

The Influence of Material Properties and Spreading Resistance in the Thermal Design of Plate Fin Heat Sinks

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The thermal design of plate fin heat sinks can benefit from optimization procedures where all design variables are simultaneously prescribed, ensuring the best thermodynamic and air flow characteristic possible. While a cursory review of the thermal network established between heat sources and sinks in typical plate fin heat sinks would indicate that the film resistance at the fluid-solid boundary dominates, it is shown that the effects of other resistance elements, such as the spreading resistance and the material resistance, although of lesser magnitude, play an important role in the optimization and selection of heat sink design conditions. An analytical model is presented for calculating the best possible design parameters for plate fin heat sinks using an entropy generation minimization procedure with constrained variable optimization. The method characterizes the contribution to entropy production of all relevant thermal resistances in the path between source and sink as well as the contribution to viscous dissipation associated with fluid flow at the boundaries of the heat sink. The minimization procedure provides a fast, convenient method for establishing the "best case" design characteristics of plate fin heat sinks given a set of prescribed boundary conditions. It is shown that heat sinks made of composite materials containing nonmetallic constituents, with a thermal conductivity as much as an order of magnitude less than typical metallic heat sinks, can provide an effective alternative where performance, cost, and manufacturability are of importance. It is also shown that the spreading resistance encountered when heat flows from a heat source to the base plate of a heat sink, while significant, can be compensated for by making appropriate design modifications to the heat sink. [DOI: 10.1115/1.2429713]

Introduction

The range of applications where heat sinks are specified as an integral part of electronics components and circuitry has increased significantly in recent years. Heat sinks have typically been used in high powered, electronic devices, such as stereo equipment, computing devices, and communications equipment, however, newer applications in the automotive industry and consumer products are reshaping the way we design and manufacture heat sinks. In addition to traditional design requirements such as thermal performance and structural integrity, the high volume, low cost requirements associated with newer heat sink markets necessitates a reexamination of the materials and methods used to manufacture heat sinks.

The most common heat sink design used in electronics applications is the plate fin heat sink due to its relative simplicity and ease of manufacture. The majority of plate fin heat sinks are produced using an extrusion process with aluminum alloys being the material of choice due to their relatively high thermal conductivity and light weight. In some instances, plate fin heat sinks requiring fin aspect ratios (fin height/fin thickness) of greater than 15:1 are fabricated from plate stock where the fins and base plate are attached using a gluing or a mechanical deformation process. Typical aluminum alloys used for fabricated fins are 6063-T5 ($k=209$ W/mK) or 1100 series aluminum ($k=219$ W/mK). While die cast aluminum components have found favor in many manufacturing sectors, the use of die cast aluminum in heat sinks is

used sparingly due to the significant reduction in thermal conductivity due to increased porosity. Typical values of thermal conductivity for cast aluminum are in the range of 100 W/mK. Copper and copper alloys can provide a high thermal conductivity ($k=400$ W/mK for pure copper) but the increase in weight and cost associated with these materials limits their use. Plastic heat sinks have been used in some lower powered applications where temperature constraints are not as severe but low cost and ease of manufacture are important. Unfortunately, the low conductivity of plastics ($k=0.2$ W/mK) does impede the widespread use of plastics as a standard heat sink material.

The use of plastic composites, combining the low cost and ease of manufacture of plastics with the high conductivity of metals or graphite fibers, is a material combination that could meet the needs of the heat sink market over the next decade. While the thermal conductivity will never approach the values associated with metals, thermal conductivities of $k=25$ to 100 W/mK can be attained.

Figure 1 shows a relative comparison of the thermal conductivity of potential heat sink materials. It can be seen that these materials vary by more than three orders of magnitude in conductivity. Given the many choices of materials and their associated manufacturing considerations, designers are faced with the task of matching system constraints, such as flow conditions, power dissipation requirements, and space availability, with thermal design specifications. Heat sinks are most often selected based on an empirically derived relationship between thermal resistance and volumetric flow rate or approach flow velocity. While the functional relationship between thermal resistance and flow rate allows heat sinks to be directly compared, it does not consider the effects of pressure drop associated with different heat sink designs. The same heat sink placed in an application with ducted or unducted

Contributed by the Electrical and Electronic Packaging Division of ASME for publication in the JOURNAL OF ELECTRONIC PACKAGING. Manuscript received December 2, 2005; final manuscript received March 31, 2006. Review conducted by Koneru Ramakrishna. Paper presented at the 35th National Heat Transfer Conference, 2001.

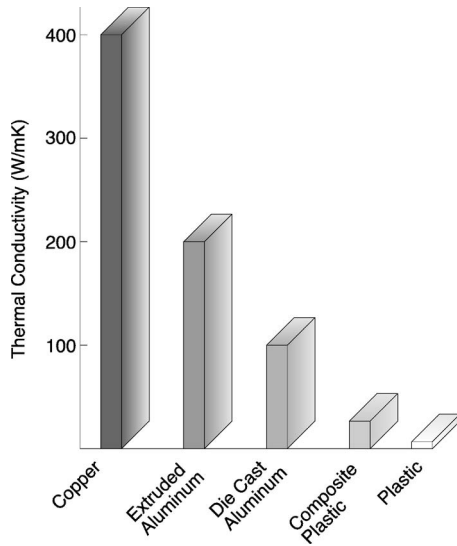


Fig. 1 Range of thermal conductivity for heat sink materials

flow will establish an operating point based on the fan performance curve and the head loss associated with the heat sink. While an optimization of thermal resistance would clearly tend towards a maximization of the fin surface area, the increased head loss associated with a reduced flow area would result in a choking of the flow between fins, producing a lower flow velocity or a reduction in the mass flow rate between fins due to flow bypass.

Heat sinks need to be selected based on a simultaneous consideration of both thermal resistance and head loss associated with viscous dissipation. While neither an optimum thermal resistance nor head loss may be achieved, the appropriate balance between these two important factors will lead to an optimized heat sink based on any limiting design constraints that are imposed.

The standard plate fin heat sink consists of a complex network of thermal resistances, as shown in Fig. 2. The analytical expression for the fully coupled network needs to be considered in any optimization procedure. It is impossible to presuppose the magnitude of any resistor elements before optimization given the coupled nature of the network and the dependence of each resistive element on geometry, thermophysical properties, and boundary conditions. While there are significant differences in the size of individual resistive elements in the thermal network established within a typical plate fin heat sink, it is clear that all resistances must be considered in order to properly optimize design variables.

The following study examines the role of material thermal conductivity and spreading resistance in the design and selection of plate fin heat sinks. It will be demonstrated that while plastic composites can have thermal conductivities one or two orders of magnitude less than traditional aluminum alloys, heat sinks manu-

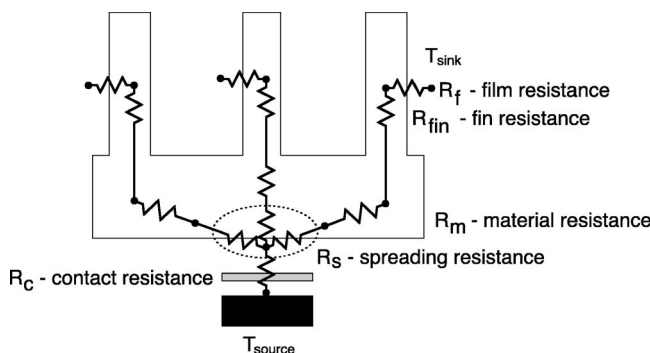


Fig. 2 Resistance network

factured using plastics offer only a nominal decrease in thermal performance when the design of the heat sink is optimized to allow for the lower thermal conductivity.

Modeling Procedure

Heat sinks are typically designed based on a measure of thermal resistance to heat flow between the heat source and the surrounding cooling medium given a known volumetric flow rate or in some instances an approach velocity. While this method of characterizing heat sinks is well accepted, it does not take into consideration the pressure drop or the resistance to fluid flow during normal operating conditions. In a ducted heat sink, the pressure drop will result in a reduction in fluid flow and a lowering of the convective coefficient over the heat transfer surfaces. The volumetric flow rate is tied to the heat sink geometry and must become an integral part of the design process. In an unducted heat sink, an increased pressure drop can result in flow bypass and in turn a significant reduction in the flow rate through the heat sink. In either case, the viscous effects must be considered as an integral part of the thermal assessment of a heat sink in order to quantify overall heat sink performance.

Parametric studies can be used to obtain relationships between thermal performance and design parameters, but these methods are generally time consuming and do not guarantee that the optimal design is obtained, only that a preferred design has been selected in the sampling of configurations tested. However, the rate of entropy generation associated with heat transfer and viscous dissipation provides a convenient measure of the thermal performance of a heat sink, as shown in Eq. (1).

$$\dot{S}_{gen} = \frac{Q^2 R_{hs}}{T_o^2} + \frac{F_d V_f}{T_o} \quad (1)$$

where the overall heat sink resistance can be treated as a series path resistor network formed between the heat source and the cooling medium, as follows:

$$R_{hs} = R_c + R_s + R_m + R_{fins} \quad (2)$$

Each component of the resistor network can be characterized in terms of the heat sink geometry, thermophysical properties, and boundary conditions.

The overall resistance of the fins can be calculated as a parallel circuit consisting of the fins and the base of the channels formed between the fins, where N is the total number of fins.

$$R_{fins} = \frac{1}{N/R_{fin} + (N-1)/R_{base}} \quad (3)$$

with the thermal resistance of the fins based on the combined internal and external resistance of rectangular, plate fins with uniform convection cooling, h as determined using the formulation of Teertstra et al. [1] for forced convection in plate fin heat sinks.

$$R_{fin} = \frac{1}{\sqrt{hPkA_c} \tanh(mH)} \quad (4)$$

$$R_{base} = \frac{1}{hbL} \quad (5)$$

and the fin parameter, m , given as

$$m = \sqrt{\frac{hP}{kA_c}} \quad (6)$$

The remaining component resistances in the network, i.e., material, contact, and spreading resistance, can be determined as follows

$$R_m = \frac{t_b}{kA_b} \quad (7)$$

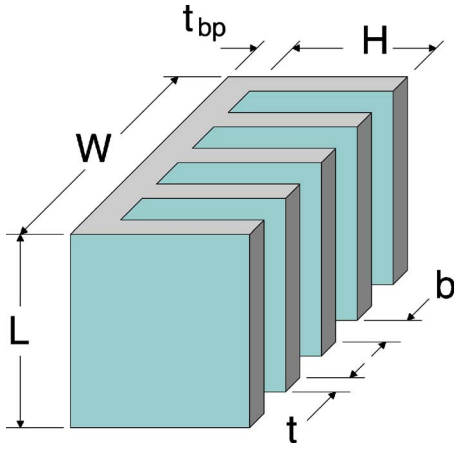


Fig. 3 Heat sink dimensions

$$R_c = \frac{1}{h_c A_b} \quad (8)$$

$$R_s = \frac{\psi_{avg}}{\sqrt{\pi k A_s}} \quad (9)$$

where the expression for R_s is based on the work of Song et al. [2], with the thermal constriction/spreading resistance determined for non-isothermal boundary conditions.

The total drag force on the heat sink is

$$\frac{F_d}{\frac{1}{2} \rho V_{ch}^2} = f_{app} N (2HL + bL) + K_c (HW) + K_e (HW) \quad (10)$$

and f_{app} is the apparent friction factor for hydrodynamically developing flow, and the channel velocity, V_{ch} , is related to the free stream velocity by

$$V_{ch} = V_f \left(1 + \frac{t}{b} \right) \quad (11)$$

The rate of entropy generation given in Eq. (1) is a direct measure of lost potential for work or, in the case of a heat sink, a reduction in the ability to transfer heat to the surrounding cooling medium. A model that establishes a relationship between entropy generation and heat sink design parameters can be optimized in such a manner that all relevant design conditions combine to produce the best possible heat sink (see Fig. 3) for the given constraints.

Bejan [3,4] developed solution procedures for single parameter optimization of thermal systems incorporating the method of entropy generation minimization, where any design parameter can be optimized while all other design conditions are set. Culham and Muzychka [5] extended this work by expanding the procedure to simultaneously determine the effects of multiple design parameters on entropy generation, leading to an optimized heat sink design. The results obtained in this work only considered unconstrained optimization of the free variables. The present work extends the previous work by addressing the issue of imposing equality and inequality constraints to the optimization procedure.

Constrained Multivariable Optimization. The general theory for constrained multivariable optimization may be found in Refs. [6–9]. The method of Lagrange multipliers may be easily applied to constrained multivariable applications. The general constrained nonlinear programming (NLP) problem takes the form: minimize (or maximize)

$$\phi(x_1, x_2, x_3, \dots, x_n) = 0 \quad (12)$$

subject to

$$g_j(x_1, x_2, x_3, \dots, x_n) = 0, \quad j = 1, \dots, m \quad (13)$$

$$h_j(x_1, x_2, x_3, \dots, x_n) \geq 0, \quad j = m + 1, \dots, p \quad (14)$$

where g_j and h_j are imposed constraints. It is often more convenient to consider the Lagrangian form of the NLP in the following manner. A new objective function is defined as follows:

$$L(x_1, \dots, x_n, \lambda_1, \dots, \lambda_p, \sigma_1, \dots, \sigma_{p-m}) = \phi(x_i) + \sum_{j=1}^m \lambda_j g_j(x_i) + \sum_{k=m+1}^p \lambda_k (h_k(x_i) - \sigma_k^2) \quad (15)$$

where λ_j are Lagrange multipliers and σ_j are slack variables. The use of slack variables enables the Lagrange multiplier method to be applied to problems with inequality constraints.

The problem is now reduced to solving the system of equations defined by

$$\frac{\partial L}{\partial x_i} = 0, \quad i = 1, \dots, n \quad (16)$$

$$\frac{\partial L}{\partial \lambda_j} = 0, \quad j = 1, \dots, p \quad (17)$$

$$\frac{\partial L}{\partial \sigma_j} = 0, \quad k = 1, \dots, p - m \quad (18)$$

The above system may be solved using numerical methods such as a multivariable Newton-Raphson method. A discussion of this method is outlined in [8] and application to the unconstrained optimization of the entropy generation rate is discussed in [5]. The constrained formulation for NLP's with inequality constraints can become quite complex. Given an NLP with n variables and p constraints with $p - m$ inequality constraints, optimization of the Lagrangian requires simultaneous solution of a system of $n + 2p - m$ equations. In most problems, the number of constraints prescribed should be judiciously chosen. For example, it is not always necessary to prescribe that all $x_i > 0$. In most problems, an optimal solution with $x_i > 0$ may be obtained if a reasonable initial guess is made while leaving the particular x_i unconstrained. In other problems, constraints such as $x_i < x_c$ may not be necessary if the optimal solution returns $x_i < x_c$ when x_i are unconstrained.

Solution to problems presented in the following section were obtained using the MAPLE V symbolic mathematics software. A simple procedure was coded that solves a system of N nonlinear equations using the multivariable Newton-Raphson method. Given the Lagrangian, L , the solution vector, $[X]$, initial guess, $[X_0]$, and maximum number of iterations, N_{max} , the procedure systematically applies the Newton-Raphson method until the desired convergence criteria and/or maximum number of iterations is achieved. The method is quite robust provided an adequate initial guess is made. Due to the nature of the numerical algorithm, problems generally encountered with a poor guess in a single parameter Newton-Raphson application result in similar divergence problems in the multiparameter optimization. In general, the physical nature of the present problem leads to very few problems.

Discussion

Culham and Muzychka [5] developed a design optimization procedure for plate fin heat sinks based on an entropy generation minimization method with unconstrained optimization. The authors have extended this procedure to include optimization with imposed constraints. The modeling procedure provides a convenient method for simultaneously selecting heat sink design variables that lead to the design that provides the best balance between heat transfer and fluid flow characteristics. In the original

Table 1 Results for case A—fixed fin thickness of 1 mm

k (W/m·K)	N	H (mm)	θ (K)	ΔP (mmH ₂ O)	η
25	29.3	35.77	15.94	6.26	0.484
50	27.4	45.72	11.82	4.73	0.519
100	26.0	50	9.17	3.93	0.625
150	25.6	50	8.18	3.69	0.705
200	25.3	50	7.65	3.56	0.757

paper by Culham and Muzychka [5], a baseline problem was examined as a means of demonstrating the capabilities and flexibility of the modeling procedure. The test case chosen involved sizing a heat sink to be used with an electronics package having a plan footprint of 50 mm by 50 mm. The objective was to select the best heat sink to fit the 50 × 50 mm footprint but not to exceed a maximum height of 50 mm. The maximum height restriction was selected to represent a typical board pitch found in a computing or telecommunications application. In addition to these overall geometric restrictions, the ambient operating environment was set to a fixed airflow of 2 m/s and 25°C. The total heat dissipation was fixed at 30 W and the contact between the heat source and the base plate of the heat sinks was assumed to be ideal. The same geometrical constraints and ambient conditions will be used here to examine the effect of changing heat sink material properties on the optimized design conditions.

Heat Sink Thermal Conductivity. It is commonly perceived that heat sink materials should be selected based solely on thermal conductivity, with aluminum and sometimes copper being the materials most often selected. But more recently a wide range of “manufactured” materials has been introduced that, while having thermal conductivities in the range of 25–100 W/mK (significantly less than aluminum and copper), do have a distinct cost and workability advantage. Several design options will be examined by determining the heat sink geometry that leads to overall optimized performance where both heat transfer and viscous effects are considered.

The first case examined, denoted as case A, will have a fixed fin thickness of $t=1$ mm plus the overall constraints of a maximum heat sink volume of 50 × 50 × 50 mm. The modeling procedure will be used to ascertain the heat sink with optimal fin height, H , and number of fins, N , that minimizes the rate of entropy generation. The problem is solved for a range of thermal conductivities between 25 and 200, which includes representative properties for a variety of materials from enhanced plastics to aluminum. Results are summarized in Table 1, where the temperature rise, θ , pressure drop, ΔP , and fin efficiency, η , are shown for the optimized heat sink determined at each level of thermal conductivity examined. It is clear that a low conductivity heat sink still results in acceptable performance in terms of operating temperature where an eightfold decrease in the thermal conductivity resulted in roughly a doubling of the temperature rise of the heat sink. It is interesting to note that as the conductivity decreases only a moderate number of additional fins are required for optimal conditions and the fins are shorter, reflecting the increased material resistance associated with long fins of low conductivity. Materials with thermal conductivities greater than 100 W/mK result in the fin height achieving a maximum value of 50 mm, as limited by the initial constraint imposed on this example. Table 1 clearly shows that heat sinks with lower thermal conductivities than normally used in plate fin heat sinks can offer a practical alternative to cooling applications where moderate increases in thermal head room are available.

The next problem examined consists of three cases (B, C, D) where the maximum heat sink temperature rise is constrained to $\theta=50^\circ\text{C}$. This requires the solution of the problem:
minimize

Table 2 Optimized design conditions for variable fin thickness

k (W/m·K)	N	H (mm)	θ (K)	ΔP (mmH ₂ O)	η
Case B: $t=1$ mm					
25	6.54	29.6	50	0.739	0.787
50	5.15	39.3	50	0.452	0.821
100	4.13	50	50	0.304	0.857
150	3.97	50	50	0.286	0.900
200	3.88	50	50	0.277	0.923
Case C: $t=2$ mm					
25	8.52	22.1	50	0.704	0.816
50	6.79	28.7	50	0.461	0.851
100	5.42	37.1	50	0.317	0.879
150	4.76	42.7	50	0.261	0.894
200	4.36	47.0	50	0.230	0.904
Case D: $t=3$ mm					
25	12.90	14.1	50	0.728	0.846
50	10.65	17.2	50	0.530	0.886
100	8.8	21.0	50	0.401	0.916
150	7.95	23.6	50	0.346	0.929
200	7.93	25.6	50	0.312	0.939

$$\phi(x_i) = S_{gen}$$

subject to

$$\theta = QR_{sink} = 50$$

in addition to the constraint that $H \leq 50$ mm. Results are computed for three prescribed values of the fin thickness, case B: $t=1$ mm, case C: $t=2$ mm, and case D: $t=3$ mm. Table 2 summarizes the optimal number of fins and fin height obtained for a range of thermal conductivities.

Table 2 shows that over the full range of thermal conductivity examined, the fin efficiency remains at or above 80%. The heat sinks with lower thermal conductivities result in optimized heat sink designs that have shorter fins, with approximately 50% more fins. In all cases the footprint of the heat sink base plate remained the same; however, through minor changes to the geometry of the heat sink, the temperature rise of the heat sink was maintained at 50°C with only a slight penalty in fin efficiency. It is clearly evident that heat sink material selection need not be limited to high conductivity materials, such as aluminum.

Finally, the constrained problem is reanalyzed, this time releasing fin thickness as a constraint. The problem now requires finding the optimal fin height, fin thickness, and number of fins that result in the prescribed temperature excess of 50°C. The results shown in Table 3 clearly demonstrate that a low conductivity heat sink provides a viable alternative to conventional heat sinks. The fin thickness for a material with a thermal conductivity of 25 W/mK is approximately three times thicker than an equivalent aluminum heat sink. While only a moderate penalty is observed in the fin efficiency of the low conductivity heat sink, the resulting pressure drop associated with the increase in the thickness of the fins has increased by a factor of 2.

Base Plate Spreading Resistance. While heat sinks are available in a wide variety of shapes and sizes, it is not always possible

Table 3 Results for case E—optimized fin thickness

k (W/m·K)	N	H (mm)	t (mm)	θ (K)	ΔP (mmH ₂ O)	η
25	9.50	22.53	1.34	50	0.546	0.757
50	9.24	23.71	0.837	50	0.379	0.790
100	8.88	23.19	0.650	50	0.327	0.861
150	8.82	23.35	0.492	50	0.299	0.876
200	8.89	23.83	0.339	50	0.278	0.864

Table 4 Effect of spreading resistance

$\frac{A_s}{A_{bp}}$	N	θ (K)	R_s	R_m	R_{fins}	R_{hs}
			(K/W)			
1.0	37.2	2.52	0.000	0.000 25	0.084	0.084
0.1	37.6	2.65	0.005	0.000 25	0.083	0.089
0.05	37.9	2.78	0.010	0.000 25	0.082	0.093
0.01	39.8	3.64	0.041	0.000 25	0.078	0.121
0.005	41.4	4.59	0.073	0.000 25	0.075	0.153
0.001	47.2	10.84	0.271	0.000 25	0.065	0.361

to have a heat sink base plate sized such that it conforms exactly to the adjoining heat producing component. In fact, it is quite common, especially in power electronics applications, to have heat sinks that are many times larger than the heat source. This introduces a thermal resistance between source and sink that can be attributed to the constriction of heat at the source as it seeks the extended surface area of the heat sink.

Culham and Muzychka [5] presented a procedure for heat sink optimization that was limited to uniform heat distribution over the base plate of a plate fin heat sink. While spreading resistance is only a single component of the overall resistive network between the source and the sink, it can be significant, especially for point sources or sources of limited size in relation to the total area of the heat sink base plate.

Table 4 presents an itemization of the components of thermal resistance encountered between a heat source of 30 W attached to a heat sink with a base plate, $20 \times 20 \times 2$ mm. The fin thickness and fin height are assumed to be 1 and 50 mm, respectively, while the conductivity of the fins and the base plate is 200 W/mK. The approach velocity is assumed to be 2 m/s. These dimensions and thermophysical properties are representative of an aluminum alloy folded fin, commonly encountered in automotive and electronic applications. With a source coverage ratio, $A_s/A_{bp}=1$, where the heat source and the base plate of the heat sink are the same size, there is no spreading resistance as heat flows in a one-dimensional manner from the heat source to the fin array. In this instance the predominant resistance in the thermal circuit between the source and the sink is the fin resistance, which is dominated by the film resistance at the solid-fluid boundary for the fins. However, as the source coverage ratio is reduced, approximating a point source, with $A_s/A_{bp}=0.001$, the spreading resistance increases to 0.271 K/W, which is 80% of the overall resistance of the heat sink.

Figure 4 shows the relationship between spreading resistance and the size of the heat source for the same heat sink geometry used in the example above. The spreading resistance remains less than 10% of the total thermal resistance of the heat sink when the size of the source is greater than 2% of the size of the base plate. As the source size falls below 2% of the base plate area, the spreading resistance increases dramatically, to more than 75% of the total thermal resistance when the source is 1/1000 of the base plate area. It is important to note that there is no optimum source coverage ratio for which spreading resistance is minimum. The spreading resistance decreases monotonically as the source coverage ratio increases.

Conclusions

The design and selection of heat sinks can often be a complicated procedure given the numerous geometric, thermophysical, and boundary conditions that are strongly coupled to provide a complex thermal network between the source and sink. The authors have presented a scientific procedure for determining optimum heat sink conditions given the simultaneous consideration of

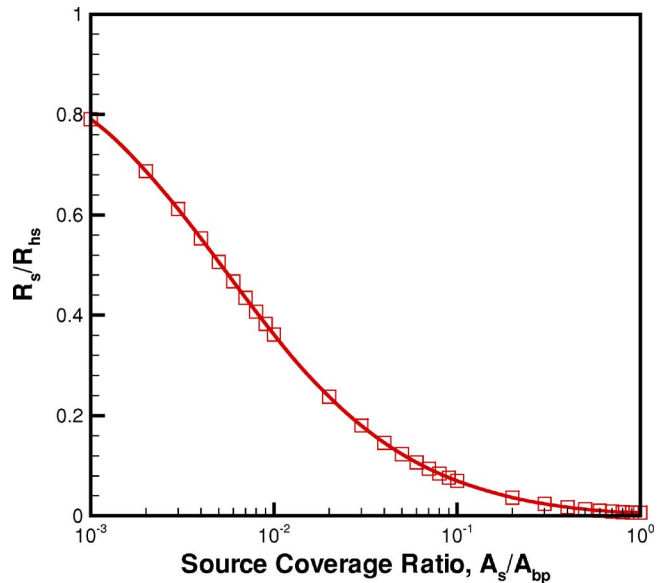


Fig. 4 Spreading resistance versus heat source coverage ($k=200$ W/mK)

both heat transfer and viscous dissipation. While the solution that results from this procedure does not optimize either heat transfer or fluid flow conditions in isolation, it does provide a theoretical optimum that ensures the best heat sink design for the given constraints.

The effect of heat sink thermal conductivity is examined with respect to its role in influencing optimum design conditions and the overall thermal performance of the heat sink. It is demonstrated that conventional heat sink materials, such as aluminum alloys, with a thermal conductivity of 200 W/mK, provide excellent heat transfer characteristics, however, manufactured composites ($k=25$ to 100 W/mK) consisting of graphite or metal particles contained within a plastic binder can be used with only a minimal loss in thermal performance. When using lower conductivity materials in heat sink applications, fin profiles are typically shorter and wider to accommodate the higher thermal resistance and in some instances there may be a marginal increase in the number of fins required to achieve optimal performance. In addition, the head loss associate with the increased flow blockage of additional fins with a thicker profile can necessitate a need to examine fan performance in these instances.

Acknowledgment

The authors acknowledge the financial support of the Manufacturing Research Corporation of Ontario and R-Theta Inc, Mississauga Ontario.

Nomenclature

- A_c = fin cross sectional area, m^2
- A_{bp} = base plate area, m^2
- b = fin spacing, m
- f, f_{app} = friction factor and apparent friction factor, respectively
- F_d = drag force, $kg \cdot m/s^2$
- g_i, h_i = imposed constraints
- h = heat transfer coefficient, W/m^2K
- H = fin height, m
- k = thermal conductivity, W/mK
- K_c, K_e = contraction and expansion loss coefficient, respectively
- L = fin length, m
- $L()$ = Lagrangian operator

m = fin parameter $\approx \sqrt{hP/kA_c}$, m^{-1}
 N = number of fins
 P = perimeter, m
 ΔP = pressure drop, mm_{H_2O}
 Q = heat flow rate, W
 R = thermal resistance, K/W
 \dot{S}_{gen} = entropy generation, W/K
 t = fin thickness, m
 t_{bp} = base plate thickness, m
 T = temperature, K
 V_{ch}, V_f = channel and approach velocity, respectively, m/s
 W = heat sink width, m
 X = solution vector
 X_0 = initial guess

Greek

η = fin efficiency
 λ_j = Lagrange multiplier
 ϕ = minimizing function
 ψ_{avg} = dimensionless spreading resistance
 ρ = density, kg/m^3
 σ_j = slack variable
 θ = temperature excess, $^{\circ}C$

Subscripts

0 = ambient

c = contact
 ch = channel
 f = film
 $fin, fins$ = single fin, multiple fins, respectively
 m = material
 s = spreading
 hs = heat sink

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