

GAUSSIAN ROUGHNESS IN THERMAL CONTACT CONDUCTANCE, MICROTUBES AND MICROFINS

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1 INTRODUCTION

The significant contributions of Professor B. B. Mikic and collaborators to fields of thermal contact conductance and contact mechanics are summarized in this technical note. Professor Mikic's research is original and fundamental. The results have been cited and utilized in many practical applications.

Mikic is one of the pioneers who contributed to the theory of Gaussian surfaces and their interactions. These concepts have been extended to model pressure drop of fully developed, laminar flow in rough surface microchannels and heat transfer augmentation in rough surface microfins.

2 THERMAL CONDUCTANCE OF ROUGH SURFACES

Mikic and Rohsenow [1] studied thermal contact conductance for various types of surface waviness and conditions. In particular; nominally flat rough surface in a vacuum, nominally flat rough surfaces in a fluid environment, smooth wavy surfaces in a vacuum environment with one of the following three types of waviness involved: spherical, cylindrical in one direction, and cylindrical in two perpendicular directions, and rough spherical wavy surfaces in a vacuum. Thermal contact resistance for two spherical wavy rough surfaces was considered as the summation of a micro and a macro thermal constriction resistance.

Mikic and Rohsenow [1], using a superposition method, derived an expression for the spreading resistance in an elemental heat channel (semi-infinite cylinder), with an

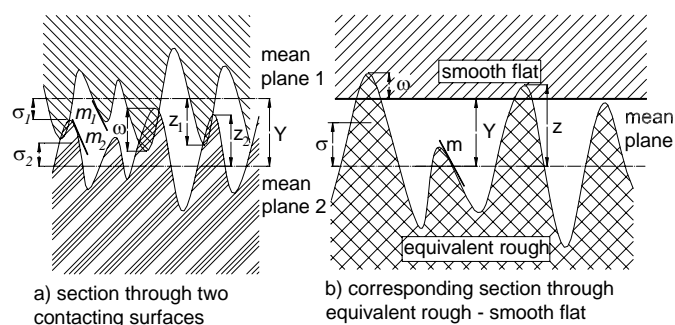


Figure 1. EQUIVALENT CONTACT OF GAUSSIAN ROUGH SURFACES

isothermal boundary condition. They developed another solution for mixed boundary conditions. They also studied thermal spreading resistance of the flux tube with a finite length. It was shown that the influence of the finite length of the elemental heat channel on the spreading resistance was negligible for all values of $l \geq b$, where l is the length and b is the radius of the flux tube. Later this expression was simplified by Cooper, Mikic, and Yovanovich [2].

Mikic and Rohsenow derived expressions for the mean size and number of microcontacts by assuming fully plastic deformation of asperities and equivalent surface approximation, see Fig. 1. They used Hertzian theory to determine the macrocontact area for smooth surfaces [1]. These relationships were used later by Cooper, Mikic, and Yovanovich (CMY) [2]. In the case of rough surface contacts, knowing that the macrocontact area would be larger than the one predicted by the Hertzian theory, they defined an effective

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macrocontact area. This area contained all the microcontact spots as if they had been uniformly distributed. Using this definition and the assumption that the mean surface would deform elastically, Mikic suggested an iterative procedure for calculating the macrocontact radius [1]. He verified his model against three sets of experiments.

Cooper, Mikic, and Yovanovich [2] developed a theory for contact of nominal flat rough surfaces, assuming plastically deformed asperities, whose height and surface slopes have Gaussian distributions, where the mean separation Y is constant throughout the contact plane. The CMY model was based on the uniform distribution of identical microcontacts inside the macrocontact area. Mikic [3] summarized the CMY relationships and reported them as follows:

$$\left. \begin{aligned} a_s &= \sqrt{\frac{8}{\pi}} \left(\frac{\sigma}{m}\right) \exp(\lambda^2) \operatorname{erfc} \lambda \\ n_s &= \frac{1}{16} \left(\frac{m}{\sigma}\right)^2 \frac{\exp(-2\lambda^2)}{\operatorname{erfc} \lambda} A_a \\ \frac{A_r}{A_a} &= \frac{1}{2} \operatorname{erfc} \lambda \end{aligned} \right\} \quad (1)$$

where σ , m , $\lambda = Y/\sqrt{2}\sigma$, n_s , a_s , A_r and A_a are roughness, surface slope, the dimensionless mean plane separation, number and average size of microcontacts, the real and the apparent contact area, respectively. Moreover, the effect of previous loading on thermal contact conductance was discussed in [3].

Later Mikic [4] derived expressions for macroscopic and microscopic thermal resistances in a vacuum, which related the micro and macro thermal resistances to arbitrary pressure distribution and surface properties. The derived relations were general in the sense that they did not require knowledge of the effective macrocontact area and they could be applied for any symmetrical cylindrical or Cartesian pressure distribution at an interface.

Mikic [5] proposed a plasticity index, $\gamma = H_{mic}/E'm$. He reported that the deformation mode of asperities depends only on material properties and the shape of the asperities, and it is not sensitive to the pressure level. Mikic performed an analysis to determine the contact pressure over the contact area based on the fact that all contact spots do not have the same contact pressure, although the average contact pressure would remain constant. For surfaces with $\gamma \geq 3$, 90% of the actual area will have the elastic contact pressure; therefore the contact will be predominantly elastic, and for $\gamma \leq 0.33$, 90% of the actual area will have the plastic contact pressure, thus the contact will be predominantly plastic. He concluded that for most engineering surfaces the asperity deformation mode is plastic and the average asperity pressure is the effective microhardness.

Mikic [5], based on the CMY model, proposed an elastic model; the elastic model was also presented in [6]. He

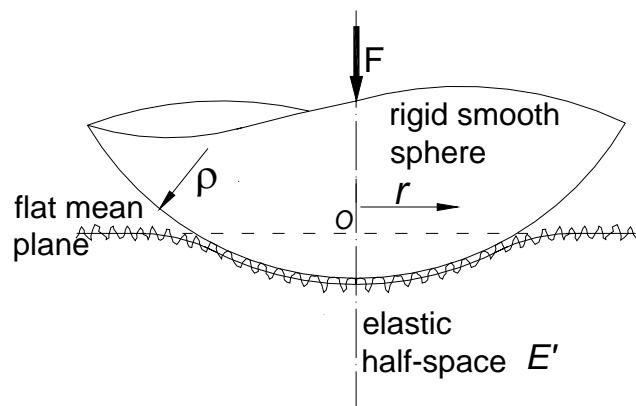


Figure 2. GEOMETRY OF ROUGH SPHERE FLAT CONTACT

assumed that the elastic real contact area is half of the plastic contact area, i.e., $A_{elastic}/A_{plastic} = 1/2$. Mikic's elastic model satisfied the linear proportionality between the applied load and the real contact area. He also proposed an effective elastic microhardness [5]:

$$H_e = \frac{E'm}{\sqrt{2}} \quad (2)$$

where

$$\frac{1}{E'} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}$$

where E and ν are the elastic modulus and Poisson's ratio, respectively. Mikic proposed an expression for calculating the thermal contact conductance of conforming rough joints assuming elastic deformation [5]

$$h_e = 1.55 k_s \left(\frac{m}{\sigma}\right) \left(\frac{P}{H_e}\right)^{0.94} \quad (3)$$

Existing plastic models such as CMY [2] did not consider the effect of elastic deformations beneath the microcontacts. These effects would be negligible if the elastic modulus of contacting bodies were infinite and/or the distance between the neighboring microcontacts was small enough so the elastic deformation was the same for all microcontacts. In reality, none of the above is true and the elastic deflection underneath a microcontact is always larger than the deformation outside the microcontact area (mean plane). Mikic [5] was the first to point out this problem and proposed a model that accounts for the elastic deformation of substrate underneath plastically deformed microcontacts.

Mikic and Roca [7] studied the contact of rough spherical bodies, Fig. 2. They developed a numerical model by assuming plastic deformation of asperities and that the height of the surface roughness has a Gaussian distribution.

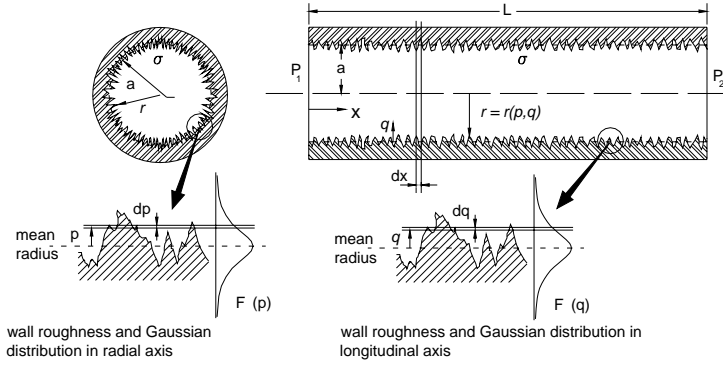


Figure 3. GEOMETRY OF ROUGH MICROTUBE

Their model showed that an increase in roughness resulted in a decrease in the contact pressure and an increase in the contact area. The modeling results of [7] were mainly a function of a non-dimensional parameter $\bar{\sigma} = \pi \sigma E' / a_H P_{0,H}$ and a weak function of $H/P_{0,H}$, where $P_{0,H}$ is the maximum pressure in the Hertzian contact and H is the joint effective hardness.

3 PRESSURE DROP ROUGH MICROTUBES

As the diameter of microtubes decreases, the surface phenomena- including the effect of wall roughness become more significant, see Fig. 3. The influence of wall roughness on the laminar, fully-developed, incompressible flow in microtubes is investigated and a novel analytical model is proposed [8]. The concept of frictional resistance is introduced and its relation to the Darcy friction coefficient is derived. The model assumes an isotropic, Gaussian distribution for wall roughness. Owing to the random nature of the wall roughness, an exact value of the local radius can not be used for rough microtubes. Instead, probabilities of different radii occurring should be computed. Two independent random variables are considered to account for deviations of the local radius of rough microtubes in the angular and longitudinal directions. The local radius is assumed to be the superposition of the two random variables. The effect of wall roughness on the frictional resistance is presented as a normalized frictional resistance or a correction factor, i.e. $R_f^* = R_f / R_{f0}$; where R_{f0} is the frictional resistance of the smooth microchannels with the same radius a . The final results are reported in the form of a compact correlation:

$$R_f^* = \begin{cases} \frac{1}{1 - 23 \epsilon^2} & \epsilon \leq 0.1 \\ \frac{1}{1 - 50 \epsilon^{2.4}} & 0.1 < \epsilon < 0.15 \end{cases} \quad (4)$$

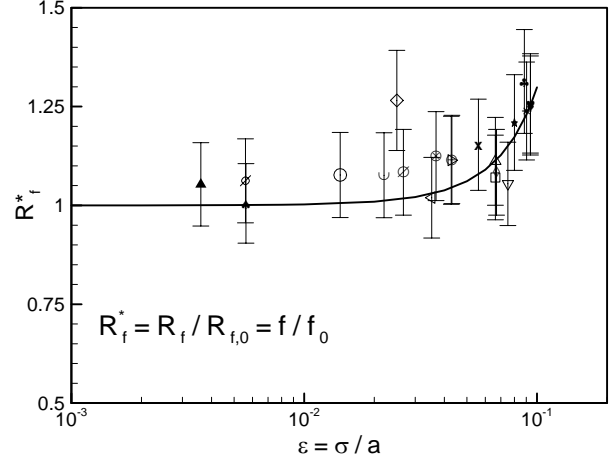


Figure 4. COMPARISON OF MODEL WITH ROUGH MICROTUBE DATA, FROM [8]

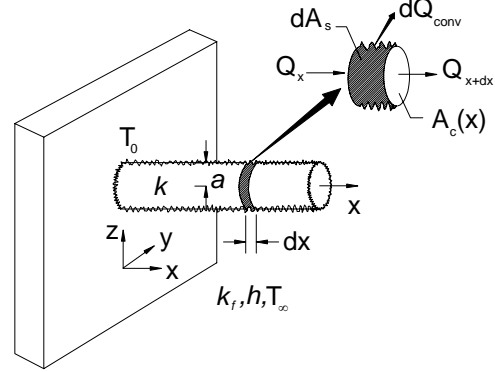


Figure 5. ENERGY BALANCE FOR AN EXTENDED ROUGH SURFACE

where ϵ is the relative wall roughness, $\epsilon = \sigma/a$.

It is found that the effect of roughness is to increase the pressure drop, compared to the Hagen-Poiseuille theory, in microtubes. As shown in Fig. 4, the published experimental data for pressure drop in microtubes, in which the wall roughness is reported, are collected and compared with the present model.

4 THERMAL PERFORMANCE OF RANDOMLY ROUGH MICROFINS

Heat transfer plays a key role in a variety of applications in MEMS such as, thermal actuators in RF devices, thermal flexure actuators, and thermal-compliant microactuators. Recently, micro pin fin heat exchangers have been

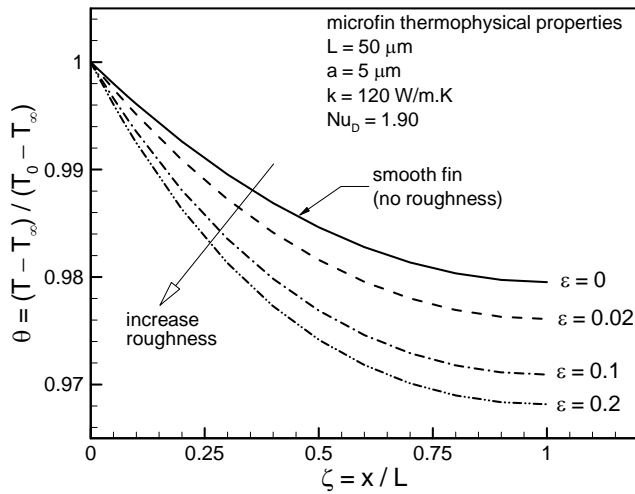


Figure 6. EFFECT OF SURFACE ROUGHNESS ON TEMPERATURE PROFILE OF PIN FIN, FROM [9]

utilized in advanced thermal management solutions ranging from gas turbine blades cooling to microelectronic chip cooling. Micro surfaces that have been fabricated by MEMS processes have some level of roughness. This surface roughness can be envisioned as another extended surface on the original extended surface, i.e., *fins on fins*, see Fig. 5.

Using the same approach that has been employed in modeling of pressure drop in rough microtubes, an analytical model is developed that takes into account the effect of roughness on a circular cylinder microfin [9]. The model assumes an isotropic, Gaussian distribution for surface roughness.

It is shown that, as a result of roughness, both cross-sectional and surface area of microfins are increased which results in an enhancement in the heat transfer rate and thus the thermal performance of microfins. Moreover, it is observed that as the surface roughness increases the mean temperature of the microfin decreases, see Fig. 6. The results of the analysis can be used to improve upon the thermal analysis and design of microfins and micro structures.

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