free}}/A_{\text{wet}}$.

Examination of the correlations developed by Wieting (1975), Joshi and Webb (1987) and Manglik and Bergles (1990, 1995) reveals that the $Re \to 0$ and $Re \to \infty$ behaviour departs from the theoretical behaviour. In the low Reynolds or laminar region, the correlations predict that as $Re \to 0$, f and j, are approximately proportional to $1/Re^{3/4}$ and $1/Re^{1/2}$, respectively. In this limit, the results should be proportional to $1/Re$ which is characteristic of fully developed duct flow. In the high Reynolds or turbulent region, the correlations predict that as $Re \to \infty$, f and j, are approximately proportional to $1/Re^{3/10}$ and $1/Re^{2/5}$, respectively. In this limit the results for f should have a Reynolds number exponent between $-1/5$ and 0 depending upon the effects fin edge thickness, and $-1/5$ for j. As a result, f and j predictions using the above correlations will be lower than expected values, outside of the $200 < Re < 10000$ range. This is due to the empirical nature of the correlation procedure. The Joshi and Webb (1987) model overcomes these limitations since it is analytically derived, however, due to its complexity and its inability to predict data in the transition region, a new model is proposed in the sections which follow.

MODEL DEVELOPEMENT

In this section the details of the model development for the OSF geometry are discussed. A

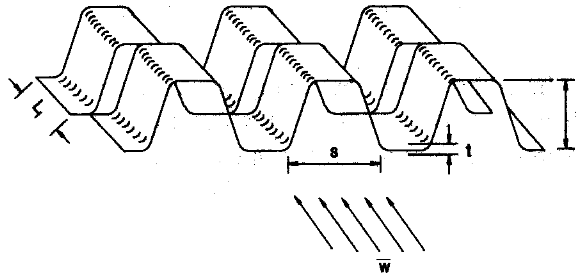

FIGURE 1 OSF Geometry.

typical OSF type geometry is shown in Figure 1. The OSF is characterized by the fin length L_f, the channel height H, the fin spacing s, and the fin thickness t. The new model will be compared with data obtained from Kays and London (1984) for nineteen configurations of the rectangular offset strip fin array. The vast majority of these data were first published by Kays (1960), Briggs and London (1961), and London and Shah (1968).

The following assumptions are made in the model development: ideal surfaces *i.e.*, no burrs or scarfed edges, uniform surface dimensions throughout the array, complete destruction of boundary layers in the wake, negligible edge contributions, perfect contact at channel walls, and isothermal surfaces. The presence of burrs in real systems cannot be ignored, and the experimental data of Kays and London (1984) are likely to be affected to some degree by the presence of burrs.

FRICTION FACTOR

Since the OSF is essentially an array of short rectangular (or other non-noncircular) ducts, the friction factor should possess characteristic behaviour of duct flow at low Reynolds numbers. In the limit $Re_{d_h} \to 0$ the value of f should approach the value for fully developed flow in a rectangular channel. In the limit of $Re_{d_h} \to \infty$ the value of f should approach a constant value which is representative of the form drag component due to the finite fin thickness. Webb and Joshi (1982) developed a simple model by combining these asymptotic limits using the Churchill and Usagi (1972) correlation method. However, this model was only valid for OSF arrays having a channel aspect ratio $s/H < 0.25$. At larger values of the channel aspect ratio $s/H > 0.25$ the model provides poor correlation of the available data. The present model overcomes the deficiencies of the Webb and Joshi (1982) model by including additional terms in the laminar and turbulent asymptotes which capture the true physical behavior of the OSF array.

Laminar Region

In the laminar region, the flow field develops within each subchannel much the same as it does in a plain channel or duct. A boundary layer is initiated on the subchannel walls and begins to grow. Depending upon the length of the subchannel, the flow may eventually become fully developed or remain partially developed when it leaves the subchannel. This suggests that an analytic result obtained by Shapiro *et al.* (1954) for the hydrodynamic entrance region may be used to model the flow. The simple result of Shapiro *et al.* (1954) for the apparent friction factor in the hydrodynamic entrance region of straight ducts is

$$f_{\text{app}} = \frac{3.44}{\sqrt{z^+}} \tag{20}$$

where $z^+ = L_f/(D_h Re_{D_h})$ is the dimensionless subchannel length, and D_h is the hydraulic diameter of the subchannel, *i.e.*, $D_h = 4A_c/P$, where A_c is the free flow cross-sectional area of the subchannel and P is the wetted perimeter of the subchannel.

Figure 2 provides a comparison of a typical set of data obtained from Kays and London (1984) with several laminar and turbulent flow solutions. Comparison of the hydrodynamic entrance solution obtained by Shapiro *et al.* (1954), Eq. (20), with OSF data from Kays and London (1984) shows that this solution overpredicts the friction factor. This overprediction is due to the fact that the entrance region solution not only accounts for the wall shear but also the increase in momentum of the inviscid core. The solution of Shapiro *et al.* (1954) also assumes a uniform entrance velocity distribution which is not present in the OSF application, (refer Fig. 3). Kays (Kays and Crawford, 1993) suggested better correlation is achieved by modelling the fin surface as a flat plate and accounting for the form drag component due to the finite edge thickness. However, this approach did not account for the low Reynolds number behavior which is apparent from Figure 2.

The proposed laminar region model is taken to be a linear superposition of the Blasius solution for a flat plate and the fully developed friction factor for a rectangular duct (Muzychka, 1999),

$$f_{\text{lam}} = \frac{(fRe_{D_h}(d_h/D_h))}{Re_{d_h}} + 1.328\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/2} \tag{21}$$

where fRe_{D_h} is the fully developed friction factor Reynolds number for the rectangular subchannel, D_h is the hydraulic diameter of the rectangular subchannel, d_h is the hydraulic diameter of the OSF array, and L_f is the fin length in the subchannel. The above model represents the correct physical behavior for $Re_{d_h} \to 0$ or $L_f > 0.05 D_h Re_{D_h}$ and as Re_{d_h} increases or L_f decreases.

The proposed model may be interpreted in the following way. Upon leaving the subchannel the flow divides and enters two new subchannels. At this point, the velocity entering the new subchannel is zero at the centerline due to wall shear in the previous subchannel, (refer to Fig. 3). Since the centerline velocity is zero, the inlet velocity distribution is not uniform, which is a requirement for the hydrodynamically developing flow solution, Eq. (20). As the flow begins to develop in the new subchannel, the velocity in the core region is much less than that which would occur if the inlet

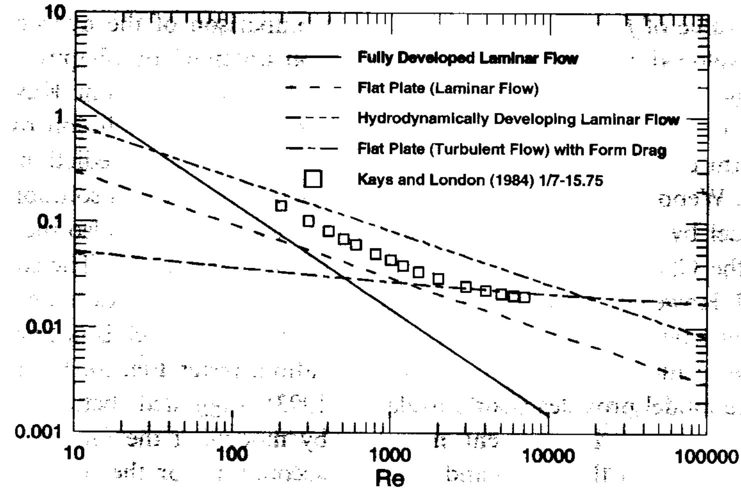

FIGURE 2 Comparison of asymptotic solutions with data of Kays and London (1984).

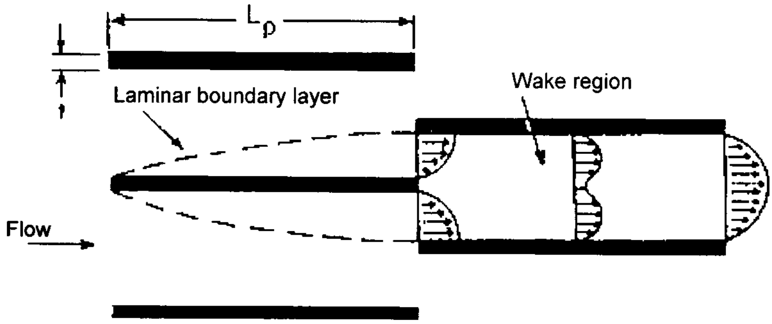

FIGURE 3 Velocity distribution in an OSF array, (Webb and Joshi, 1982).

distribution were uniform. As a result of this lag, the boundary layer development is not affected by the inviscid core in the same way, and is similar to boundary layer development over a flat plate. In Shapiro *et al.* (1954) the authors point out that in the entrance region of a duct very near the inlet, boundary layer growth is very similar to that of a flat plate. However, as the core begins to accelerate, the boundary layer is unable to develop at the same rate as that for an isolated flat plate, since it is affected by the accelerating core.

Turbulent Region

The turbulent or non-laminar region may be modelled in a similar manner as done for the laminar region. The non-laminar region is generally characterized as having a turbulent wake (Joshi and Webb, 1987). In this region, the boundary layers which are formed are predominantly laminar but due to the wake effect, the behavior of the OSF is similar to that of a turbulent boundary layer. Performing a force balance on a subchannel results in the following expression (Kays and Crawford, 1993):

$$\frac{\bar{\tau}}{(1/2)\rho\bar{w}^2} = \frac{\tau_{\text{wall}}}{(1/2)\rho\bar{w}^2}\left(\frac{A_{\text{wall}}}{A_{\text{wet}}}\right) + C_D\left(\frac{A_{\text{profile}}}{A_{\text{wet}}}\right) \tag{22}$$

where $A_{\text{wall}} = 2L_f(H+s)$, $A_{\text{wet}} \approx 2L_f(H+s)$, and $A_{\text{profile}} = Ht + st/2$.

This expression may be written in terms of the turbulent boundary layer friction factor and drag coefficient given earlier. Equation (22) may be further simplified by assuming that $A_{\text{wall}}/A_{\text{wet}} \approx 1$ since the contribution of fin thickness to the total surface area, A_{wet}, is usually small, $< 5\%$, to give

$$f_{\text{tur}} = 0.074\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/5} + \frac{(Ht + st/2)}{2L_f(H+s)}C_D \quad (23)$$

where H is the channel height, s is the width of the sub-channel, t is the thickness of the fin, and C_D is the form drag coefficient taken to be $C_D = 0.88$. Figure 2 shows the turbulent friction factor asymptote along with data for a typical OSF array from Kays and London (1984). The data are in good agreement with the above expression in the region defined by $Re_{d_h} > 3000$.

Transition Region

A general model which predicts the friction factor over the entire range of Reynolds numbers is developed by combining the laminar and turbulent friction factors using the Churchill and Usagi (1972) correlation method.

The proposed model which is valid for the laminar-transition-turbulent region is given by

$$f = \left[\left\{\frac{(fRe_{D_h}(d_h/D_h))}{Re_{d_h}} + 1.328\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/2}\right\}^n + \left\{0.074\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/5} + \frac{(Ht+st/2)}{2L_f(H+s)}C_D\right\}^n\right]^{1/n} \quad (24)$$

where n is the correlation parameter. Values for n which minimize the RMS percent difference between predicted and experimental results for each data set have been found to vary between $1.3 < n < 5$. A single value of $n \approx 3$ provides excellent correlation for all 19 data sets from Kays and London (1984). This results in an RMS error of 11.64 percent for all of the friction factor data. Joshi and Webb (1987) reported an RMS percent difference of 16.8 percent for all friction factor data. Comparison of the proposed model with data from Kays and London (1984) and the correlations developed by Manglik and Bergles (1990, 1995) will be provided after the development of a model for the analogous Colburn j factor.

COLBURN j FACTOR

The Colburn j factor model is developed in essentially the same manner as the Fanning friction factor model. Since the OSF is essentially an array of short rectangular (or non-noncircular) ducts, the Colburn j factor should possess characteristic behaviour of duct flow at low Reynolds numbers. In the limit $Re_{d_h} \to 0$ the value of j should approach the value for fully developed flow in a rectangular channel. In the limit $Re_{d_h} \to \infty$ the value of j should approach a value typical of turbulent boundary layer flow.

Laminar Region

In the laminar region, the Colburn j factor should possess characteristics of laminar duct flow. Kays (Kays and Crawford, 1993) suggested that the OSF can be modelled as a series of flat plates and that the j factor can be predicted by the analytical solution for a flat plate given by

$$j = 0.664(Re_{L_f})^{-1/2} \quad (25)$$

where L_f is the fin length.

However, this relation tends to overpredict the data of Kays and London (1984), as shown in Figure 4, and it has been suggested by Shah (1985), that better agreement with the data for OSF arrays is obtained using thermally developing flow solutions, such as the generalized Leveque type

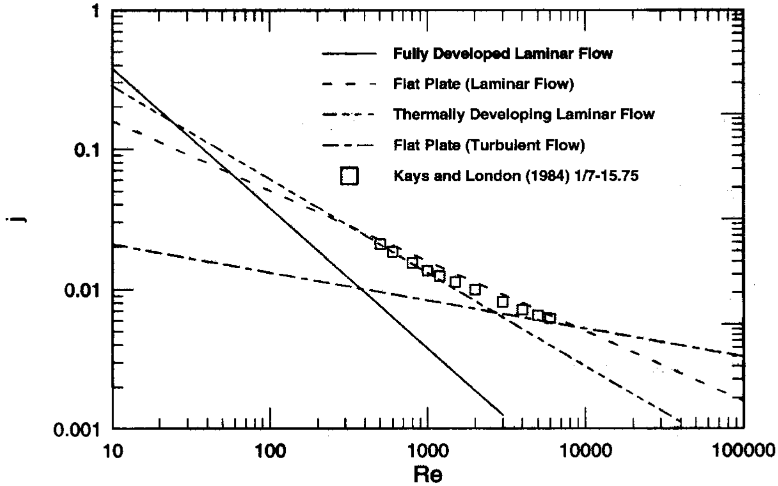

FIGURE 4 Comparison of asymptotic solutions with data of Kays and London (1984).

solution proposed by Shah and London (1978):

$$j = 0.641\left[\frac{f\,Re_{D_h}}{Re_{D_h}^2}\frac{D_h}{L_f}\right]^{1/3} \tag{26}$$

where $f\,Re_{D_h}$ is the fully developed friction factor Reynolds number group for the subchannel. This result implies that a fully developed hydrodynamic boundary layer exists, whereas the flat plate model assumes that both hydrodynamic and thermal boundary layers develop simultaneously. In the laminar region the thermally developing flow solution underpredicts the data of Kays and London (1984). Both the flat plate model and the thermally developing flow asymptotic solutions are shown in Figure 4 along with the turbulent boundary layer model and the fully developed laminar duct flow limit which is satisfied in the limit $Re_{d_h} \to 0$. It is clear from Figure 4 that both the fully developed laminar flow asymptote and the thermally developing flow asymptote characterize the laminar flow region.

If both the low Reynolds number limit $Re_{d_h} \to 0$ and the thermally developing flow asymptote are taken into consideration, then the model developed by Muzychka and Yovanovich (1998) for thermally developing flows in non-circular ducts gives:

$$j_{\mathrm{lam}} = \left[\left(\frac{Nu_{D_h}(d_h/D_h)}{Re_{d_h}Pr^{1/3}}\right)^5 + \left(0.641\frac{f\,Re_{D_h}^{1/3}}{Re_{d_h}^{2/3}}\left(\frac{d_h^2}{D_hL_f}\right)^{1/3}\right)^5\right]^{1/5} \tag{27}$$

where $f\,Re_{D_h}$ is the fully developed friction factor Reynolds number for the subchannel, Nu_{D_h} is the fully developed Nusselt number for the subchannel, D_h is the hydraulic diameter of the subchannel, d_h is the hydraulic diameter of the array, and Pr is the Prandtl number. The above model represents the correct physical behavior for $Re_{d_h} \to 0$ or $L_f > 0.05D_hRe_{D_h}$ and represents the correct physical behavior as Re_{d_h} increases or L_f decreases. Equation (27) also reveals that for low Reynolds number flows, Prandtl number effects begin to emerge.

The proposed model may be interpreted in the following way. The data show that for $Re_{d_h} < 1000$ the flow is laminar. As the Reynolds number is decreased, the flow becomes more hydrodynamically developed. A typical OSF array has a fin length which is on the order of $1-10$ mm, while the subchannel has a hydraulic diameter on the order of $2-3$ mm. Using the approximate expression for predicting entry lengths, $L_{hy} \approx 0.05 D_h Re_{D_h}$, suggests that for most arrays the flow becomes hydrodynamically developed for $Re_{D_h} < 100$. This suggests that if $Re_{D_h} < 100$, the thermal boundary layer is more likely to be in a state of thermally developing flow. At $Re_{D_h} > 100$ the flow is more likely to be in a state of simultaneously developing flow and thus the flat plate model is more applicable. However, examination of Figure 4 shows that the results for Eqs. (25) and (26) are very close to each other in the region $100 < Re_{D_h} < 1000$ and both may predict results within 20 percent. Thus a model composed of the fully developed and thermally developed limits is more appropriate than one composed of the fully developed limit and the flat plate limit.

Turbulent Region

The turbulent or non-laminar region may be modelled in a similar manner as done for the friction factor. The non-laminar region is generally characterized as having a turbulent wake. In this region, the boundary layers which are formed are predominantly laminar but due to the wake effect, the behavior of the OSF is similar to that of a turbulent boundary layer. In the turbulent region the Colburn j factor is modelled using the Reynolds analogy $j = f/2$. Using the turbulent skin friction relation presented earlier gives:

$$j_{\text{tur}} = 0.037\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/5} \tag{28}$$

where L_f is the fin length and d_h is the hydraulic diameter of the array. The turbulent model is shown in Figure 4 along with data from Kays and London (1984). In the region defined by $Re_{d_h} > 5000$, the data agree well with Eq. (28).

Transition Region

A general model which predicts the Colburn j factor over the entire range of Reynolds numbers is developed by combining the the laminar and turbulent models using the Churchill and Usagi (1972) correlation method.

The resulting model which is valid for the laminar-transition-turbulent region is given by

$$j = \left[\left\{\left(\frac{Nu_{D_h}(d_h/D_h)}{Re_{d_h}Pr^{1/3}}\right)^5 + \left(\frac{0.641 f\, Re_{D_h}^{1/3}}{Re_{d_h}^{2/3}}\left(\frac{d_h^2}{D_h L_f}\right)^{1/3}\right)^5\right\}^{m/5} + \left\{0.037\left(Re_{d_h}\frac{L_f}{d_h}\right)^{-1/5}\right\}^m\right]^{1/m} \tag{29}$$

where m is the correlation parameter. Values for m which minimize the RMS percent difference between predicted and experimental results have been found to vary between $2 < m < 5$. A single value of $m \approx 7/2$ provides excellent correlation for 19 data sets from Kays and London (1984). This results in an RMS error of 14.7 percent for all of the Colburn j factor data. Joshi and Webb (1987) reported an RMS percent difference of 11.5 percent for all Colburn j factor data. Comparisons of the proposed model with the data from Kays and London (1984) and the correlations developed by Manglik and Bergles (1995) are given in the next section.

COMPARISON OF MODELS WITH DATA

The proposed models are now compared with nineteen sets of data for the OSF configuration which are available in tabular and graphical form in Kays and London (1984). Table II presents a summary of the optimal value of the correlation parameters for combining the laminar and turbulent models for each OSF data set. The values reported in Table II result in the root mean square (RMS) percent difference being minimized. Excellent correlation is obtained for the friction

TABLE II Comparison of models with data with optimal value of blending parameter

	f		j	
Designation	n	RMS	m	RMS
1/8-15.61	1.8	7.13	3.2	9.17
1/8-19.86	2.2	3.09	2.9	10.79
1/9-22.68	2.3	4.28	4.8	8.28
1/9-24.12	1.7	4.42	3.6	17.76
1/9-25.01	1.5	5.62	2.5	5.43
1/10-19.35	5.0	8.11	3.6	10.44
1/10-19.74	1.9	7.98	4.6	26.35
1/10-27.03	1.5	6.70	2.0	1.83
3/32-12.22	1.3	4.34	4.7	21.19
1/2-11.94 D	5.0	3.28	4.1	1.83
1/4-15.40 D	5.0	4.68	5.0	12.77
1/6-12.18 D	4.7	6.64	4.5	19.03
1/7-15.75 D	5.0	6.03	3.5	2.09
1/8-16.00 D	2.9	1.71	2.2	4.85
1/8-16.12 D	2.9	1.26	4.8	15.14
1/8-19.82 D	1.3	5.62	2.3	2.89
1/8-20.06 D	2.3	3.72	2.6	4.17
1/8-16.12 T	1.8	6.76	3.8	13.03
501 MOD	3.6	4.71	4.8	17.79

factor data for most configurations. Examination of the data from Kays and London (1984) shows that the 1/8-15.61, 1/9-24.12, 1/9-25.01, 1/10-19.74, and 1/8-19.82 *D* devices show possible effects of burred edges at high Reynolds numbers, Muzychka (1999). If the fin edges are burred, an

TABLE III Comparison of models with data with fixed value of blending parameter

	f(Eq. (24), n = 3)		j(Eq. (29), m = 7/2)	
Designation	RMS	(min/max)	RMS	(min/max)
1/8-15.61	8.89	0.47/18.33	9.41	−13.90/11.95
1/8-19.86	4.61	2.96/7.35	10.63	−18.33/11.97
1/9-22.68	4.10	−1.11/7.48	9.49	−13.47/0.14
1/9-24.12	10.19	4.25/19.28	17.92	−29.06/10.87
1/9-25.01	13.33	9.17/22.52	8.17	−4.20/16.69
1/10-19.35	15.41	−19.54/−5.38	10.60	−19.73/7.12
1/10-19.74	9.92	−2.71/24.34	28.26	−45.35/2.50
1/10-27.03	13.58	8.76/18.30	10.58	6.29/16.07
3/32-12.22	16.65	12.13/20.11	22.60	−50.49/−2.03
1/2-11.94 D	10.85	−14.51/−2.45	2.40	−3.64/4.29
1/4-15.40 D	13.18	−16.41/−4.19	15.28	−24.10/0.10
1/6-12.18 D	13.87	−18.22/1.15	20.17	−39.46/−1.54
1/7-15.75 D	15.43	−20.81/−2.74	2.10	−2.53/4.25
1/8-16.00 D	4.06	−6.61/4.48	10.35	0.48/15.68
1/8-16.12 D	1.53	−2.13/−0.43	17.05	−24.87/−0.92
1/8-19.82 D	20.56	13.29/30.77	8.91	2.50/14.61
1/8-20.06 D	3.34	−0.38/7.30	7.16	−0.35/13.16
1/8-16.12 T	9.60	3.77/15.64	13.53	−21.55/5.11
501 MOD	4.76	−7.31/5.29	19.14	−29.57/8.14

effective increase in the fin thickness will result in higher form drag contributions.

Good agreement between the proposed model and the OSF data is also achieved for all but five data sets for the Colburn *j* factor. Examination of the data from Kays and London (1984) shows that for the 1/9-24.12, 1/10-19.74, 3/32-12.22, 1/6-12.18 *D*, and 501 MOD devices, the model overpredicts at low values of the Reynolds number. At higher values of the Reynolds number, the data agree well with the model. This discrepancy may be due to experimental error. The experimental data provided in Kays and London (1984) were obtained using air ($Pr \approx 0.71$) as a test fluid. At low velocities, tests conducted with air may experience a phenomena referred to as "rollover", (Shah, 1985). This phenomena results in measurements for the *j* factor which are lower than expected. Other factors, as outlined by Shah (1985), which may explain the trends in these data, are the effect of passage to passage non-uniformity and the presence of burrs.

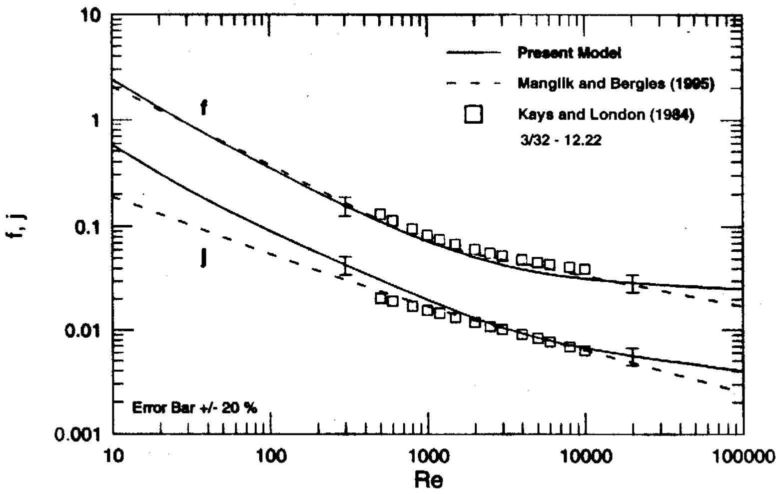

FIGURE 5 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 3/32-12.22.

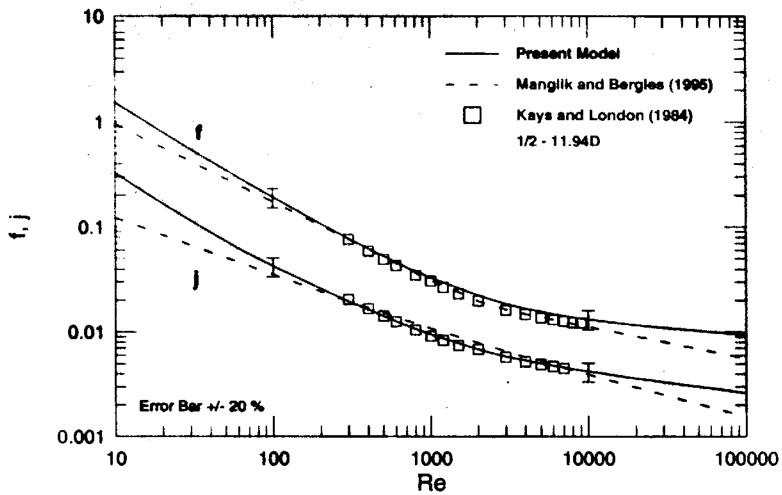

FIGURE 6 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/2-11.94D.

Table III presents the RMS and (min/max) values of the percent differences for the case where a fixed value of the correlation parameter is used for all cases. The majority of the friction factor data are predicted within ± 20 percent. Five data sets for the Colburn j factor data have predictions exceeding ± 30 percent. In these five cases the large errors only occur in the region with $Re < 1000$, as shown in Figure 5. Figures 5–14 compare the proposed models, Eqs. (24) and (29), with the correlations developed by Manglik and Bergles (1995) with ten sets of data from Kays and London (1984). Further comparisons may be found in Muzychka (1999). The trends exihibited by these plots is typical of the remaining data from Kays and London (1984). In almost all cases the correlations of Manglik and Bergles (1995) underpredict in regions of small and large Reynolds numbers. Since the correlations are empirical fits, they are only valid within the range of the

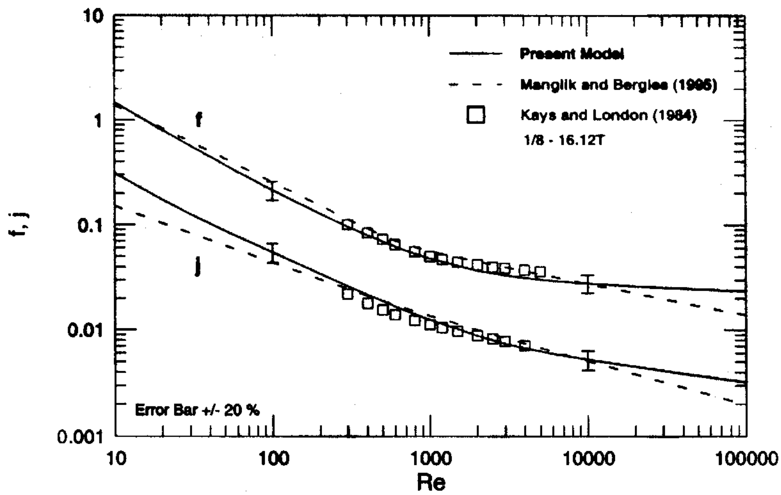

FIGURE 7 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/8-16.12T.

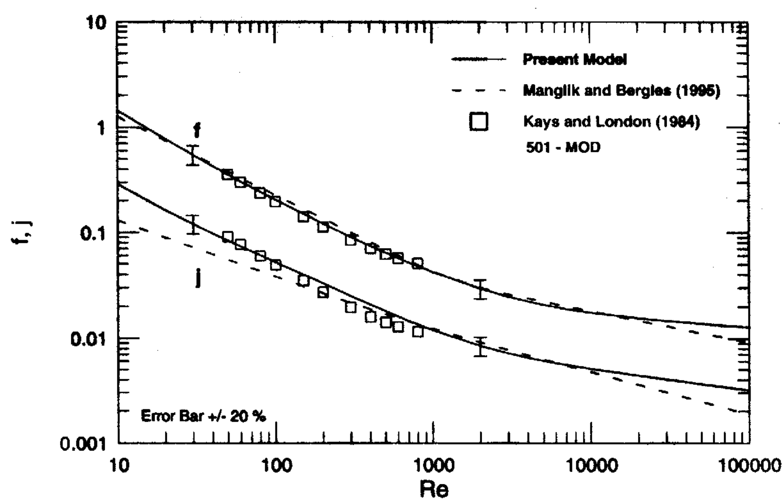

FIGURE 8 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 501-MOD.

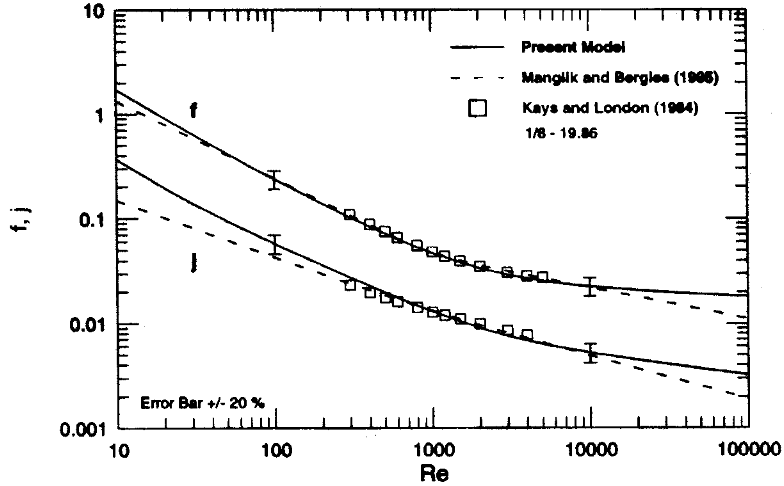

FIGURE 9 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/8-19.86.

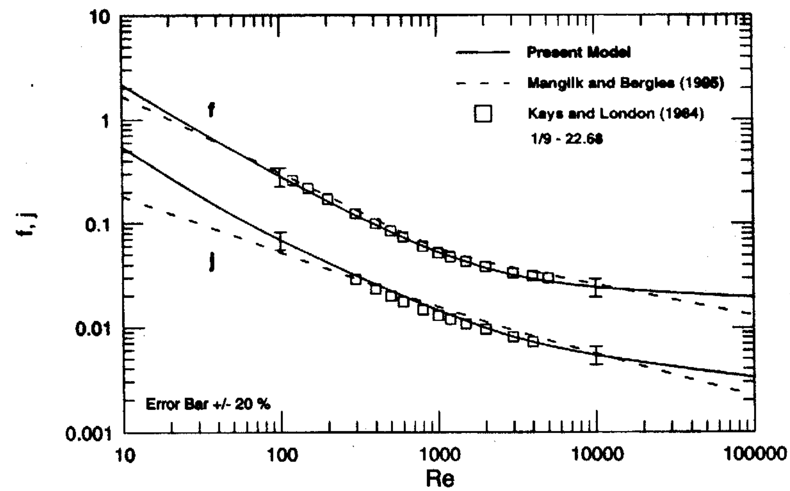

FIGURE 10 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/9-22.68.

experimental data from which they were developed. In a number of cases the empirical correlations of Manglik and Bergles (1995) provide poor correlation over the range of the available data. It is clear from Figures 5 – 14 that the proposed model captures the correct physical behavior of the data. With the exception of five j data sets and two f data sets, in which the model either overpredicts or underpredicts the data, excellent agreement between the proposed model and experimental data is achieved over the entire range of Reynolds numbers.

EFFECT OF SUBCHANNEL SHAPE

Most OSF arrays are composed of rectangular sub-channels. However, in many applications, other shapes such as trapezoidal and sinusoidal may arise. The models developed for both the

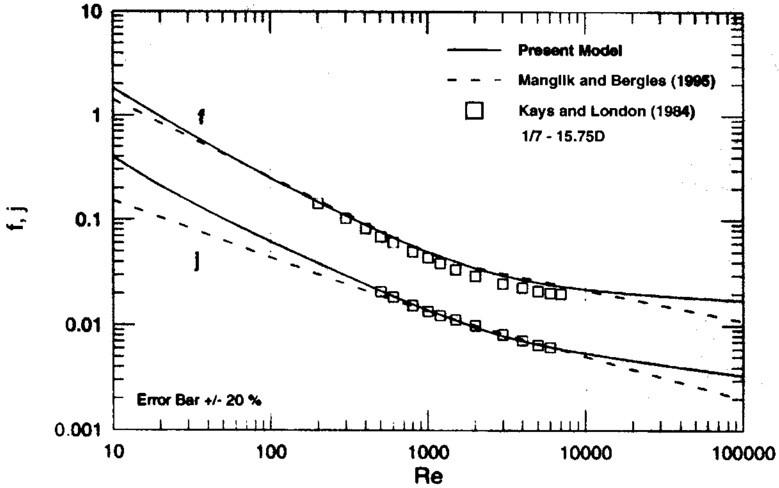

FIGURE 11 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/7-15.75D.

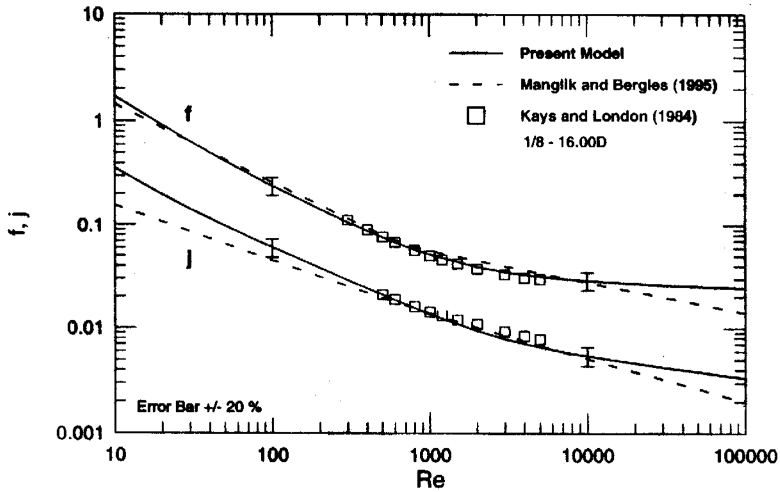

FIGURE 12 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/8-16.00D.

friction factor and the Colburn j factor may be applied to the these and other non-circular passage shapes, by simply providing the appropriate value for the fRe_{fd} and Nu_{fd} which appear in the models.

For a rectangular offset strip fin array the values for fRe_{D_h} and Nu_{D_h} are given by following results for fully developed flow in rectangular ducts, Shah (1985):

$$fRe_{D_h} = 23.94 - 30.05\varepsilon + 32.37\varepsilon^2 - 12.08\varepsilon^3 \quad (30)$$

and

$$Nu_{D_h} = 7.45 - 16.9\varepsilon + 22.1\varepsilon^2 - 9.75\varepsilon^3 \quad (31)$$

where $0 < \varepsilon = s/H < 1$ for a rectangular subchannel. If subchannel is a geometry other than rectangular, fRe and Nu should be replaced by appropriate values for the particular geometry.

These two dimensionless parameters can be accurately predicted using the models developed by

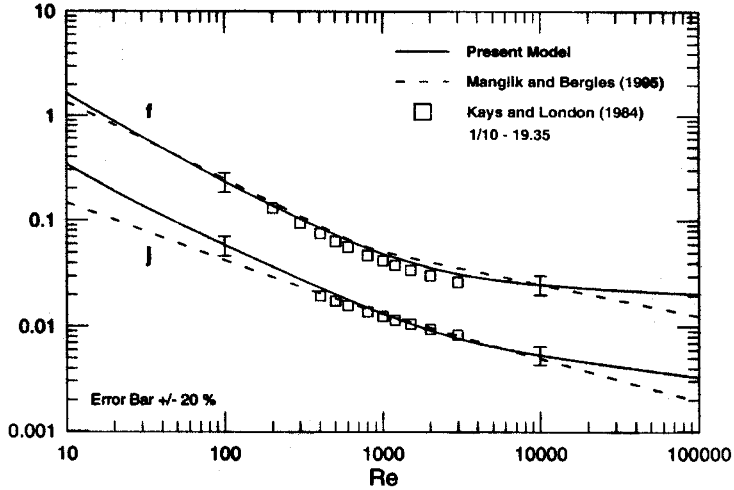

FIGURE 13 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/10-19.35.

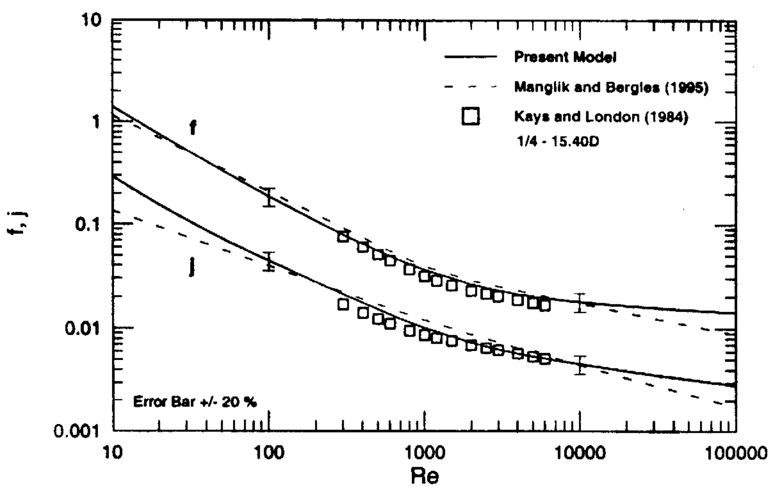

FIGURE 14 Comparison of Eqs. (24) and (29) with data of Kays and London (1984) for 1/4-15.40D.

Muzychka and Yovanovich (1998) or Muzychka (1999). The geometric factor in Eq. (23), ($A_{profile}/A_{wet}$), modifying the form drag coefficient, C_D, should also be adjusted based upon the actual profile area for the non-rectangular subchannel.

SUMMARY AND CONCLUSIONS

New models for predicting the f and j characteristics of the offset strip fin array have been developed. These models which are analytically based, are much simpler than the analytical models of Joshi and Webb (1987), and provide excellent correlation of nineteen sets of OSF data from Kays and London (1984) over the full range of Reynolds numbers. Model predictions are within ± 20 percent for 96 percent of the f data and 82 percent of the j data. The proposed models account for the correct low Reynolds number behaviour which is presently not accounted for in the widely used multiple regression correlations.

Since the models are analytically based, they are valid over the entire range of Reynolds number unlike the empirical models which should only be used within the range of the experimental data. Finally, simple extension of the present models for OSF arrays composed of non-rectangular sub-channels was also discussed.

Acknowledgements

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NOMENCLATURE

A	area, m^2
C_D	drag coefficient
D_h	hydraulic diameter of subchannel, $\equiv 4A/P$
d_h	hydraulic diameter of OSF array, $\equiv 4V_{free}/A_{wet}$
f	friction factor $\equiv \bar{\tau}/((1/2)\rho\bar{w}^2)$
H	channel or fin height, m
h	heat transfer coefficient, W/m^2K
j	Colburn factor, $\equiv St_{\mathcal{L}}Pr^{2/3}$
k	thermal conductivity, W/mK
L	length of channel, m
L_f	fin length, m
$\mathcal{L}$	characteristic length scale, m
m, n	correlation parameter
$Nu_{\mathcal{L}}$	Nusselt number, $\equiv h\mathcal{L}/k$
p	pressure, Pa
Pr	Prandtl number, $\equiv \nu/\alpha$
$Re_{\mathcal{L}}$	Reynolds number, $\equiv \bar{w}\mathcal{L}/\nu$
s	fin spacing, m
S_o	effective fin length, m
t	fin thickness, m
V	volume, m^3
$\bar{w}$	average velocity, m/s
$z^+_{\mathcal{L}}$	dimensionless duct length, $\equiv z/\mathcal{L}Re_{\mathcal{L}}$

Greek Symbols

α	thermal diffusivity, m^2/s
ε	aspect ratio, $\equiv s/H$
μ	dynamic viscosity, Ns/m^2
ν	kinematic viscosity, m^2/s
ρ	fluid density, kg/m^3
τ	wall shear stress, N/m^2

Subscripts

app	apparent
d_h, D_h	based upon hydraulic diameter
f	fin
hy	hydrodynamic
L_f	based upon the length L_f
lam	laminar
tur	turbulent

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