Fluid Flow and Heat Transfer in Power-Law Fluids Across Circular Cylinders: Analytical Study

An integral approach of the boundary layer analysis is employed for the modeling of fluid flow around and heat transfer from infinite circular cylinders in power-law fluids. The Von Karman-Pohlhausen method is used to solve the momentum integral equation whereas the energy integral equation is solved for both isothermal and isoflux boundary conditions. A fourth-order velocity profile in the hydrodynamic boundary layer and a third-order temperature profile in the thermal boundary layer are used to solve both integral equations. Closed form expressions are obtained for the drag and heat transfer coefficients that can be used for a wide range of the power-law index, and generalized Reynolds and Prandtl numbers. It is found that pseudoplastic fluids offer less skin friction and higher heat transfer coefficients than dilatant fluids. As a result, the drag coefficients decrease and the heat transfer increases with the decrease in power-law index. Comparison of the analytical models with available experimental/numerical data proves the applicability of the integral approach for power-law fluids. [DOI: 10.1115/1.2241747]

Introduction

Many practical situations need a knowledge of fluid flow around and heat transfer from horizontal cylinders subjected to cross flow of non-Newtonian fluids. These fluids are classified by different authors in different ways. One important classification is the purely viscous fluids made to investigate heat transfer to power-law fluids. Wolf and Shah et al. [21] performed a series of experiments to study the flow of non-Newtonian fluids around a cylinder. They measured shedding frequency, the formation length \(l_f\) and the pressure distribution around a cylinder and determined the shedding regimes and the drag coefficients.

D’Alessio and Pascal [32] investigated numerically the steady power-law flow around a circular cylinder at three different Reynolds numbers \(Re_{dp}=5, 20, \) and \(40\) using a first-order accurate difference method for a fixed blockage ratio. They found that the
critical Reynolds number, wake length, separation angle, and drag coefficient depend on the power-law index. Chhabra et al. [33] extended that work by using a more accurate second-order finite difference method, more refined computational meshes, and greater blockage ratio and power-law index ranges in order to investigate the effect of blockage on drag coefficient, wake length, separation angle, and flow patterns over wide ranges of conditions. Agarwal et al. [34] investigated numerically the momentum and thermal boundary layers for power-law fluids over a thin needle under wide ranges of kinematic and physical conditions. They reported extensive results on axial velocity profiles, shear stress, and skin friction distribution on the surface of the needle, and thermal boundary layer thicknesses in the following ranges of conditions: 0.2 ≤ n ≤ 1.6, 1 ≤ Pr ≤ 1000, Re < 10^6 and for three needle sizes.

In this study, an approximate method, based on the Karman-Pohlhausen integral momentum and energy equations, is used to study the fluid flow and heat transfer in power-law fluids across a single circular cylinder.

Analysis

Consider a uniform flow of a non-Newtonian (power-law) fluid past a fixed circular cylinder of diameter D, with vanishing circulation around it, as shown in Fig. 1. The approaching velocity of the fluid is \(U_{\text{app}}\) and the ambient temperature is assumed to be \(T_w\). The surface temperature of the wall is \(T_w(>T_s)\) in the case of the isothermal cylinder and the heat flux is \(q\) for the isoflux boundary condition. The flow is assumed to be laminar, steady, and two dimensional. The fluid is assumed to be incompressible with constant thermophysical and rheological properties. The potential flow velocity just outside the boundary layer is denoted by \(U(s)\).

Using an order-of-magnitude analysis (Khan [35]), the reduced equations of continuity, momentum, and energy in the curvilinear system of coordinates (Fig. 1) for a power-law fluid can be written as:

**Continuity:**

\[
\frac{\partial u}{\partial s} + \frac{\partial u}{\partial \eta} = 0
\]  

**s-Momentum:**

\[
\frac{u}{\partial s} + \frac{\partial u}{\partial \eta} = \frac{1}{\rho} \frac{dP}{ds} + \frac{1}{\rho} \frac{\partial \tau_w}{\partial \eta}
\]  

**\(\eta\)-Momentum:**

\[
\frac{dP}{d\eta} = 0
\]  

**Energy:**

\[
\frac{\partial T}{\partial s} + \frac{u}{\partial \eta} \frac{\partial T}{\partial \eta} = \alpha \frac{\partial^2 T}{\partial \eta^2}
\]  

with

\[
- \frac{1}{\rho} \frac{dP}{ds} = U(s) \frac{dU(s)}{ds}
\]  

and

\[
\tau_w = m \left( \frac{\partial u}{\partial \eta} \right)^n_{\eta=0}
\]  

where \(m\) is a consistency index for non-Newtonian viscosity and \(n\) is called power-law index, that is < 1 for pseudoplastic, = 1 for Newtonian, and > 1 for dilatant fluids.

Hydrodynamic Boundary Conditions

The assumptions of no slip boundary condition at the cylinder wall (\(\eta=0\), at \(\eta=0\)), no mass flow through the cylinder wall (\(\eta=0\) at \(\eta=0\)), and the potential flow just outside the boundary layer (\(\eta=U(s)\) at \(\eta=\delta(s)\)) give the following complete set of hydrodynamic conditions:

(i) at the cylinder surface, i.e., at \(\eta=0\)

\[
u = 0 \quad \frac{\partial^2 u}{\partial \eta^2} = \frac{U(s) \frac{dU(s)}{ds}}{n \gamma \left( \frac{\partial u}{\partial \eta} \right)^{n-1}}
\]  

where \(\gamma = m/\rho\),

(ii) at the edge of the boundary layer, i.e., at \(\eta=\delta(s)\)

\[
u = U(s), \quad \frac{\partial u}{\partial \eta} = 0 \quad \frac{\partial^2 u}{\partial \eta^2} = 0
\]  

These conditions will help in determining the velocity distribution inside the boundary layer.

Thermal Boundary Conditions

The assumptions of uniform wall temperature (UWT) and uniform wall flux (UWF) boundary conditions give the following complete set thermal conditions:

(i) at the cylinder surface, i.e., at \(\eta=0\)

\[
\frac{\partial T}{\partial \eta} = 0, \quad T = T_w
\]  

for UWT

\[
\frac{\partial T}{\partial \eta} = \frac{q}{k_f}
\]  

for UWF

(ii) at the edge of thermal boundary layer, i.e., at \(\eta=\delta_T\)

\[
T = T_a \quad \frac{\partial T}{\partial \eta} = 0
\]  

Using these thermal conditions, the temperature distributions inside the thermal boundary layer can be determined.

Velocity Distribution

Assuming a thin hydrodynamic boundary layer around the cylinder, the velocity distribution inside the boundary layer, satisfying Eqs. (10) and (11), can be approximated by a fourth-order polynomial as suggested by Pohlhausen [36]:

\[
u = (2 \eta^2 - 2 \eta^2 + 3 \eta^2 \eta^3) \frac{\lambda}{6} \eta^2 - 3 \eta^2 + 3 \eta^2 - \eta^2)
\]  

where \(0 \leq \eta \leq \eta = \eta / \delta(s) \leq 1\) and \(\lambda\) is the pressure gradient parameter, given by
With the help of velocity profiles, Schlichting [37] showed that the parameter $\lambda$ is restricted to the range $-12 \leq \lambda \leq 12$.

**Temperature Distribution**

Assuming a thin thermal boundary layer around the cylinder, the temperature distribution in the thermal boundary layer, satisfying Eqs. (12) and (13), can be approximated, in terms of $\eta_T = \eta/\delta_T$, by a third-order polynomial

$$
\frac{T - T_a}{T_w - T_a} = 1 - \frac{3}{2} \eta_T + \frac{1}{2} \eta_T^2
$$

(16)

for the isothermal boundary condition and

$$
T - T_a = 2\eta_T^2 \left( \frac{3}{2} \eta_T + \frac{1}{2} \eta_T^2 \right)
$$

(17)

for the isoflux boundary condition.

**Boundary Layer Parameters**

The momentum integral equation can be written as

$$
U^2 \frac{d\delta_1}{ds} + (2\delta_2 + \delta_1) U \frac{dU}{ds} = \frac{1}{\rho} \tau_w
$$

(18)

where $\delta_1$ and $\delta_2$ are the displacement and momentum boundary layer thicknesses and are given by

$$
\delta_1 = \delta \left[ \int_0^1 \left[ 1 - \frac{u}{U(s)} \right] d\eta_H \right]
$$

(19)

and

$$
\delta_2 = \delta \left[ \int_0^1 \frac{u}{U(s)} \left[ 1 - \frac{u}{U(s)} \right] d\eta_H \right]
$$

(20)

Using velocity distribution from Eq. (14), Eqs. (19) and (20) can be written as

$$
\delta_1 = \frac{\delta}{10} \left( 3 - \frac{\lambda}{12} \right)
$$

(21)

and

$$
\delta_2 = \frac{\delta}{63} \left( \frac{37}{5} - \frac{\lambda}{15} - \frac{\lambda^2}{144} \right)
$$

(22)

Simplifying and arranging the terms in Eq. (18), we get

$$
\frac{dU}{U^{n-1}} \frac{d\delta_1}{ds} + \left( \frac{\delta_2}{\delta_1} \right) U^{n-1} \frac{dU}{ds} = \frac{\delta_2}{\delta_1} \left( \frac{2 + \lambda}{6} \right)^n
$$

Assuming

$$
\frac{Z}{U} = \frac{\delta_2^{n-1} U^{1-n}}{\gamma} \quad K = \frac{dU}{ds}
$$

Eq. (23) can be reduced to a nonlinear differential equation of the first order for $Z$, which can be written as

$$
\frac{dZ}{ds} = \frac{F}{U}
$$

(24)

where

$$
F = \frac{K}{n} \left( \frac{n+1}{n} \right) \left( \frac{2 + \lambda}{6} \right)^n
$$

(25)

At the stagnation point $s=0$, $U=0$. Since $dZ/ds$ cannot be infinite, $F$ must be zero at the stagnation point. Hence

$$
\frac{(n+1)}{n} \left( \frac{2 + \lambda}{6} \right)^n = - (1 + 3n) \lambda \left( \frac{\delta_2}{\delta_1} \right) - (n+1) \lambda \left( \frac{\delta_1}{\delta} \right) = 0
$$

(26)

which gives the values of the pressure gradient parameter $\lambda$ for different values of $n$ at the stagnation point. Due to limitations of the method used in this study, no root of Eq. (26) could be found in the range $-12 \leq \lambda \leq 12$ for $n \leq 0.895$. Bizzell and Slattery [9] could calculate the roots for $0.7358 \leq n \leq 1.0$ only, whereas Mizushima and Usui [14] calculated the roots in the range $0.895 \leq n \leq 1.19$. In the present study, the values of $\lambda$ are calculated for $n \geq 0.895$. These values are plotted in Fig. 2 as a function of $n$.

Following Walz [38], the function $F$ can be approximated by a straight line

$$
F = a - bK
$$

(27)

where the constants $a$ and $b$ are determined for each power index $n$. From Eq. (25), the values of $K$ and $F$ are obtained for different values of lambda. Following the method of Walz [38] and Schlichting [37], straight lines are obtained between the point of maximum velocity ($K=0$) and the stagnation point ($F=0$) and then correlated to obtain

$$
a = 0.45n^{-1.28} \quad b = 6.2n^{0.8}
$$

(28)

These correlations are valid between the stagnation point ($F=0$) and the point of maximum velocity ($K=0$). So Eq. (24) can be written as

$$
U(s) \frac{dZ}{ds} = a - bK
$$

(29)

Using potential flow velocity outside the boundary layer for a circular cylinder and rearranging the terms, Eq. (29) can be solved for the local dimensionless momentum thickness:
The first parameter of interest is fluid friction which manifests itself in the form of the drag force \( F_D \), where \( F_D \) is the sum of the skin friction drag \( D_f \) and pressure drag \( D_p \). Skin friction drag is due to viscous shear forces produced at the cylinder surface, predominantly in those regions where the boundary layer is attached. In dimensionless form, it can be written as

\[
\frac{D_f}{D} = \left[ \frac{\tau_\theta}{\rho U_{app}^2} \right]_{\text{circumference}}
\]

Using Eqs. (9) and (14) and simplifying, we get

\[
C_f = \frac{2}{\text{Re}_{Dp}^{1/(n+1)}} \left[ \frac{\lambda + 12\sin \theta}{3} \right]^n \left[ \frac{\sin 2\theta \left( \frac{2 + \lambda}{6} \right)}{2^{n-2} n \lambda \sin^n \theta} \right]^{1/(n+1)}
\]  

The friction drag coefficient can be defined as

\[
C_{Df} = \int_0^\theta C_f \sin \theta d\theta = \int_0^\theta C_f \sin \theta d\theta + \int_\theta^\pi C_f \sin \theta d\theta
\]

Since the shear stress on the cylinder surface after boundary layer separation is very small, the second integral can be neglected and the friction drag coefficient can be written as

\[
C_{Df} = \int_0^\theta C_f \sin \theta d\theta
\]

The calculations were performed for different values of \( n \) and the results are correlated in terms of \( n \) and the generalized Reynolds number \( \text{Re}_{Dp} \) in to a single correlation

\[
C_{Df} = \frac{5.786n^{0.32}}{\text{Re}_{Dp}^{1/(n+1)}}
\]

which gives friction drag coefficient for the flow of a power-law fluid over a circular cylinder in an infinite medium. It is interesting to note here that for \( n = 1 \), Eq. (37) gives the friction drag coefficient for the flow of Newtonian fluid over a circular cylinder (Khan et al. [3]).

Pressure drag is due to the unbalanced pressures which exist between the relatively high pressures on the upstream surfaces and the lower pressures on the downstream surfaces. In dimensionless form, it can be written as

\[
C_{Dp} = \int_0^\pi C_p \cos \theta d\theta
\]

where \( C_p \) is the pressure coefficient and can be defined as

\[
C_p = \frac{\Delta P}{1/2 \rho U_{app}^2}
\]

The pressure difference \( \Delta P \) can be obtained by integrating \( \theta \)-momentum equation with respect to \( \theta \). Following Shibu et al. [39], the \( \theta \)-momentum equation, for power-law fluids, can be written as

\[
\frac{\partial u_\theta}{\partial r} + \frac{u_u \partial u_\theta}{r \partial \theta} + \frac{u_\theta \partial u_u}{r \partial \theta} = -\frac{1}{2r \partial \theta} \frac{\partial C_p}{\partial \theta} + \frac{2\tau_\theta}{r \partial \theta} \left( \frac{\partial}{\partial \theta} \left( 2\tau_\theta \right) + \frac{1}{r} \frac{\partial \tau_\theta}{\partial \theta} \right)
\]

where

\[
\tau_\theta = 2 \eta \varepsilon_\theta \quad \tau_{\theta \theta} = 2 \eta \varepsilon_{\theta \theta}
\]

\[
\eta = (2\Pi)^{(n-1)/2} 
\]

with

\[
\varepsilon_\theta = \frac{1}{\Pi} \frac{\partial u_\theta}{\partial r} + \frac{u_u}{r} \quad \Pi = \varepsilon_r^2 + \varepsilon_\theta^2 + 2 \varepsilon_r \varepsilon_\theta
\]

and

\[
u_u = \cos \theta \left( 1 - \frac{1}{r} \right) 
\]

Using Eqs. (9) and (14) and simplifying, we get

\[
C_f = \frac{2}{\text{Re}_{Dp}^{1/(n+1)}} \left[ \frac{\lambda + 12\sin \theta}{3} \right]^n \left[ \frac{\sin 2\theta \left( \frac{2 + \lambda}{6} \right)}{2^{n-2} n \lambda \sin^n \theta} \right]^{1/(n+1)}
\]
cylinder $R$, the stress components by $m(U_{app}/R)^n$, and the second invariant of the rate of deformation tensor, $\Pi_2$, using $(U_{app}/R)^2$. Using derivatives of the velocity components and an order-of-magnitude analysis (Khan [35]), Eq. (40) can be reduced to
\[
\frac{\partial C_p}{\partial \theta} = -4 \sin 2\theta - \frac{23n}{Re_{dp}} \sin \theta \tag{41}
\]
Integrating it with respect to $\theta$, we get
\[
C_p = 2(1 - \cos 2\theta) + \frac{23n}{Re_{dp}}(1 - \cos \theta) \tag{42}
\]
Using Eq. (42) in Eq. (38), the pressure drag coefficients are calculated for different values of $n$ up to the separation point and correlated in terms of $n$ and $Re_{dp}$ to give
\[
C_{dp} = \frac{1.26n^{3.25}}{Re_{dp}} + 1.28\left[1 - \exp(-2.4n)\right] \tag{43}
\]
The total drag coefficient $C_D$ can be written as the sum of both drag coefficients
\[
C_D = \frac{5.786n^{0.32}}{Re_{dp}^{1/(n+1)}} + \frac{1.26n^{3.25}}{Re_{dp}} + 1.28\left[1 - \exp(-2.4n)\right] \tag{44}
\]
which agrees with the drag coefficient of a Newtonian fluid ($n = 1$) over a circular cylinder (Khan et al. [3]).

**Heat Transfer**

The second parameter of interest is the dimensionless average heat transfer coefficient, $Nu_D$ for large Prandtl numbers. This parameter is determined by integrating Eq. (7) from the cylinder surface to the thermal boundary layer edge. Assuming the presence of a thin thermal boundary layer $\delta_T$ along the cylinder surface, the energy integral equation for the isothermal boundary condition can be written as
\[
\frac{d}{ds} \int_0^{\delta_T} (T - T_w) u d\eta = -\alpha \frac{dT}{d\eta}\bigg|_{\eta = 0} \tag{45}
\]
Using velocity and temperature profiles Eqs. (14) and (16), and assuming $\xi = \delta_T/\delta = 1$, Eq. (45) can be simplified to
\[
\delta_T \frac{d}{ds} [U(s) \delta_T \xi(\lambda + 12)] = 90\alpha \tag{46}
\]
Heat transfer analysis is divided into two parts due to discontinuity at $\theta = \pi/2$. So Eq. (46) is rewritten separately for the two regions (Fig. 1), i.e.
\[
\delta_T \frac{d}{ds} [U(s) \delta_T \xi(\lambda + 12)] = 90\alpha \tag{47}
\]
for region I, and
\[
\delta_T \frac{d}{ds} [U(s) \delta_T \xi(\lambda_2 + 12)] = 90\alpha \tag{48}
\]
for region II. Integrating Eqs. (47) and (48), in the respective regions, with respect to $s$, one can obtain local thermal boundary layer thicknesses
\[
\left(\frac{\delta_T(\theta)}{D}\right) \cdot Re_{dp}^{1/(n+1)} Pr_p^{1/3} = \left\{\begin{array}{ll}
\sqrt{45F_1(n, \theta)} & \text{for region I} \\
\sqrt{45F_2(n, \theta)} & \text{for region II}
\end{array}\right. \tag{49}
\]
where the Prandtl number, $Pr_p$, for a power-law fluid is defined as
\[
Pr_p = \frac{U_{app}D}{\alpha} Re_{dp}^{-2/(n+1)} \tag{50}
\]
The functions $F_1(n, \theta)$ and $F_2(n, \theta)$ in Eq. (49) are given by
\[
F_1(n, \theta) = \frac{f_1(\theta)}{\sin^2 \theta(\lambda_1 + 12)^n} \left[\frac{2^{n-2}n \lambda_1 \sin \theta}{\sin 2\theta(\lambda_1 + 12)^n 6}ight]^{1/(n+1)} \tag{51}
\]
\[
F_2(n, \theta) = \frac{f_2(\theta)}{\sin^2 \theta(\lambda_2 + 12)^n} \left[\frac{2^{n-2}n \lambda_2 \sin \theta}{\sin 2\theta(\lambda_2 + 12)^n 6}ight]^{1/(n+1)} \tag{52}
\]
with
\[
f_1(\theta) = \int_0^\theta \sin \theta(\lambda_1 + 12)d\theta \tag{53}
\]
\[
f_2(\theta) = \int_\theta^{\tilde{\theta}_1} \sin \theta(\lambda_2 + 12)d\theta \tag{54}
\]
\[
f_3(\theta) = \frac{f_1(\theta)}{\lambda_1 + 12} + \frac{f_2(\theta)}{\lambda_2 + 12} \tag{55}
\]
The local heat transfer coefficients, for the isothermal boundary condition, in both the regions can be written as
\[
h_1(\theta) = \frac{3k_f}{2\delta_T} \quad h_2(\theta) = \frac{3k_f}{2\delta_T} \tag{56}
\]
The first term on the right-hand side shows the heat transfer from the front stagnation point to the separation point and can be written as
\[
\int_0^{\theta_1} h(\theta)d\theta = \frac{1}{\pi} \int_0^{\theta_1} h(\theta)d\theta + \frac{1}{\pi} \int_{\theta_1}^\pi h(\theta)d\theta \tag{57}
\]
where $\theta_1$ is the angle where the values of the pressure gradient parameter are zero. This angle is found to be very close to $\pi/2$ for all values of $n$. So, the dimensionless average heat transfer coefficient of the cylinder from the front stagnation point to the separation point can be obtained, using Eqs. (49)–(53), for different values of $n$ and then correlated them to obtain a single expression in terms of $Re_{dp}$, and $Pr_p$. This expression is given by
\[
Nu_{dp} = 0.593n^{0.17} Re_{dp}^{1/(n+1)} Pr_p^{1/3} \tag{58}
\]
The second term on the right-hand side of Eq. (55) gives the heat transfer from the separation point to the rear stagnation point. The integral analysis can predict only heat transfer values from the front stagnation point to the separation point. The experiments (Zukauskas and Ziezgda [40], Fand and Keswani [41], and Nakamura and Igarashi [42] among others for Newtonian fluids and Rao [27] for non-Newtonian fluids) show that the heat transfer from the rear portion of the cylinder increases with Reynolds number. For Newtonian fluids, Van der Hegge Zijnen [43] demonstrated that the average heat transferred from the rear portion of the cylinder can be determined from $Nu_{dp} = 0.001 Re_{dp}$ that shows the weak dependence of average heat transfer from the rear portion of the cylinder on Reynolds number. The same weak depen-
dence can be observed from Rao [27] experiments for non-
Newtonian fluids.

Thus, the total average dimensionless heat transfer, for isother-
mal boundary condition, can be written as

\[ \text{Nu}_{D|\text{isothermal}} = 0.593n^{-0.17} \text{Re}_{Dp}^{1/(n+1)} \text{Pr}_p^{1/3} + 0.001 \text{Re}_{Dp} \]  

(58)

For the isoflux boundary condition, the energy integral equation can be written as

\[ \frac{d}{ds} \int_0^{\delta_f} (T - T_w) \, \frac{d\eta}{\rho c_p} = \frac{q}{\rho c_p} \]  

(59)

Assuming constant heat flux and thermophysical properties and
using Eqs. (14) and (17), Eq. (58) can be simplified to

\[ \frac{d}{ds} \left[ (U(s) \delta_f) (\lambda + 12) \right] = 90 \frac{k_f}{\rho c_p} \]  

(60)

Rewriting Eq. (60) for the two regions in the same way as Eq. (46), one can obtain local thermal boundary layer thicknesses \( \delta_{\lambda_1} \) and \( \delta_{\lambda_2} \) under isoflux boundary condition. The local surface temperatures for the two regions can then be obtained from temperature distribution

\[ \Delta T_I(\theta) = \frac{2q \delta_{\lambda_1}}{3k_f} \quad \text{and} \quad \Delta T_{II}(\theta) = \frac{2q \delta_{\lambda_2}}{3k_f} \]  

(61)

The local heat transfer coefficient can now be obtained from its
definition as

\[ h_I(\theta) = \frac{q}{\Delta T_I(\theta)} \quad \text{and} \quad h_{II}(\theta) = \frac{q}{\Delta T_{II}(\theta)} \]  

(62)

which give the local Nusselt number for the cross flow over a
cylinder with constant flux

\[ \text{Nu}_{D}(\theta)|_{\text{isoflux}} = \frac{3}{2} \left( \frac{2}{45G_1(n, \theta)} \right)^{1/(n+1)} \]  

(63)

where

\[ G_1(n, \theta) = \frac{\theta}{\sin \theta (\lambda_1 + 12)} \left[ \frac{2^{n-2} n \lambda_1 \sin^n \theta}{\sin^2 \theta \left( \frac{\lambda_1 + 12}{6} \right)^{1-n}} \right]^{1/(n+1)} \]  

(64)

\[ G_2(n, \theta) = \frac{g(\theta)}{\sin \theta} \left[ \frac{2^{n-2} n \lambda_2 \sin^n \theta}{\sin^2 \theta \left( \frac{\lambda_1 + 12}{6} \right)^{1-n}} \right]^{1/(n+1)} \]  

(65)

with

\[ g(\theta) = \left[ \frac{\theta}{\lambda_1 + 12} + \frac{\theta - \pi/2}{\lambda_2 + 12} \right] \]

Following the same procedure for the average heat transfer coefficient as mentioned earlier, one can obtain the average Nusselt number for an isoflux cylinder as

\[ \text{Nu}_{D|\text{isoflux}} = 0.627n^{-0.19} \text{Re}_{Dp}^{1/(n+1)} \text{Pr}_p^{1/3} \]  

(66)

Combining the results for both thermal boundary conditions, we have

\[ \frac{\text{Nu}_{D}}{\text{Re}_{Dp}^{1/(n+1)} \text{Pr}_p^{1/3}} = \begin{cases} 0.593n^{-0.17} & \text{for UWT} \\ 0.627n^{-0.19} & \text{for UWF} \end{cases} \]  

(67)

These correlations agree with the heat transfer coefficients for a
Newtonian fluid \((n=1)\) over a circular cylinder (Khan et al. [3]).

**Results and Discussion**

**Flow Characteristics.** The dimensionless local skin shear stress, \( C_f \), is plotted in Fig. 4 for different power-law fluids. It shows that \( C_f \) is zero at the stagnation point for each fluid and reaches a maximum at \( \theta \approx 58^\circ \).

The increase in shear stress is caused by the deformation of the velocity profiles in the boundary layer, a higher velocity gradient at the wall, and a thicker boundary layer. In the region of decreasing \( C_f \) preceding the separation point, the pressure gradient decreases further and finally \( C_f \) falls close to zero around the separation point, where boundary-layer separation occurs. Beyond this point, \( C_f \) remains close to zero up to the rear stagnation point. It also shows that the skin friction increases with the increase in power-law index \( n \). Thus pseudoplastic (shear-thinning) fluids offer less skin friction than dilatant (shear-thickening) fluids, which is in accordance with the numerical results of Agarwal et al. [34]. The results of Newtonian fluids \((n=1)\) are compared with the numerical data of Schönauer [44], which shows good agreement for the entire range.

The variation of the total drag coefficient \( C_D \) with the power-law index \( n \) for different Reynolds number is illustrated in Fig. 5.
It shows that for a given Reynolds number, the drag coefficient \( C_D \) increases linearly with \( n \) and for a given fluid it decreases with the increase in Reynolds number. The drag coefficients for pseudoplastic fluids are found to be lower than dilatant fluids.

The effect of Reynolds number on the drag coefficients for different fluids is shown in Fig. 6. Since there are no other experimental/numerical results to compare with pseudoplastic/dilatant fluids, comparisons are made with the experimental results of Wieselsberger [45] for the air \((n=1)\) only. The comparison shows good agreement with the Newtonian case.

**Heat Transfer Characteristics.** The heat transfer parameter (HTP) \( \frac{\text{Nu}_D}{\text{Re}_D^{1/(n+1)} \text{Pr}_D^{1/3}} \) is presented in Fig. 7 for both the isothermal and isoflux boundary conditions. It shows that HTP decreases with the increase in the power law index \( n \). Thus pseudoplastic fluids transfer more heat than dilatant fluids for the same thermal boundary condition. The isoflux boundary condition gives a higher heat transfer coefficient for both types of fluids.

Figure 8 shows the comparison of the average heat transfer coefficients \( \frac{\text{Nu}_D}{\text{Pr}_D^{1/3}} \) versus \( \text{Re}_D \) for Newtonian fluids. Here, the experimental results of Hilpert [46] and McAdams [47] for air are compared with the present model for \( n=1 \). The average heat transfer coefficients are also presented for non-Newtonian fluids. Experimental results of Takahashi et al. [23] and Mizushina et al. [14] for different CMC (carboxy methyl cellulose) solutions are compared with the present models for isothermal boundary condition. The comparison is found to be in good agreement for all fluids.

**Conclusions**

An integral approach is employed to investigate the fluid flow and heat transfer from an isolated circular cylinder submerged in power-law fluids. Closed form solutions are developed for both the drag and heat transfer coefficients in terms of generalized Reynolds and Prandtl numbers. The correlations of heat transfer are developed for both isothermal and isoflux boundary conditions. It is found that pseudoplastic fluids offer less skin friction and higher heat transfer coefficients than dilatant fluids. Furthermore, the drag coefficients decrease and the heat transfer increases with the decrease in power-law index. It is shown that the present results are in good agreement with the available suitable data for the full laminar range of Reynolds number in the absence of free stream turbulence and blockage effects.

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**Nomenclature**

- \( C_D \): total drag coefficient
- \( C_{DF} \): friction drag coefficient
- \( C_{DP} \): pressure drag coefficient
- \( C_f \): skin friction coefficient \( = 2 \eta / \rho U^2 \)
- \( C_p \): pressure coefficient \( = 2 \Delta P / \rho U^2 \)
- \( c_p \): specific heat of the fluid \((\text{J/kg·K})\)
- \( D \): cylinder diameter \((\text{m})\)
- \( k \): thermal conductivity \((\text{W/m·K})\)
- \( h \): average heat transfer coefficient \((\text{W/m}^2·\text{K})\)
- \( m \): consistency index for non-Newtonian viscosity \((\text{Pa}·\text{s})\)
- \( n \): power-law index
- \( \text{Nu}_D \): average Nusselt number based on the diameter of the cylinder \( = hD/k_f \)
Greek Symbols

\[ \begin{align*}
Pr_p &= \text{Prandtl number for power-law fluids} \\
&= (U_{app}/D)\sigma Re_{pp}^{2/(\gamma+1)} \\
P &= \text{pressure (N/m}^2) \\
q &= \text{heat flux (W/m}^2) \\
Re_{DP} &= \text{generalized Reynolds number based on the diameter of the cylinder} = D q^{1/2} / \eta m \\
s &= \text{distance along the curved surface of the circular cylinder measured from the forward stagnation point (m)} \\
T &= \text{temperature (°C)} \\
U_{app} &= \text{approach velocity (m/s)} \\
U(s) &= \text{potential flow velocity just outside the boundary layer} = 2U_{app}\sin \theta (m/s) \\
u &= \text{\(s\)-component of velocity in the boundary layer (m/s)} \\
v &= \text{\(\eta\)-component of velocity in the boundary layer (m/s)}
\end{align*} \]

Subscripts

\[ \begin{align*}
\alpha &= \text{ambient} \\
\delta &= \text{fluid or friction} \\
H &= \text{hydrodynamic} \\
p &= \text{pressure} \\
s &= \text{separation} \\
T &= \text{thermal or temperature} \\
w &= \text{wall}
\end{align*} \]

References


