

Optimization of Microchannel Heat Sinks Using Entropy Generation Minimization Method

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Abstract

In this study, an entropy generation minimization (EGM) procedure is employed to optimize the overall performance of microchannel heat sinks. This allows the combined effects of thermal resistance and pressure drop to be assessed simultaneously as the heat sink interacts with the surrounding flow field. New general expressions for the entropy generation rate are developed by considering an appropriate control volume and applying mass, energy, and entropy balances. The effect of channel aspect ratio, fin spacing ratio, heat sink material, Knudsen numbers and accommodation coefficients on the entropy generation rate is investigated in the slip flow region. Analytical/empirical correlations are used for heat transfer and friction coefficients, where the characteristic length is used as the hydraulic diameter of the channel. A parametric study is also performed to show the effects of different design variables on the overall performance of microchannel heat sinks.

Nomenclature

A	=	total heating surface area [m^2]
A_c	=	cross-section area of a single fin [m^2]
D_h	=	hydraulic diameter [m]
f	=	friction factor
G	=	volume flow rate [m^3/s]
g, l	=	equality and inequality constraints
H_c	=	channel height [m]
h	=	specific enthalpy of the fluid [J/kg]
h_{av}	=	average heat transfer coefficient [$W/m^2 \cdot K$]
j	=	number of imposed constraints
Kn	=	Knudsen number $\equiv \lambda/D_h$
k	=	thermal conductivity of solid [$W/m \cdot K$]
k_{ce}	=	sum of entrance and exit losses
k_{eq}	=	ratio of thermal conductivity of fluid to solid $\equiv k_f/k$
k_f	=	thermal conductivity of fluid [$W/m \cdot K$]
$\mathcal{L}$	=	Lagrangian function
L	=	length of channel in flow direction [m]
m	=	fin parameter [m^{-1}]
$\dot{m}$	=	total mass flow rate [kg/s]
N	=	total number of microchannels
n	=	number of design variables
Nu_{D_h}	=	Nusselt number based on hydraulic diameter $\equiv D_h h_{av}/k_f$
Pe_{D_h}	=	Peclet number based on hydraulic diameter $\equiv D_h U_{av}/\alpha$
Pr	=	Prandtl number

Q_b	=	heat transfer rate from the base [W]
Q_{fin}	=	heat transfer rate from the fin [W]
q	=	heat flux [W/m^2]
R	=	resistance [K/W]
Re_{D_h}	=	Reynolds number based on hydraulic diameter $\equiv D_h U_{av}/\nu$
S_{gen}	=	total entropy generation rate [W/K]
s	=	specific entropy of fluid [$J/kg \cdot K$]
T	=	absolute temperature [K]
t	=	thickness [m]
U_{av}	=	average velocity in channels [m/s]
u	=	specific internal energy [J/kg]
v	=	specific volume of fluid [m^3/kg]
W	=	width of heat sink [m]
w_c	=	half of the channel width [m]
w_w	=	half of the fin thickness [m]
x_i	=	design variables

Greek Symbols

ΔP	=	pressure drop across microchannel [Pa]
α	=	thermal diffusivity [m^2/s] or constant defined by Eq. (23)
α_c	=	channel aspect ratio $\equiv 2w_c/H_c$
α_{hs}	=	heat sink aspect ratio $\equiv L/2w_c$
β	=	fin spacing ratio $\equiv w_c/w_w$
γ	=	ratio of specific heats $\equiv c_p/c_v$
λ, χ	=	mean free path [m] or Lagrangian multipliers
μ	=	absolute viscosity of fluid [$kg/m \cdot s$]
ν	=	kinematic viscosity of fluid [m^2/s]
ρ	=	fluid density [kg/m^3]
σ	=	tangential momentum accommodation coefficient (TMAC)
σ_t	=	energy accommodation coefficient
ζ_u	=	slip velocity coefficient
ζ_t	=	temperature jump coefficient

Subscripts

a	=	ambient
av	=	average
b	=	base surface
cap	=	capacity
$conv$	=	convective
f	=	fluid
fin	=	single fin
hs	=	heat sink
in	=	entrance
out	=	exit
s	=	slip

th = thermal
 w = wall

Introduction

One of the important aspects of electronics packaging is the thermal management of electronic devices. The trend in the electronic industry toward denser and more powerful products requires a higher level of performance from cooling technology. After the pioneering work of Tuckerman and Pease [1], microchannels have received considerable attention especially in microelectronics. Microchannel heat sinks provide a powerful means for dissipating high heat flux with small allowable temperature rise. Due to an increase in surface area and a decrease in the convective resistance at the solid/fluid interface, heat transfer is enhanced in microchannels. These heat sinks can be applied in many important fields like microelectronics, aviation and aerospace, medical treatment, biological engineering, materials sciences, cooling of high temperature superconductors, thermal control of film deposition, and cooling of powerful laser mirrors. The two important characteristics of microchannel heat sinks are the high heat transfer coefficients and lower friction factors.

In the present study an entropy generation minimization (EGM) criterion is used to determine the overall performance of a microchannel heat sink, which allows the combined effect of thermal resistance and pressure drop to be assessed through the simultaneous interaction with the heat sink. A general expression for the entropy generation rate is obtained by using the conservation equations for mass and energy with the entropy balance.

Literature Review

The concept of “micro-channel heat sinks” was first introduced by Tuckerman and Pease [1]. Following this work, several experimental, numerical and theoretical studies on the optimization of micro-channel heat sinks have been carried out. These studies are reviewed in this section.

Steinke and Kandlikar [2] presented a comprehensive review of conventional single-phase heat transfer enhancement techniques. They discussed several passive and active enhancement techniques for minichannels and microchannels. Some of their proposed enhancement techniques include fluid additives, secondary flows, vibrations, and flow pulsations.

Kandlikar and Grande [3] explored the cooling limits of the plain rectangular microchannels with water cooling for high heat flux dissipation and illustrated the need for enhanced microchannels. They described a simplified and well-established fabrication process to fabricate three-dimensional microchannels. They also demonstrated the efficacy of the fabrication process in fabricating complex microstructures within a microchannel.

Knight et al. [4, 5], Perret et al. [6, 7], Kim [8], Upadhye and Kandlikar [9], Kandlikar and Grande [3] and Liu and Garimella [10] developed theoretical optimization procedures to minimize the overall thermal resistance of microchannel heat sinks for a given pressure drop, whereas

Singhal et al. [11], and Kandlikar and Upadhye [12] carried out analyses to optimize the channel configuration that yields a minimum pressure drop for a given heat load. They presented solution procedures for laminar/turbulent flow and generalized their results in terms of different heat sink parameters.

Kleiner et al. [13], Aranyosi et al. [14], and Harris et al. [15] investigated theoretically and experimentally the performance of microchannel heat sinks. They modeled the performance in terms of thermal resistance, pressure drop, and pumping power as a function of heat sink dimensions, tube dimensions, and air flow rate. Their results show an enhancement in heat removal capability compared to traditional forced air-cooling schemes.

Garimella and Singhal [16] and Jang and Kim [17] analyzed experimentally the pumping requirements of microchannel heat sinks and optimized the size of the microchannels for minimum pumping requirements. Jang and Kim [17] showed that the cooling performance of an optimized microchannel heat sink subject to an impinging jet is enhanced by about 21% compared to that of the optimized microchannel heat sink with parallel flow under the fixed-pumping-power condition.

Choquette et al. [18], Zhimin and Fah [19], Meysenc et al. [20], Chong et al. [21], Liao et al. [22], Ryu et al. [23], Wei and Joshi [24], and Jean-Antoine et al. [25] performed numerical optimizations of thermal performance of microchannel heat sinks for given pumping power/pressure drop. Zhimin and Fah [17] considered laminar, turbulent, developed and developing flow and heat transfer in the analysis. Using self-developed software, they investigated the effects of heat sink channel aspect ratio, fin width to channel width ratio and the channel width on the performance of heat sink. They found that the channel width is the most important parameter and governs the performance of a microchannel heat sink. Min et al. [26] showed numerically that the tip clearance can improve the cooling performance of a microchannel heat sink when tip clearance is smaller than a channel width. Delsman et al. [27] performed a CFD study for the optimization of the plate geometry to reach the design target regarding the quality of the flow distribution.

Haddad et al. [28] investigated numerically the entropy generation due to steady laminar forced convection fluid flow through parallel plate microchannels. They discussed the effect of Knudsen, Reynolds, Prandtl, Eckert numbers and the nondimensional temperature difference on entropy generation within the microchannel. The entropy generation within the microchannel is found to decrease as Knudsen number increases, and it is found to increase as Reynolds, Prandtl, Eckert numbers and the nondimensional temperature difference increase. The contribution of the viscous dissipation in the total entropy generation increases as Knudsen number increases over wide ranges of the flow controlling parameters.

It is obvious from the literature survey that all the studies related to optimization of microchannel heat sinks are used to minimize thermal resistance for a given pres-

sure drop or to minimize pumping power for a specified thermal resistance. No study exists to optimize both thermal and hydraulic resistances simultaneously. In this study, an entropy generation minimization, EGM, method is applied as a unique measure to study the optimization of thermal and hydraulic resistances simultaneously. All

relevant design parameters for microchannel heat sinks, including geometric parameters, material properties and flow conditions are optimized simultaneously by minimizing the entropy generation rate S_{gen} subject to manufacturing and design constraints.

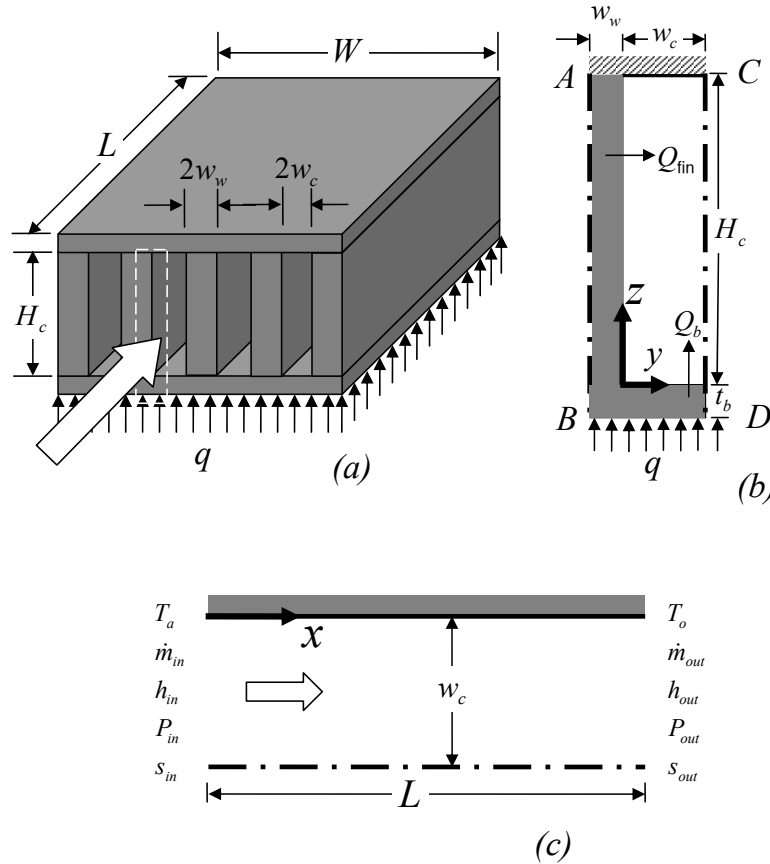


Fig. 1 Microchannel heat sink: geometry, control volume and definitions.

Model Development

The geometry of a microchannel heat sink is shown in Figure 1 (a). The length of the heat sink is L and the width is W . The top surface is insulated and the bottom surface is uniformly heated. A coolant passes through a number of microchannels along the x -axis and takes heat away from the heat dissipating electronic component attached below. There are N channels and each channel has a height H_c and width $2w_c$. The thickness of each fin is $2w_w$ whereas the thickness of the base is t_b . The temperature of the channel walls is assumed to be T_w with an inlet water temperature of T_a . At the channel wall, the slip flow velocity and temperature jump boundary conditions were applied to calculate friction and heat transfer coefficients.

Taking advantage of symmetry, a control volume is selected for developing the entropy generation model, as shown in Figure 1 (b). The length of the control volume is taken as unity for convenience and the width and height are taken as $w_w + w_c$ and $H_c + t_b$ respectively. This control volume includes half of the fin and half of the channel along with the base. The side surfaces AB and CD and the top surface AC of this CV can be regarded as

impermeable, adiabatic and shear free (no mass transfer and shear work transfer across these surfaces). The heat flux over the bottom surface BD of the CV is q . The ambient temperature is T_a and the surface temperature of the channel wall is T_w . The bulk properties of the coolant are represented by u_{in} , P_{in} , s_{in} at the inlet and by u_{out} , P_{out} , s_{out} at the outlet respectively. The irreversibility of this system is due to heat transfer across the finite temperature difference $T_b - T_a$ and to friction. Heat transfer in the control volume (Fig. 1b) is a conjugate, combining heat conduction in the fin and convection to the cooling fluid. To simplify the analysis, the following assumptions were employed:

Assumptions

1. Uniform heat flux on the bottom surface.
2. Smooth surface of the channel
3. Adiabatic fin tips
4. Isotropic material
5. Fully developed heat and fluid flow
6. Steady, laminar and 2-D flow

7. Slip flow (i.e., $0.001 \leq Kn \leq 0.1$) with negligible creep effects
8. Incompressible fluid with constant thermophysical properties
9. Negligible axial conduction in both the fin and fluid
10. Changes in kinetic and potential energies are negligible

The mass rate balance for the steady state reduces to

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m} \quad (1)$$

Assuming negligible changes in the kinetic and potential energies, the first law of thermodynamics for a steady state condition can be written as

$$Q = \dot{m} \left[\underbrace{(u_{out} + P_{out}v_{out})}_{h_{out}} - \underbrace{(u_{in} + P_{in}v_{in})}_{h_{in}} \right] \quad (2)$$

The combination of specific internal and flow energies is defined as specific enthalpy; therefore, the energy rate balance reduces further to:

$$Q = \dot{m}(h_{out} - h_{in}) \quad (3)$$

From the second law of thermodynamics

$$\frac{dS_{cv}}{dt} = \dot{m}(s_{in} - s_{out}) + \frac{Q_{fin}}{T_b} + \frac{Q_b}{T_b} + S_{gen} \quad (4)$$

For steady state, $\frac{dS_{cv}}{dt} = 0$, and the total heat transferred from the bottom of the heat sink $Q = Q_{fin} + Q_b$, so the entropy rate balance reduces to

$$S_{gen} = \dot{m}(s_{out} - s_{in}) - \frac{Q}{T_b} \quad (5)$$

where T_b represents the absolute temperature of the heat sink base.

Integrating Gibb's equation from inlet to the outlet gives

$$h_{out} - h_{in} = T_a(s_{out} - s_{in}) + \frac{1}{\rho}(P_{out} - P_{in}) \quad (6)$$

Combining Eqs. (3, 5 and 6), the entropy generation rate can be written as:

$$S_{gen} = Q \left[\frac{1}{T_a} - \frac{1}{T_b} \right] + \frac{\dot{m}\Delta P}{\rho T_a} \quad (7)$$

Rearranging the terms and writing $\theta_b = T_b - T_a$, we have

$$S_{gen} = \frac{Q\theta_b}{T_a T_b} + \frac{\dot{m}\Delta P}{\rho T_a} \quad (8)$$

As $\theta_b = QR_{th}$, the entropy generation rate can be written as:

$$S_{gen} = \frac{Q^2 R_{th}}{T_a T_b} + \frac{\dot{m}\Delta P}{\rho T_a} \quad (9)$$

where R_{th} is the total thermal resistance and ΔP is the total pressure drop across the channel. This expression describes the entropy generation rate model completely and

it shows that the entropy generation rate depends on the total thermal resistance R_{th} and the pressure drop across the channel, provided that the heat load and ambient conditions are specified. If G is the volume flow rate, then the total mass flow rate can be written as

$$\dot{m} = \rho G \quad (10)$$

The average velocity in the channel, U_{av} is given by:

$$U_{av} = \frac{\dot{m}}{N\rho(2w_c)H_c} \quad (11)$$

where N is the number of channels given by

$$N = \frac{W - w_w}{w_c + w_w} \quad (12)$$

Thermal Resistance

The total thermal resistance is defined as

$$\begin{aligned} R_{th} &= \frac{\theta_b}{Q} \\ &= \frac{T_b - T_f}{Q} + \frac{T_f - T_a}{Q} \\ &= R_{conv} + R_{fluid} \end{aligned} \quad (13)$$

where T_b is the heat sink base temperature and T_f is the bulk fluid temperature. These temperatures are given by

$$T_b = T_a + QR_{th} \quad (14)$$

$$T_f = T_a + \frac{Q}{2\dot{m}C_p} \quad (15)$$

Assuming $h_{fin} = h_{base} = h_{av}$, the convective and fluid thermal resistances can be written as

$$R_{conv} = \frac{1}{h_{av}A} \quad (16)$$

$$R_{fluid} = \frac{1}{\dot{m}C_p} \quad (17)$$

where A is the total heat surface area and is given by

$$A = 2NL(w_c + H_c\eta_{fin}) \quad (18)$$

with

$$\eta_{fin} = \frac{\tanh(mH_c)}{mH_c} \quad (19)$$

$$m = \sqrt{\frac{2h_{av}}{kw_w}} \quad (20)$$

Using slip flow velocity and temperature jump boundary conditions, Khan [29] solved the energy equation and developed the following theoretical model for the dimensionless heat transfer coefficient for a parallel plate microchannel:

$$Nu_{D_h} = \frac{h_{av}D_h}{k_f} = \frac{140}{17(1 + \alpha_c)K_s} \quad (21)$$

where

$$K_s = 1 - \frac{6}{17} \left(\frac{U_s}{U_{av}} \right) + \frac{2}{51} \left(\frac{U_s}{U_{av}} \right)^2 - \frac{140}{17} \zeta_t \quad (22)$$

$$\alpha_c = \frac{2w_c}{H_c} \quad (23)$$

with

$$\frac{U_s}{U_{av}} = \frac{6\alpha}{1+6\alpha} \quad (24)$$

$$\alpha = \frac{2\zeta_u}{1+\alpha_c} \quad (25)$$

$$\zeta_u = \left(\frac{2-\sigma}{\sigma} \right) Kn \quad (26)$$

$$\zeta_t = \left(\frac{2-\sigma_t}{\sigma_t} \right) \cdot \frac{2\gamma}{\gamma+1} \cdot \frac{Kn}{Pr} \quad (27)$$

Using Eqs. (14)-(21), Eq. (13) can be written as

$$R_{th} = \frac{2C_3\alpha_{hs}}{k_f LC_1 C_2} \quad (28)$$

where

$$C_1 = N\alpha_{hs}(2\eta_{fin} + \alpha_c) \quad (29)$$

$$C_2 = (1 + \alpha_c)/\alpha_c \quad (30)$$

$$C_3 = \frac{1}{Nu_{D_h}} + \frac{C_1}{Pe_{D_h}} \quad (31)$$

with

$$\alpha_{hs} = \frac{L}{2w_c} \quad (32)$$

$$\eta_{fin} = \frac{\tanh(mH_c)}{mH_c} \quad (33)$$

$$mH_c = \frac{1}{\alpha_c} \sqrt{2\beta k_{eq} Nu_{D_h} (1 + \alpha_c)} \quad (34)$$

$$k_{eq} = \frac{k_f}{k} \quad (35)$$

$$\beta = w_c/w_w \quad (36)$$

Pressure Drop

The pressure drop associated with flow across the channel is given by:

$$\Delta P = \frac{\rho U_{av}^2}{2} \left[k_{ce} + \left(f \frac{L}{D_h} \right) \right] \quad (37)$$

where the friction factor f depends on the Reynolds number, channel aspect ratio and slip velocity coefficient (Khan et al. [29]), and can be written as:

$$f = \frac{24}{Re_{D_h}} \left(\frac{1}{1+6\alpha} \right) \left(\frac{1}{1+\alpha_c} \right) \quad (38)$$

Kleiner et al. [13] used experimental data from Kays and London [30] and derived the following empirical correlation for the entrance and exit losses k_{ce} in terms of channel width and fin thickness:

$$k_{ce} = 1.79 - 2.32 \left(\frac{w_c}{w_c + w_w} \right) + 0.53 \left(\frac{w_c}{w_c + w_w} \right)^2 \quad (39)$$

Entropy Generation Rate

Substituting Eqs. (28) and (37) in to Eq. (9), we get

$$S_{gen} = \frac{Q^2}{T_a T_b} \cdot \frac{2\alpha_{hs} C_3}{L k_f C_1 C_2} + \frac{\rho w_c H_c U_{av}^3}{T_a} C_4 \quad (40)$$

$$= S_{gen,h} + S_{gen,f} \quad (41)$$

where $S_{gen,h}$ and $S_{gen,f}$ show the entropy generation rates due to heat transfer and fluid friction respectively and C_4 is given by

$$C_4 = \left[k_{ce} + \left(f \frac{L}{D_h} \right) \right] \quad (42)$$

Optimization Procedure

The problem considered in this study is to minimize the entropy generation rate, given by Eq. (9) or (40), for the optimal overall performance of microchannel heat sinks. If $f(\mathbf{x})$ represent the entropy generation rate that is to be minimized subject to equality constraints $g_j(x_1, x_2, \dots, x_n) = 0$ and inequality constraints $l_k(x_1, x_2, \dots, x_n) \geq 0$, then the complete mathematical formulation of the optimization problem may be written in the following form:

$$\text{minimize } f(\mathbf{x}) = S_{gen}(\mathbf{x}) \quad (43)$$

subject to the equality constraints:

$$g_j(\mathbf{x}) = 0, \quad j = 1, 2, \dots, m \quad (44)$$

and inequality constraints

$$l_j(\mathbf{x}) \geq 0, \quad j = m+1, \dots, n \quad (45)$$

where g_j and l_j are the imposed equality and inequality constraints and $\mathbf{x}$ denotes the vector of the design variables $(x_1, x_2, x_3, \dots, x_n)^T$. The objective function can be redefined by using the Lagrangian function as follows:

$$\mathcal{L}(\mathbf{x}, \lambda, \chi) = f(\mathbf{x}) + \sum_{j=1}^m \lambda_j g_j(\mathbf{x}) - \sum_{j=m+1}^n \chi_j l_j(\mathbf{x}) \quad (46)$$

where λ_j and χ_j are the Lagrange multipliers. The λ_j can be positive or negative but the χ_j must be ≥ 0 . The necessary condition for $\mathbf{x}^*$ to be a local minimum of the problem, under consideration, is that the Hessian matrix of $\mathcal{L}$ should be positive semidefinite, i.e.

$$\mathbf{v}^T \nabla^2 [(\mathbf{x}^*, \lambda^*, \chi^*)] \mathbf{v} \geq 0 \quad (47)$$

For a local minimum to be a global minimum, all the eigenvalues of the Hessian matrix should be ≥ 0 .

A system of non-linear equations is obtained, which can be solved using numerical methods such as a multivariable Newton-Raphson method. This method has been described in Stoecker [31] and applied by Culham and Muzychka [32], and Culham et al. [33] to study the optimization of plate fin heat sinks and by Khan et al. ([34], [35]) for pin fin heat sinks and tube banks. In this

study, the same approach is used to optimize the overall performance of a microchannel heat sink in such a manner that all relevant design conditions combine to produce the best possible heat sink for the given constraints. The optimized results are then presented in graphical form.

Parameter	Assumed Values
Channel or heat sink length, L (mm)	51
Width of heat sink, W (mm)	51
Channel height, H_c (mm)	1.7
Channel width, $2w_c$ (mm)	0.25
Fin thickness, $2w_w$ (mm)	0.14
Thermal conductivity of solid ($W/m \cdot K$)	148
Thermal conductivity of air ($W/m \cdot K$)	0.0261
Density of air (kg/m^3)	1.1614
Specific heat of air ($J/kg \cdot K$)	1007
Kinematic viscosity (m^2/s)	1.58×10^{-5}
Prandtl number (air)	0.71
Heat flux (W/cm^2)	15
Volume flow rate (m^3/s)	0.007
Ambient temperature ($^{\circ}C$)	27
Tangential momentum accommodation coefficient	0.85
Thermal energy coefficient	0.85

Table 1: Assumed Parameter Values

Kn	G	Optimized Design Variables		Performance Parameters		
	m^3/s	Channel Aspect Ratio α_c	Fin Spacing Ratio β	R_{hs} K/W	ΔP Pa	S_{gen} W/K
0.1	0.005	0.164	3.06	0.201	924	0.463
	0.007	0.194	4.99	0.160	902	0.398
	0.009	0.222	7.31	0.142	895	0.370
0.01	0.005	0.172	3.13	0.224	1549	0.511
	0.007	0.203	5.16	0.189	1485	0.462
	0.009	0.233	7.59	0.177	1441	0.451
0.001	0.005	0.175	3.16	0.229	1673	0.521
	0.007	0.207	5.20	0.195	1591	0.476
	0.009	0.237	7.67	0.185	1537	0.468

Table 2: Results of Optimization

Case Studies and Discussion

The assumed parameter values (Kleiner et al. [13]), given in Table 1, are used as the default case to determine the overall performance of microchannel heat sinks. The fluid properties are evaluated at the ambient temperature. The results obtained are shown in Table 2.

Microchannel width, height and fin thickness are optimized in terms of channel aspect ratio and fin spacing ratio for three different volume flow rates and Knudsen numbers in the slip flow region. The corresponding values

of the total thermal resistance, pressure drop and entropy generation rate are tabulated. It is observed that both channel aspect and fin spacing ratios increase with the increase in volume flow rate and Knudsen numbers in the slip flow region. The total thermal resistance and the pressure drop are found to decrease with the increase in volume flow rates and increase with the decrease in Knudsen numbers.

A parametric study is carried out to investigate the dependence of the optimal dimensions of the microchannel heat sink on the thermal and hydraulic performance.

The current study attempts to find an optimal flow rate, channel width, heat sink material and the effect of tangential momentum and energy coefficients. Figure 2 shows the variation of the total entropy generation rate as a function of volume flow rate for three different values of Knudsen numbers. This figure shows that as the Knudsen number increases the optimal entropy generation rate decreases due to the increase in velocity slip and temperature jump at the wall that lead to reduced heat transfer and momentum transfer from the wall to the fluid. It also shows that the optimal flow rate increases with the increase in Knudsen number.

The friction factor and the Nusselt number, in the laminar flow region, depends on the channel aspect ratio α_c . The effect of this aspect ratio is shown in Fig. 3 for three different Knudsen numbers in the slip flow region. The increase in aspect ratio increase the cross-sectional area available for flow and the total surface area available for convective heat transfer, which reduces thermal and hydraulic resistances. This figure also shows the decrease in the optimum entropy generation rate with an increase in Knudsen number.

The fin spacing ratio β plays an important role in the heat transfer analysis. It should be greater than 1 to ensure that there is flow in the microchannel. The effect of this ratio is shown in Fig. 4 for the same Knudsen numbers in the slip flow region. For each Knudsen number, the fin spacing ratio is optimized to get a minimum entropy generation rate. It can be observed that the optimum fin spacing ratio is a very weak function

of the Knudsen number, however, optimal entropy generation rate depends upon the Knudsen number and decreases with an increase in the Knudsen number. It shows that lower fin spacing ratios are appropriate in the case of microchannel heat sinks.

The total entropy generation rate, S_{gen} , includes the contributions due to heat transfer and viscous dissipation. The thermal conductivity of the heat sink material affects only the contribution due to heat transfer, $S_{gen,h}$, whereas the contribution due to viscous dissipation, $S_{gen,f}$, remains unchanged. Figure 5 shows the effect of thermal conductivity on the $S_{gen,h}$. Again the entropy generation rate decreases with the increase in Knudsen numbers. This figure shows that the entropy generation rate due to heat transfer decreases sharply from 25 $W/m \cdot K$ to 180 $W/m \cdot K$ and then becomes almost constant. The accommodation coefficients model the momentum and energy exchange of the gas molecules impinging on the walls. They are dependent on the specific gas and the surface quality and are tabulated in Schaaf and Chambre [36]. Very low values of σ and σ_t will increase the slip on the walls considerably even for small Knudsen number flows, due to the $2-\sigma/\sigma$ factor in Eqs. (26) and (27). The effect of these accommodation coefficients is shown in Fig. 6. This figure shows that for high Knudsen numbers, in the slip flow region, there is considerable change in the total entropy generation rate but as the Knudsen number decreases, the losses due to heat transfer and fluid friction

become negligible.

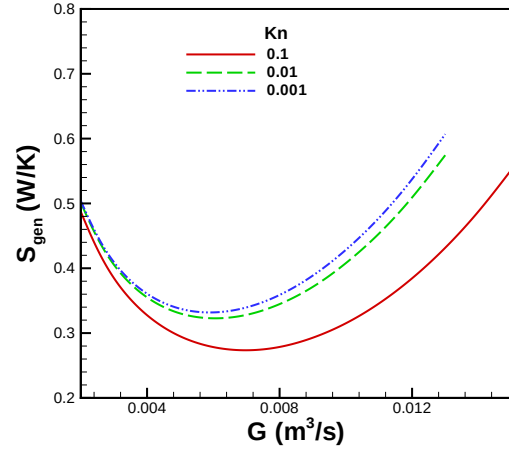


Fig. 2 Effect of Volume Flow Rate on S_{gen} .

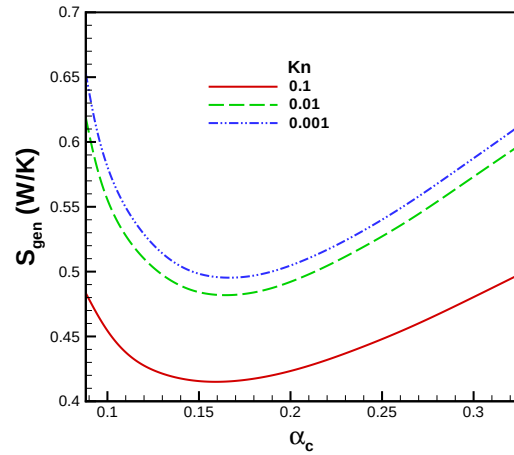


Fig. 3 Effect of Channel Aspect Ratio on S_{gen} .

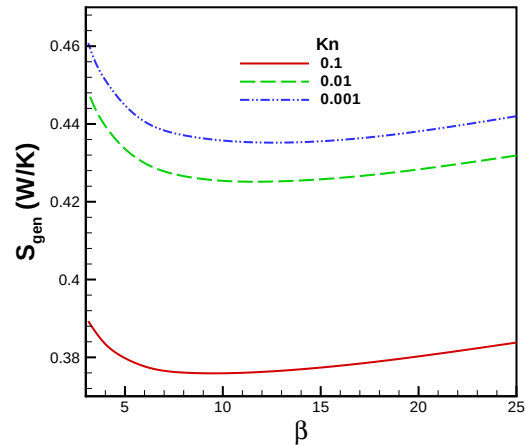


Fig. 4 Effect of Fin Spacing Ratio on S_{gen} .

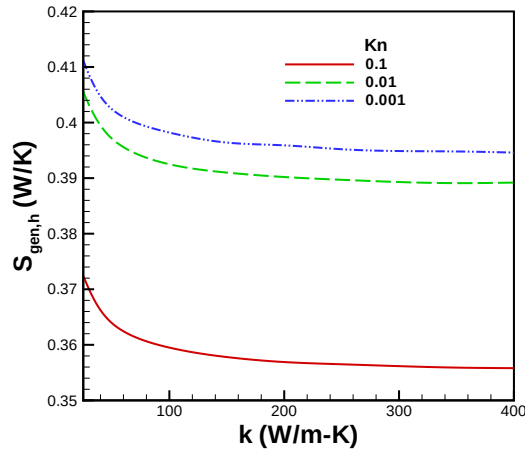


Fig. 5 Effect of Heat Sink Material on S_{gen} .

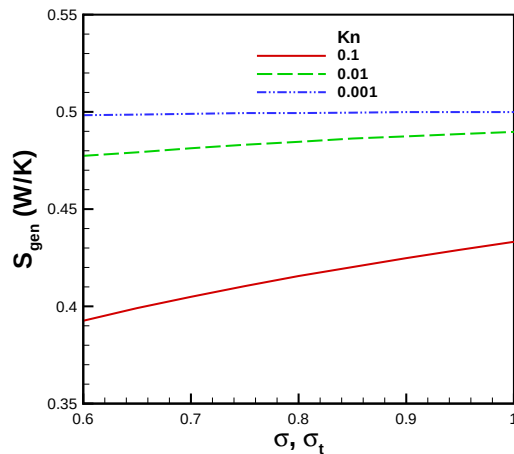


Fig. 6 Effect of Accommodation Coefficients on S_{gen} .

Conclusions

Based on the results of case studies and parametric optimization, it can be concluded that:

1. Thermal resistance and pressure drop across the microchannel decrease with an increase in volume flow rate and increase with a decrease in Knudsen numbers in the slip flow region.
2. The optimum channel aspect and fin spacing ratios increase with the volume flow rate to allow the decrease in thermal resistance and pressure drop.
3. The optimum entropy generation rate decreases with the increase in Knudsen numbers in the slip flow region.
4. Due to slip flow and temperature jump boundary conditions, fluid friction decreases and heat transfer increases in the microchannels which decreases the total entropy generation rate.
5. A low thermal conductivity heat sink with a large number of microchannels gives acceptable performance in terms of entropy generation rate.

6. Lower tangential momentum and energy accommodation coefficients results in higher entropy generation rates.

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References

1. Tuckerman, D. B., and Pease, R. F. W., High-Performance Heat Sinking for VLSI, IEEE Electron Device Letter, EDL-2, pp. 126-129, 1981.
2. Steinke, M. E. and Kandlikar, S. G., "Single-Phase Heat Transfer Enhancement Techniques in Microchannel and Minichannel Flows," Proceedings of the Second International Conference on Microchannels and Minichannels, Rochester, NY, USA, June 17-19, 2004.
3. Kandlikar, S. G., and Grande, W. J., "Evaluation of Single Phase Flow in Microchannels for High Flux Chip Cooling - Thermohydraulic Performance Evaluation and Fabrication Technology," Heat Transfer Engineering, Vol. 25, No. 8, pp. 5-16, 2004.
4. Knight, R.W., Goodling, J.S., Hall, D.J., "Optimal Thermal Design of Forced Convection Heat Sinks-Analytical," Journal of Electronic Packaging, Vol. 113, No. 3, pp. 313-321, 1991.
5. Knight, R. W., Hall, D. J., Goodling, J. S., and Jaeger, R.C., "Heat Sink Optimization with Application to Micro-channels," IEEE Transactions on Components, Hybrids, and Manufacturing Technology, Vol. 15, No. 5, pp. 832-842, 1992.
6. Perret, C., Schaeffer, Ch., and Boussey, J., "Microchannel Integrated Heat Sinks in Silicon Technology," Proceedings of the 1998 IEEE Industry Applications Conference, Oct 12-15 1998, St.Louis, MO, USA , Vol. 2, pp. 1051-1055, 1998.
7. Perret, C., Boussey, J., Schaeffer, C., and Coyaud, M., "Analytic Modeling, Optimization, and Realization of Cooling Devices in Silicon Technology," IEEE Transactions on Components and Packaging Technologies, Vol. 23, Issue 4, pp. 665 - 672, 2000.
8. Kim, S. J., "Methods for Thermal Optimization of Microchannel Heat Sinks," Heat Transfer Engineering, Vol. 25, No. 1, pp. 37-49, 2004.
9. Upadhye, H. R., and Kandlikar, S. G., "Optimization of Microchannel Geometry for Direct Chip Cooling Using Single Phase Heat Transfer," Proceedings of the Second International Conference on Microchannels and Minichannels, pp. 679-685, 2004.
10. Liu, D., and Garimella, S. V., "Analysis and Optimization of the Thermal Performance of Microchannel Heat Sinks," International Journal of Numerical Methods for Heat and Fluid Flow, Vol. 15, No. 1, pp. 7-26, 2005.

11. Singhal, V., Liu, D., and Garimella, S. V., "Analysis of Pumping Requirements for Microchannel Cooling Systems," International Electronic Packaging Technical Conference and Exhibition, Jul 6-11 2003, Maui, HI, Advances in Electronic Packaging, Vol. 2, 2003, pp. 473-479.
12. Kandlikar, S.G., and Upadhye, H.R., "Extending The Heat Flux Limit With Enhanced Microchannels in Direct Single-Phase Cooling of Computer Chips," IEEE Twenty First Annual Symposium on Semiconductor Thermal Measurement and Management, 15-17 March, pp. 8 - 15, 2005.
13. Kleiner, M.B., Kuhn, S.A., and Habberger, K., "High Performance Forced Air Cooling Scheme Employing Microchannel Heat Exchangers," IEEE Transactions on Components, Packaging, and Manufacturing Technology, Part A, Vol. 18, Issue 4, pp. 795 - 804, 1995.
14. Aranyosi, A., Bolle, L.M.R., and Buyse, H.A., "Compact Air-Cooled Heat Sinks for Power Packages," IEEE Transactions on Components, Packaging, and Manufacturing Technology, Part A, Vol. 20, Issue 4, pp. 442 - 451, 1997.
15. Harris, C., Despa, M., and Kelly, K., "Design and Fabrication of a Cross Flow Micro Heat Exchanger," Journal of Microelectromechanical Systems, Vol. 9, No. 4, pp. 502-508, 2000.
16. Garimella, S. V., and Singhal, V. , "Single-Phase Flow and Heat Transport and Pumping Considerations in Microchannel Heat Sinks," Heat Transfer Engineering, Vol. 25, No. 1, pp. 15-25, 2004.
17. Jang, S. P., and Kim, S. J., "Fluid Flow and Thermal Characteristics of a Microchannel Heat Sink Subject to an Impinging Air Jet," Journal of Heat Transfer, Vol. 127, No. 7, pp. 770-779, 2005.
18. Choquette, S.F., Faghri, M., Charmchi, M., and Asako, Y., "Optimum Design of Microchannel Heat Sinks," American Society of Mechanical Engineers, Dynamic Systems and Control Division (Publication) DSC, Vol. 59, Micro-Electro-Mechanical Systems (MEMS), pp. 115-126, 1996.
19. Zhimin, W., and Fah, C. K., "Optimum Thermal Design of Microchannel Heat Sinks," Proceedings of the 1997 1st Electronic Packaging Technology Conference, EPTC, Oct 8-10 1997, Singapore, pp. 123-129, 1997.
20. Meysenc, L., Saludjian, L., Bricard, A., Rael, S., and Schaeffer, C., "A High Heat Flux IGBT Micro Exchanger Setup," IEEE Transactions on Components, Packaging, and Manufacturing Technology, Part A, Vol. 20, Issue 3, pp. 334 - 341, 1997.
21. Chong, S.H., Ooi, K.T., and Wong, T.N., "Optimization of Single and Double Layer Counter Flow Microchannel Heat Sinks," Applied Thermal Engineering, Vol. 22, No. 14, pp. 1569-1585, 2002.
22. Liao, X.S., Liu, Y., Ning, Y.Q., Cheng, D.M., Wang, L., and Wang, L.J., "The Optimal Design of Structure Parameters for Microchannel Heat Sink," Proceedings of SPIE - The International Society for Optical Engineering, Vol. 4914, pp. 181-186, 2002.
23. Ryu, J.H., Choi, D.H., and Kim, S.J., "Numerical Optimization of the Thermal Performance of a Microchannel Heat Sink," International Journal of Heat and Mass Transfer, Vol. 45, No. 13, pp. 2823-2827, 2002.
24. Wei, X. and Joshi, Y., "Optimization Study of Stacked Micro-Channel Heat Sinks for Micro-Electronic Cooling," IEEE Transactions On Components and Packaging Technologies, Vol. 26, No. 1, pp. 55-61, 2003.
25. Jean-Antoine, G., Christophe, B., and Bernard, T., "Extruded Microchannel-Structured Heat Exchangers," Heat Transfer Engineering, Vol. 26, No. 3, pp. 56-63, 2005.
26. Min, J. Y., Jang, S. P., and Kim, S. J., "Effect of Tip Clearance on the Cooling Performance of a Microchannel Heat Sink," International Journal of Heat and Mass Transfer, Vol. 47, No. 5, pp. 1099-1103, 2004.
27. Delsman, E.R., Pierik, A., De Croon, M.H.J.M.; Kramer, G.J., and Schouten, J.C., "Microchannel Plate Geometry Optimization for Even Flow Distribution at High Flow Rates," Chemical Engineering Research and Design, Vol. 82, No. 2, p 267-273, 2004.
28. Haddad, O, Abuzaid, M., and Al-Nimr, M., "Entropy Generation Due to Laminar Incompressible Forced Convection Flow Through Parallel-Plates Microchannel," Entropy, Vol. 6, No. 5, pp. 413-426.
29. Khan, W. A., Yovanovich, M. M., and Culham, J. R., "Analytical Modeling of Fluid Flow and Heat Transfer in Microchannel Heat Sinks," ready to submit in International Journal of Heat and Mass Transfer.
30. Kays, W. M. and London, A. L., "Compact Heat Exchangers," McGraw Hill, New York, 1964.
31. Stoecker, W. F., Design of Thermal Systems, McGraw-Hill, New York, 1989.
32. Culham, R. J. and Muzychka, Y. S., "Optimization of Plate Fin Heat Sinks Using Entropy Generation Minimization," IEEE Transactions on Components and Packaging Technologies, Vol. 24, No. 2, pp. 159-165, 2001.
33. Culham, R. J., Khan, W. A., Yovanovich, M. M., and Muzychka, Y. S., "The Influence of Material Properties And Spreading Resistance in the Thermal Design of Plate Fin Heat Sinks," Proceedings of NHTC'01, 35th National Heat Transfer Conference, June 10-12, Anaheim, California, pp. 240-246, 2001.
34. Khan, W. A., Culham, J. R., and Yovanovich, M. M., "Optimization of Pin-Fin Heat Sinks Using Entropy Generation Minimization," IEEE Transactions on Components and Packaging Technologies, Vol. 28, No. 2, pp. 247-254, 2005.
35. Khan, W. A., Culham, J. R., and Yovanovich, M. M., "Optimal Design of Tube Banks in Crossflow Using Entropy Generation Minimization Method," presented at 44th AIAA Aerospace Sciences Meeting and Exhibit, Reno, Nevada, 9 - 12 Jan 2006.
36. Schaaf, S.A. and Chambre, P.L., "Flow of Rarefied Gases," Princeton University Press, 1961.