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THE ROLE OF FIN GEOMETRY IN HEAT SINK PERFORMANCE

Waqar A. Khan
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1
Email: wkhan@mhtlab.uwaterloo.ca

J. R. Culham
M. M. Yovanovich
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1
http://www.mhtlab.uwaterloo.ca

ABSTRACT

The following study will examine the effect on overall thermal/fluid performance associated with different fin geometries, including, rectangular plates as well as square, circular and elliptical pin fins. The use of EGM allows the combined effect of thermal resistance and pressure drop to be assessed through the simultaneous interaction with the heat sink. A general expression for the entropy generation rate is obtained by using the conservations equations for mass, energy, and entropy. The formulation for the dimensionless entropy generation rate is developed in terms of dimensionless variables, including the aspect ratio, Reynolds number, Nusselt number and the drag coefficient. Selected fin geometries are examined for the minimum entropy generation rate corresponding to different parameters including axis ratio, aspect ratio, and approach velocity. The results clearly indicate that the preferred fin profile is very dependent on these parameters.

NOMENCLATURE

$\mathcal{L}$	=	characteristic length of the fin $[m]$
A_c	=	cross sectional area of the fin $[m^2]$
A_p	=	planform area for drag force $[m^2]$
A_w	=	wetted surface area of the fin $[m^2]$
a, b	=	semi major and minor axis length of the elliptical fin $[m]$
B	=	duty parameter $\equiv \rho\nu^3 kT_\infty/Q^2$
C_D	=	total drag coefficient
d	=	pin diameter $[m]$
e	=	eccentricity in case of elliptical geometry $\equiv \sqrt{1-\epsilon^2}$
k	=	thermal conductivity $[W/mK]$
k_{eq}	=	ratio of thermal conductivity of fluid to the thermal conductivity of the fin material$\equiv k_f/k$
$\bar{h}$	=	average heat transfer coefficient $[W/m^2K]$
H	=	fin height $[m]$
L	=	length of the flat plate $[m]$
m	=	fin performance parameter $[m^{-1}]$
N_s	=	dimensionless total entropy generation rate
N_{sf}	=	fluid flow irreversibility
N_{sh}	=	heat transfer irreversibility
$Nu_\mathcal{L}$	=	Nusselt number based on the characteristic length of the fin $\equiv \bar{h}\mathcal{L}/k_f$
P	=	perimeter of the fin $[m]$
Q	=	total base heat flow rate $[W]$
$Re_\mathcal{L}$	=	Reynolds number based on the characteristic length of the fin $\equiv U_\infty\mathcal{L}/\nu$
$\dot{S}_{gen}$	=	total entropy generation rate $[W/K]$
s	=	side of a square fin $[m]$
t	=	thickness of the flat plate $[m]$

Subscripts

f	=	fluid
∞	=	free stream conditions
w	=	wall

Greek Symbols

ϵ	=	axis ratio of elliptical fin$\equiv b/a$
ϵ_1	=	ratio of the plate sides$\equiv t/L$
γ	=	aspect ratio of the fin$\equiv H/\mathcal{L}$
μ	=	absolute viscosity of the fluid $[kg/m\,s]$

ν = kinematic viscosity of the fluid $[m^2/s]$
ρ = density of the fluid $[kg/m^3]$

INTRODUCTION

While heat sinks are routinely used in most electronics applications, the rationale for selecting a particular design of heat sink or more specifically a particular fin cross sectional profile, remains somewhat uncertain. Most often these types of selection procedures are based exclusively on performance evaluations consisting of formulations for extended surface heat transfer found in most fundamental heat transfer text books. Unfortunately, these formulations do not consider the role of pressure drop in determining the local fin velocity or heat transfer coefficient and therefore the resulting heat transfer calculations rarely pertain to actual flow conditions. The effects of viscous dissipation associated with flow past fins of arbitrary cross section can be conveniently coupled with the thermal resistance to heat flow in forced convection by using entropy generation minimization (EGM).

A careful review of the literature reveals that no theoretical study exists which compares the overall performance of the different fin geometries (selected in this study) based on the thermal as well as the hydraulic resistance . Recently, McIntyre et al. (2001?) compared the performance of three pin fins (cylindrical, square, and elliptical) on the basis of thermal resistance only. They showed that, for the same constant value of heat transfer coefficient and the cross-sectional or frontal area, the square cross section outperforms the circular cross section and by adjusting the axis ratio of the elliptical fin, it could meet and eventually surpass the performance of the square fin. Behnia et al. (1998) compared numerically the heat transfer performance of various commonly used fin geometries (circular, square, rectangular and elliptical). They fixed the fin cross-sectional area per unit base area, the wetted surface area per unit base area, and the flow passage area for all geometries. They found that circular pin fins outperform square pin fins and elliptical fins outperform plate fins. They also found that elliptical fins work best at lower values of pressure drop and pumping work whereas round pin fins offer highest performance at higher values. Li et al. (1998) showed experimentally that the heat transfer rate with elliptical pin fins is higher than that with circular pin fins while the resistance of the former is much lower than that of the latter in the Reynolds number range from 1000 to 10000. Chapman et al. (1994) investigated experimentally the parallel plate fins and cross-cut pin fins in low air flow environments and compared these fins with elliptical pin fin heat sinks. They used equal volume heat sinks in their experiments. They found that the overall thermal resistance of the parallel plate fin was lower than the other two designs, whereas the heat transfer coefficient was higher for elliptical pin fins than the other two designs. Ota et al. (1983, 1984) studied experimentally heat transfer and flow around an elliptical cylinder of axes ratios 1 : 2 and 1 : 3. Their experimental results show that heat transfer coefficient of the elliptical cylinder is higher than that of a circular one with equal circumference and the pressure drag coefficients of the former are much lower than that of the later. Poulikakos and Bejan (1982) established a theoretical framework to determine the optimum fin dimensions for minimum entropy generation in forced convection. They first developed an expression for the entropy generation rate for a general fin and then applied it to select the optimum dimensions of pin fins, rectangular plate fins, plate fins with trapezoidal cross section, and triangular plate fins with rectangular cross section. Their study seems to be inconclusive as to which geometry offers advantages over others.

This study will show in a graphical manner the relationship between the entropy generation rate and the range of approach velocities commonly found in microelectronic applications. The results will allow designers to quickly and easily assess the merits of pin fin geometries for specific design conditions.

ANALYSIS

This study is based on the following assumptions:

1. The fins are isothermal with adiabatic tip.
2. The airflow is normal to the fins.
3. The flow is steady, laminar and two dimensional.
4. The radiation heat transfer is negligible.
5. The fluid is considered incompressible with constant properties.
6. There is no contact resistance where the base of the fin joins the prime surface.
7. There are no heat sources within the fin itself.

Consider a fin of arbitrary cross section which is immersed in a uniform stream of air with velocity U_∞ and absolute temperature T_∞. The fin is assumed to be isothermal at temperature T_w. Performing mass, energy and entropy balance for this fin, one can write the dimensionless entropy generation rate as follows:

$$\dot{S}_{gen} = \frac{Q^2 R_{th}}{T_\infty^2} + \frac{F_D U_\infty}{T_\infty} \tag{1}$$

where R_{th} is the thermal resistance of the fin and F_D is the drag force which is the sum of the skin friction drag F_f

and pressure drag F_p. The thermal resistance and the drag force for the fin of arbitrary cross section can be written as

$$R_{th} = \frac{1}{k\, A_c\, m\, \tanh(mH)} \tag{2}$$

and

$$F_D = C_D \left(\frac{1}{2}\rho U_\infty^2\right) A_p \tag{3}$$

where C_D is drag coefficient and is given by:

$$C_D = \frac{C_1}{\sqrt{Re_{\mathcal{L}}}} + C_2 + \frac{C_3}{Re_{\mathcal{L}}} \tag{4}$$

where C_1, C_2, and C_3 are the constants depending upon the geometry. These constants are tabulated in Table 1. The fin performance parameter in Eq. (2) is given by:

$$m = \sqrt{\frac{\bar{h}P}{kA_c}} \tag{5}$$

where $\bar{h}$ is the average heat transfer coefficient. Since the numerical value of $\bar{h}$ in a system depends on the characteristic length of the surface ($\mathcal{L}$), the approach velocity (U_∞) as well as the physical properties of the fluid (ρ, μ, c_p, k_f), the functional relationship for the average dimensionless heat transfer coefficient can be written as:

$$Nu_{\mathcal{L}} = f(Re_{\mathcal{L}}, Pr) \tag{6}$$

So, the average Nusselt number for the selected geometries can be written as:

$$Nu_{\mathcal{L}} = C_4 Re_{\mathcal{L}}^n Pr^{1/3} \tag{7}$$

where C_4 is another constant depending upon the geometry, and n is the index (see Table 1). For isothermal boundary conditions, the dimensionless entropy generation rate, also called the entropy generation number, can be defined as:

$$N_s = \frac{\dot{S}_{gen}}{(Q^2 U_\infty / k\nu T_\infty^2)} = N_{sh} + N_{sf} \tag{8}$$

For any arbitrary cross section, this entropy generation number can be written as:

$$N_s = \frac{1}{Re_{\mathcal{L}}\sqrt{C_5\, Nu_{\mathcal{L}}\, k_{eq}}\, \tanh(\gamma\, \sqrt{C_6\, Nu_{\mathcal{L}}\, k_{eq}})} + \frac{1}{2}\, C_D\; B\, \gamma\, Re_{\mathcal{L}}^2 \tag{9}$$

where B is a fixed dimensionless duty parameter that accounts for the importance of fluid friction irreversibility relative to heat transfer irreversibility and C_5 and C_6 are the constants depending on the geometry of the fin and are given by:

$$C_5 = \frac{P\, A_c}{\mathcal{L}^3} \qquad \text{and} \qquad C_6 = \frac{P\, \mathcal{L}}{A_c}$$

The values of these constants for the selected geometries are given in Table 1. Equation (9) shows that, for any given fin geometry, heat duty and a stream of constant thermophysical property fluid, the total dimensionless entropy generation rate will be a function of Reynolds number which in turn depends on the characteristic length $\mathcal{L}$ and the approach velocity, U_∞.
The cross sections for rectangular plate fin (RPF), circular pin fin (CPF), square pin fin (SPF), and elliptical pin fin (EPF) are shown in Fig. 1 and a summary of different parameters for the selected geometries is given in Table 1.

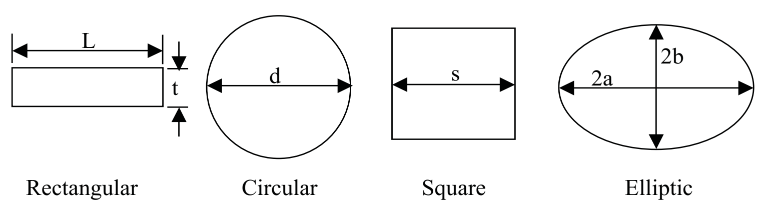

Figure 1. Cross Sections of Selected Geometries

RESULTS AND DISCUSSION

The effects of the axis ratio on the drag force and heat transfer from the selected geometries having the same wetted surface area are shown in Figs. 2 and 3.Clearly the square cross section is the worst choice from both points of views due to highest drag force and the lowest heat transfer rate. The drag force for the elliptical geometry decreases monotonically from the circular geometry ($\epsilon = 1$) to the

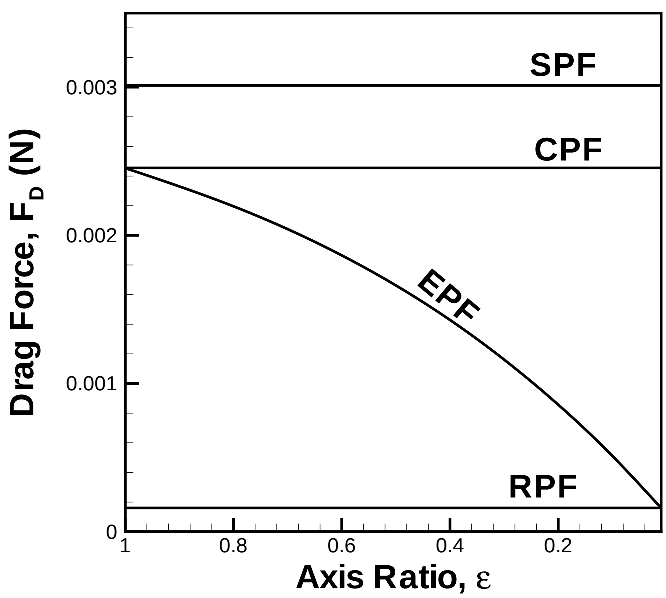

Figure 2. Effect of the Axis Ratio on the Drag Force

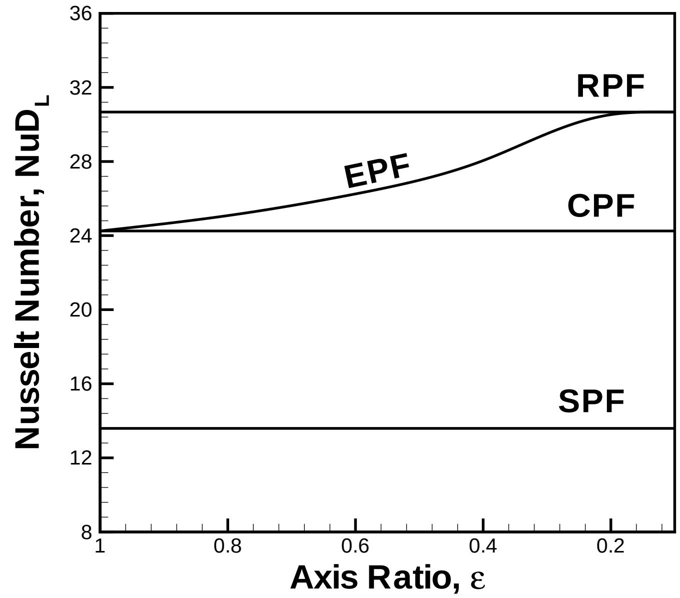

Figure 3. Effect of the Axis Ratio on the Dimensionless Heat Transfer Coefficients

flat plate ($\epsilon = 0.01$). The heat transfer rate from elliptical geometry increases from $\epsilon = 1$ (circular geometry) to $\epsilon = 0.1$ and then becomes constant.

So the elliptical geometry with low axis ratio $\epsilon = 0.1$ could be the best choice from both points of views of drag force and heat transfer.

Figure 4 shows the effects of approach velocity on the drag force for the selected geometries when the wetted surface area is kept constant. It is clear that, for high approach velocities, flat plate is superior to other geometries considered due to lowest drag force.

The elliptical geometry with low axis ratios is the next favorable geometry as far as drag force is concerned. It should be noted that, there is no optimum approach velocity to provide a minimum drag force for any geometry, since it increases monotonically with the approach velocity. It is interesting to note that, for very low approach velocities, the importance of geometry disappears and all the curves approach to the same value as U_∞ approaches zero.

Figure 5 shows the variation of dimensionless entropy generation rate, N_s, with the approach velocity, U_∞, for the selected geometries. The wetted surface area of each geometry, A_w, and the ambient temperature, T_∞, are kept constant. As the approach velocity increases, the best choice moves from elliptical geometry to the flat plate. The square geometry gives the highest entropy generation rate for the entire range of the approach velocities. It should be noted that each geometry has its own optimum for N_s which moves from square geometry to the flat plate. In general, for low approach velocities, the choice of geometry moves from circular to elliptical and for higher velocities it moves from elliptical geometry to flat plate.

The dimensionless total entropy generation rate, N_s, includes the contributions due to heat transfer and viscous friction. As the approach velocity is increased, the contribution due to heat transfer, N_{sh}, decreases and that of viscous friction, N_{sf}, increases for each of the geometry considered. This behavior is shown in Fig. 6 for the circular geometry. An optimal approach velocity U_∞ results away from which the dimensionless total entropy generation rate would increase. The optimal U_∞ exists for all geometries depending upon the wetted surface area.

The effect of the wetted surface area on the dimensionless total entropy generation rate is shown in Fig. 7. It is clear from Fig. 7 that the square geometry is the worst choice from the point of view of entropy generation rate for the entire range of surface areas. For smaller surface areas and low approach velocities, the circular geometry is the best

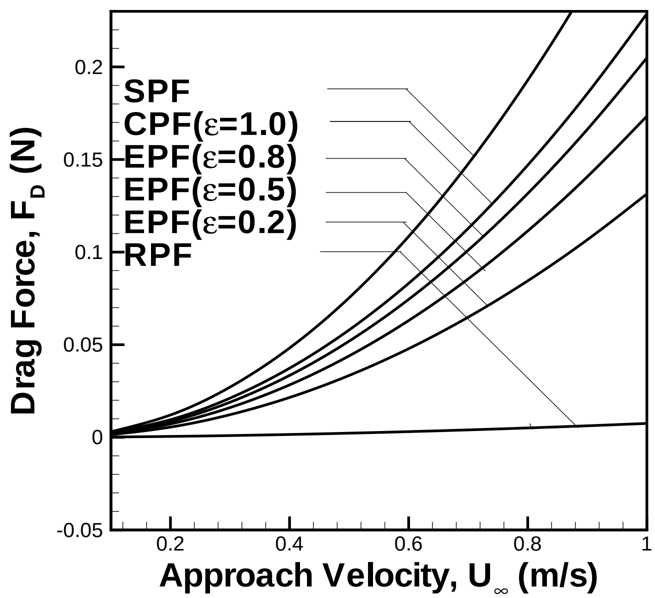

Figure 4. Effect of the Approach Velocity on the Drag Force

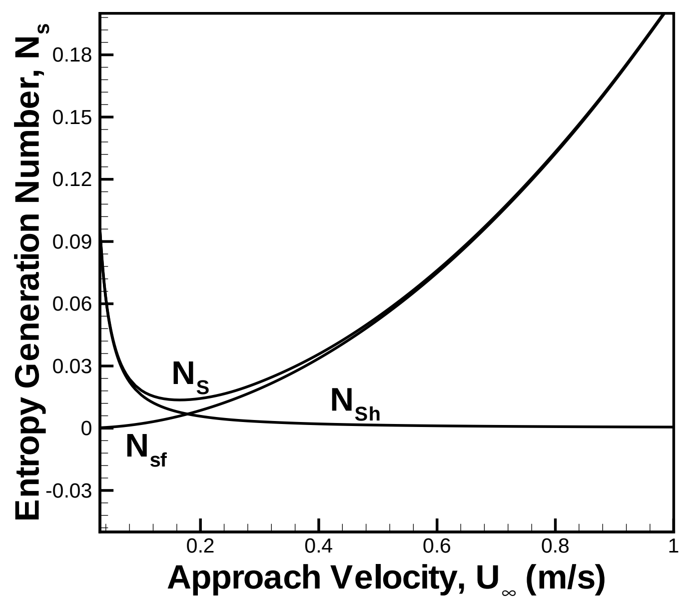

Figure 6. Effect of Approach Velocity on the Dimensionless Entropy Generation Rates for the Circular Geometry

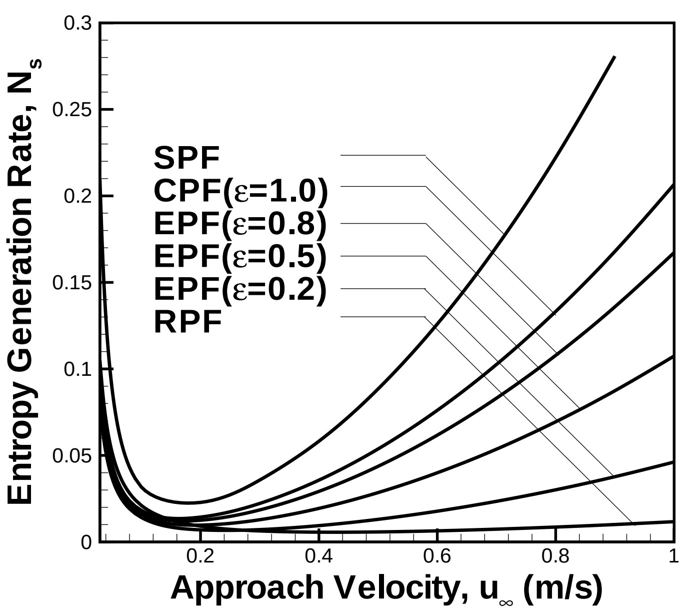

Figure 5. Dimensionless Entropy Generation Rate vs Approach Velocity

choice, but as the surface area and the approach velocity increase, the choice moves first from circular to elliptical geometry and then to flat plate. For each surface area, a minima exists for the dimensionless total entropy generation rate which depends upon the approach velocity. The effects of the axis ratio on the dimensionless total entropy generation rate for the selected geometries, are shown in Fig. 8. As expected, the plate, circular, and square geometries have constant N_s, but, for elliptical geometry, it decreases from $\epsilon = 1$ (circular geometry) to $\epsilon = 0.01$ (flat plate).

The effect of the aspect ratio on the dimensionless entropy generation rate for different geometries is shown in Fig. 9. Again, each geometry has its own optimum point for the minimum entropy generation rate which decreases from the square geometry to the flat plate. It is observed that, for low approach velocities and smaller aspect ratios, the circular geometry gives better results from the point of view of minimum entropy generation rate but the choice of geometry moves with the approach velocity as well as the aspect ratio.

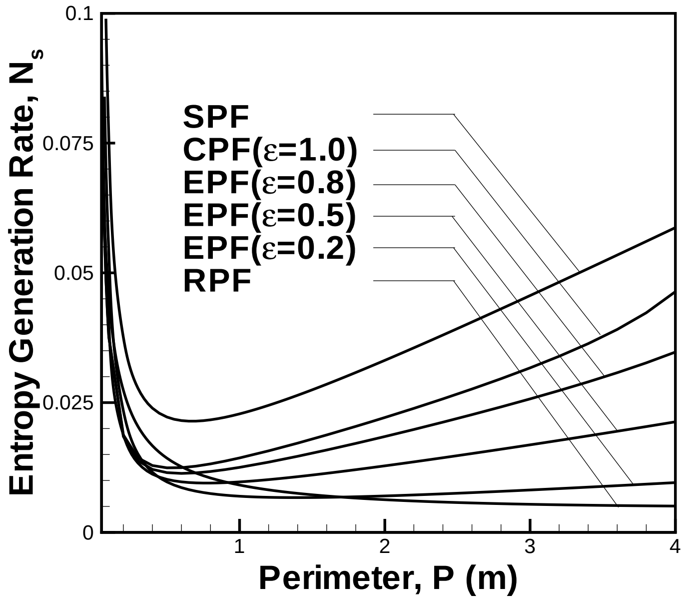

Figure 7. Effect of the Perimeter on the Dimensionless Entropy Generation Rate

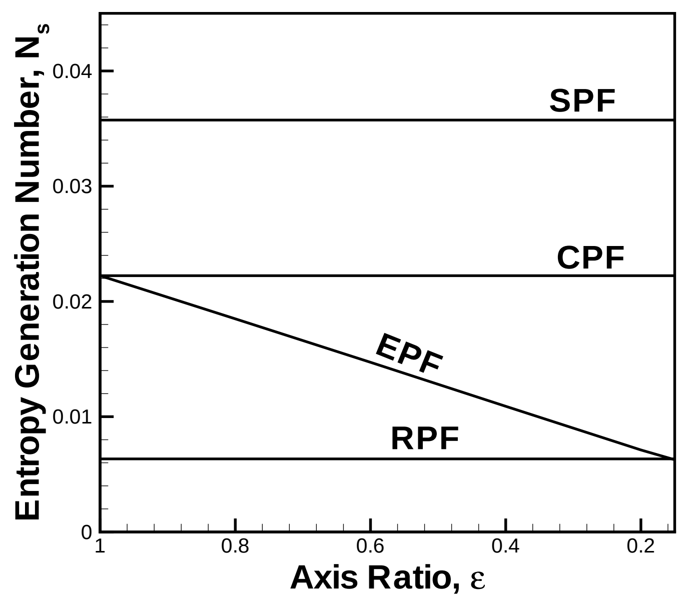

Figure 8. Effect of the Axis Ratio on the Dimensionless Entropy Generation Rate

CONCLUSIONS

Different fin geometries having the same wetted surface area are compared from the point of views of heat transfer, drag force, and dimensionless total entropy generation rate. Optimum dimensionless entropy generation rate exists for each geometry corresponding to approach velocity, wetted surface area, and the aspect ratio. No Optimum dimensionless entropy generation rate exists for the axis ratio of the elliptical geometry when the approach velocity is taken as the parameter or vice versa. The square geometry is found to be the worst choice from the point of view of heat transfer and drag force and hence from the point of view of total entropy generation rate. Whereas, the circular geometry appears as the best from the point of view of the dimensionless total entropy generation rate for low approach velocities and small wetted surface areas. The flat plate gives the best results from the point of view of total entropy generation rate for higher approach velocities and large surface areas. The elliptical geometry is the next favorable geometry from the point of view of total entropy generation rate for higher approach velocities and with low axis ratios. It offers higher heat transfer coefficients and lower drag force as the axis ratio is decreased and the approach velocity is increased. Elliptical geometry could perform better than circular geometry at medium approach velocities for larger surface areas and flat plate could outperform elliptical geometry at higher approach velocities for the same areas with high aspect ratios. However, for small surface areas and low velocities, flat plates are not a good selection from the point of view of entropy generation rate.

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Table 1. PARAMETERS FOR DIFFERENT GEOMETRIES.

Parameters	Geometry			
	Plate	**Circular**	**Square**	**Elliptical**
$\mathcal{L}$	L	d	s	$2a$
A_c	tL	$\pi d^2/4$	s^2	$\pi\, a\, b$
A_p	LH	dH	sH	$2\, a\, H$
P	$2(L+t)$	$\pi\, d$	$4\, s$	$4\, aE(e)$
C_1	1.357	5.781	0	$-4.1(0.67 - \exp(0.733\epsilon))$
C_2	0	1.152	2	$1.1526\epsilon^{0.951}$
C_3	0	1.26	0	$\frac{-8.556740253 + 9.9240913\epsilon^{.40208311}}{.87880008e - 1 + \epsilon^{.40208311}}$
C_4	0.75	0.593	0.102	$0.75 - 0.16\exp(-0.018\epsilon^{-3.1})$
C_5	$2\epsilon_1(1+\epsilon_1)$	$\pi^2/4$	4	$\pi^4\, \epsilon\, /16E^2(e)$
C_6	$2(1+\epsilon_1)/\epsilon_1$	4	4	$16E^2(e)/\pi^2\, \epsilon$
n	1/2	1/2	0.675	1/2

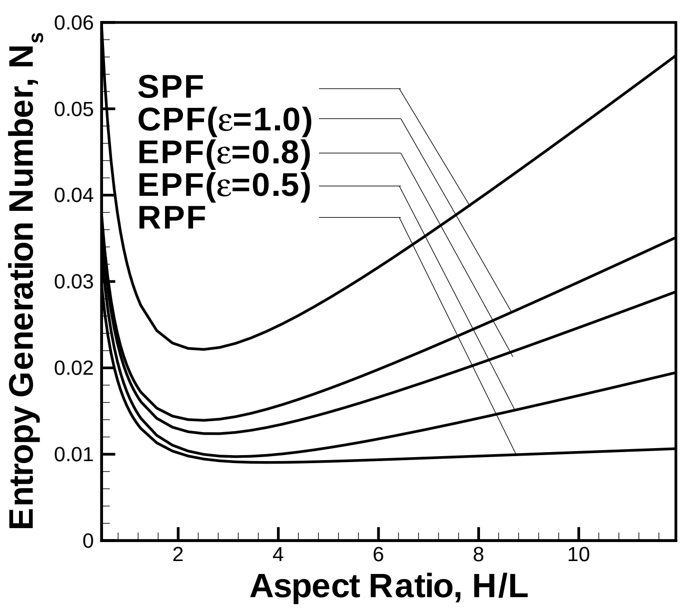

Figure 9. Effect of the Aspect Ratio on the Dimensionless Entropy Generation Rate

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