Optimization of Microchannel Heat Sinks Using Entropy Generation Minimization Method

Waqar Ahmed Khan, J. Richard Culham, *Member, IEEE*, and M. Michael Yovanovich

***Abstract*—In this paper, an entropy generation minimization (EGM) procedure is employed to optimize the overall performance of microchannel heat sinks. This allows the combined effects of thermal resistance and pressure drop to be assessed simultaneously as the heat sink interacts with the surrounding flow field. New general expressions for the entropy generation rate are developed by considering an appropriate control volume and applying mass, energy, and entropy balances. The effect of channel aspect ratio, fin spacing ratio, heat sink material, Knudsen numbers, and accommodation coefficients on the entropy generation rate is investigated in the slip flow region. Analytical/empirical correlations are used for heat transfer and friction coefficients, where the characteristic length is used as the hydraulic diameter of the channel. A parametric study is also performed to show the effects of different design variables on the overall performance of microchannel heat sinks.**

***Index Terms*—Analytical model, entropy generation minimization, microchannel heat sinks, optimization.**

NOMENCLATURE

A	Total heating surface area [mm^2].
A_c	Cross-section area of a single fin [mm^2].
D_h	Hydraulic diameter [mm].
f	Friction factor.
G	Volume flow rate [m^3/s].
g, l	Equality and inequality constraints.
H_c	Channel height [m].
h	Specific enthalpy of the fluid [J/kg].
h_{av}	Average heat transfer coefficient [W/m^2·K].
j	Number of imposed constraints.
Kn	Knudsen number $\equiv \lambda/D_h$.
k	Thermal conductivity of solid [W/m·K].
k_{ce}	Sum of entrance and exit losses.
k_{eq}	Ratio of thermal conductivity of fluid to solid $\equiv k_f/k$.
k_f	Thermal conductivity of fluid [W/m·K].
$\mathcal{L}$	Lagrangian function.
L	Length of channel in flow direction [mm].
m	Fin parameter [m^{-1}].
$\dot{m}$	Total mass flow rate [kg/s].
N	Total number of microchannels.
n	Number of design variables.
Nu_{D_h}	Nusselt number based on hydraulic diameter $\equiv D_h h_{av}/k_f$.
P	Pressure [Pa].
Pe_{D_h}	Peclet number based on hydraulic diameter $\equiv D_h U_{av}/\alpha$.
Pr	Prandtl number.
Q_b	Heat transfer rate from the base [W].
Q_{fin}	Heat transfer rate from the fin [W].
q	Heat flux [W/m^2].
R	Resistance [K/W].
Re_{D_h}	Reynolds number based on hydraulic diameter $\equiv D_h U_{av}/\nu$.
S_{gen}	Total entropy generation rate [W/K].
s	Specific entropy of fluid [J/kg·K].
T	Absolute temperature [K].
t	Thickness [m].
U_{av}	Average velocity in channels [m/s].
u	Specific internal energy [J/kg].
v	Specific volume of fluid [m^3/kg].
W	Width of heat sink [mm].
w_c	Half of the channel width [mm].
w_w	Half of the fin thickness [mm].
x_i	Design variables.

GREEK SYMBOLS

ΔP	Pressure drop across microchannel [Pa].
α	Thermal diffusivity [m^2/s] or constant defined by (23).
α_c	Channel aspect ratio $\equiv 2w_c/H_c$.
α_{hs}	Heat sink aspect ratio $\equiv L/2w_c$.
β	Fin spacing ratio $\equiv w_c/w_w$.
γ	Ratio of specific heats $\equiv c_p/c_v$.

Manuscript received March 20, 2006; revised July 26, 2006. Current version published July 22, 2009. This work was supported by the Centre for Microelectronics Assembly and Packaging and Natural Sciences and Engineering Research Council of Canada. This work was recommended for publication by Associate Editor B. Sammakia upon evaluation of the reviewers comments.

W. A. Khan is with the Department of Engineering Sciences, National University of Sciences and Technology, PN Engineering College, PNS Jauhar, Karachi 75350, Pakistan.

J. R. Culham and M. M. Yovanovich are with the Microelectronics Heat Transfer Laboratory, Department of Mechanical Engineering, University of Waterloo, Waterloo, ON N2L 3G1 Canada (e-mail: rix@mhtlab.uwaterloo.ca).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCAPT.2009.2022586

1521-3331/$25.00 © 2009 IEEE

λ, χ Mean free path [m] or Lagrangian multipliers.
μ Absolute viscosity of fluid [kg/m·s].
ν Kinematic viscosity of fluid [m^2/s].
ρ Fluid density [kg/m^3].
σ Tangential momentum accommodation coefficient (TMAC).
σ_t Energy accommodation coefficient.
ζ_u Slip velocity coefficient.
ζ_t Temperature jump coefficient.

SUBSCRIPTS

a Ambient.
av Average.
b Base surface.
c Channel.
cap Capacity.
ce Contraction and expansion.
conv Convective.
f Fluid.
fin Single fin.
h Hydraulic.
hs Heat sink.
in Entrance.
out Exit.
s Slip.
th Thermal.
w Wall.

I. INTRODUCTION

ONE of the important aspects of electronics packaging is the thermal management of electronic devices. The trend in the electronic industry toward denser and more powerful products requires a higher level of performance from cooling technology. After the pioneering work of Tuckerman and Pease [1], microchannels have received considerable attention especially in microelectronics. Microchannel heat sinks provide a powerful means for dissipating high heat flux with small allowable temperature rise. Due to an increase in surface area and a decrease in the convective resistance at the solid/fluid interface, heat transfer is enhanced in microchannels. These heat sinks can be applied in many important fields like microelectronics, aviation and aerospace, medical treatment, biological engineering, materials sciences, cooling of high-temperature superconductors, thermal control of film deposition, and cooling of powerful laser mirrors. The two important characteristics of microchannel heat sinks are the high heat transfer coefficients and lower friction factors.

In this paper, an entropy generation minimization (EGM) criterion is used to determine the overall performance of a microchannel heat sink, which allows the combined effect of thermal resistance and pressure drop to be assessed through the simultaneous interaction with the heat sink. A general expression for the entropy generation rate is obtained by using the conservation equations for mass and energy with the entropy balance.

II. LITERATURE REVIEW

The concept of "microchannel heat sinks" was first introduced by Tuckerman and Pease [1]. Following this work, several experimental, numerical, and theoretical studies on the optimization of microchannel heat sinks have been carried out. These studies are reviewed in this section.

Steinke and Kandlikar [2] presented a comprehensive review of conventional single-phase heat transfer enhancement techniques. They discussed several passive and active enhancement techniques for minichannels and microchannels. Some of their proposed enhancement techniques include fluid additives, secondary flows, vibrations, and flow pulsations.

Kandlikar and Grande [3] explored the cooling limits of the plain rectangular microchannels with water cooling for high heat flux dissipation and illustrated the need for enhanced microchannels. They described a simplified and well-established fabrication process to fabricate 3-D microchannels. They also demonstrated the efficacy of the fabrication process in fabricating complex microstructures within a microchannel.

Knight *et al.* [4], [5], Perret *et al.* [6], [7], Kim [8], Upadhye and Kandilkar [9], Kandlikar and Grande [3], and Liu and Garimella [10] developed theoretical optimization procedures to minimize the overall thermal resistance of microchannel heat sinks for a given pressure drop, whereas Singhal *et al.* [11], and Kandlikar and Upadhye [12] carried out analyses to optimize the channel configuration that yields a minimum pressure drop for a given heat load. They presented solution procedures for laminar/turbulent flow and generalized their results in terms of different heat sink parameters.

Kleiner *et al.* [13], Aranyosi *et al.* [14], and Harris *et al.* [15] investigated theoretically and experimentally the performance of microchannel heat sinks. They modeled the performance in terms of thermal resistance, pressure drop, and pumping power as a function of heat sink dimensions, tube dimensions, and air flow rate. Their results show an enhancement in heat removal capability compared to traditional forced air-cooling schemes.

Garimella and Singhal [16] and Jang and Kim [17] analyzed experimentally the pumping requirements of microchannel heat sinks and optimized the size of the microchannels for minimum pumping requirements. Jang and Kim [17] showed that the cooling performance of an optimized microchannel heat sink subject to an impinging jet is enhanced by about 21% compared to that of the optimized microchannel heat sink with parallel flow under the fixed-pumping-power condition.

Choquette *et al.* [18], Zhimin and Fah [19], Meysenc *et al.* [20], Chong *et al.* [21], Liao *et al.* [22], Ryu *et al.* [23], Wei and Joshi [24], and Jean-Antoine *et al.* [25] performed numerical optimizations of thermal performance of microchannel heat sinks for given pumping power/pressure drop. Zhimin and Fah [17] considered laminar, turbulent, developed, and developing flow and heat transfer in the analysis. Using self-developed software, they investigated the effects of heat sink channel aspect ratio, fin-width-to-channel-width ratio, and the channel width on the performance of heat sink. They found that the channel width is the most important parameter and governs the performance of a microchannel heat sink. Min *et al.* [26] showed numerically that the tip clearance can improve the cooling performance of a microchannel heat sink when tip clearance is smaller than a channel width. Delsman *et al.* [27] performed a CFD study for the optimization of the plate geometry to reach the design target regarding the quality of the flow distribution.

Haddad *et al.* [28] investigated numerically the entropy generation due to steady laminar forced convection fluid flow through parallel plate microchannels. They discussed the effect of Knudsen, Reynolds, Prandtl, and Eckert numbers and the nondimensional temperature difference on entropy generation within the microchannel. The entropy generation within the microchannel is found to decrease as Knudsen number increases, and it is found to increase as Reynolds, Prandtl, and Eckert numbers and the nondimensional temperature difference increase. The contribution of the viscous dissipation in the total entropy generation increases as Knudsen number increases over wide ranges of the flow controlling parameters.

It is obvious from the literature survey that all the studies related to optimization of microchannel heat sinks are used to minimize thermal resistance for a given pressure drop or to minimize pumping power for a specified thermal resistance. No study exists to optimize both thermal and hydraulic resistances simultaneously. In this study, an EGM method is applied as a unique measure to study the optimization of thermal and hydraulic resistances simultaneously. All relevant design parameters for microchannel heat sinks, including geometric parameters, material properties, and flow conditions are optimized simultaneously by minimizing the entropy generation rate S_{gen} subject to manufacturing and design constraints.

III. Model Development

The geometry of a microchannel heat sink is shown in Fig. 1(a). The length of the heat sink is L and the width is W. The top surface is insulated and the bottom surface is uniformly heated. A coolant passes through a number of microchannels along the x-axis and takes heat away from the heat dissipating electronic component attached below. There are N channels and each channel has a height H_c and width $2w_c$. The thickness of each fin is $2w_w$ whereas the thickness of the base is t_b. The temperature of the channel walls is assumed to be T_w with an inlet water temperature of T_a. At the channel wall, the slip flow velocity and temperature jump boundary conditions were applied to calculate friction and heat transfer coefficients.

Taking advantage of symmetry, a control volume is selected for developing the entropy generation model, as shown in Fig. 1(b). The length of the control volume is taken as unity for convenience and the width and height are taken as $w_w + w_c$ and $H_c + t_b$, respectively. This control volume includes half of the fin and half of the channel along with the base. The side surfaces AB and CD and the top surface AC of this CV can be regarded as impermeable, adiabatic, and shear free (no mass transfer and shear work transfer across these surfaces). The heat flux over the bottom surface BD of the CV is q. The ambient temperature is T_a and the surface temperature of the channel wall is T_w. The bulk properties of the coolant are represented

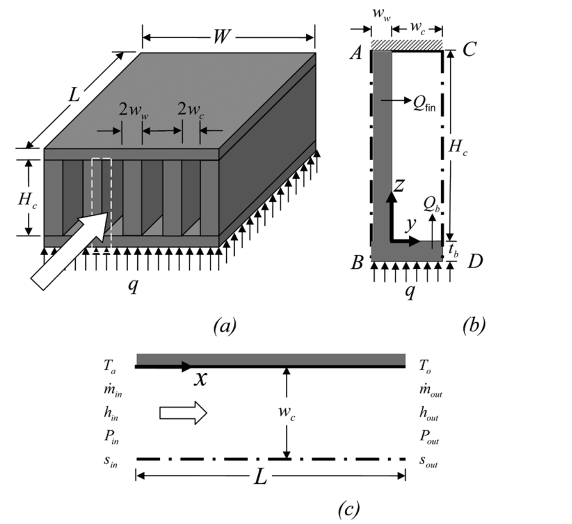


Fig. 1. Microchannel heat sink: geometry, control volume, and definitions.

by u_{in}, P_{in}, and s_{in} at the inlet and by u_{out}, P_{out}, and s_{out} at the outlet, respectively. The irreversibility of this system is due to heat transfer across the finite temperature difference $T_b - T_a$ and to friction. Heat transfer in the control volume [Fig. 1(b)] is a conjugate, combining heat conduction in the fin and convection to the cooling fluid. To simplify the analysis, the following assumptions were employed.

1) Uniform heat flux on the bottom surface.
2) Smooth surface of the channel.
3) Adiabatic fin tips.
4) Isotropic material.
5) Fully developed heat and fluid flow.
6) Steady and laminar flow.
7) Slip flow (i.e., $0.001 \leq Kn \leq 0.1$) with negligible creep effects.
8) Incompressible fluid with constant thermophysical properties.
9) Negligible axial conduction in both the fin and fluid.
10) Changes in kinetic and potential energies are negligible.

The mass rate balance for the steady state reduces to

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m}. \tag{1}$$

Assuming negligible changes in the kinetic and potential energies, the first law of thermodynamics for a steady-state condition can be written as

$$Q = \dot{m}[\underbrace{(u_{\text{out}} + P_{\text{out}}v_{\text{out}})}_{h_{\text{out}}} - \underbrace{(u_{\text{in}} + P_{\text{in}}v_{\text{in}})}_{h_{\text{in}}}]. \tag{2}$$

The combination of specific internal and flow energies is defined as specific enthalpy; therefore, the energy rate balance reduces further to

$$Q = \dot{m}(h_{\text{out}} - h_{\text{in}}). \tag{3}$$

From the second law of thermodynamics

$$\frac{dS_{cv}}{dt} = \dot{m}(s_{\text{in}} - s_{\text{out}}) + \frac{Q_{\text{fin}}}{T_b} + \frac{Q_b}{T_b} + S_{\text{gen}}. \tag{4}$$

For steady state, $dS_{cv}/dt = 0$, and the total heat transferred from the bottom of the heat sink $Q = Q_{\text{fin}} + Q_b$, so the entropy rate balance reduces to

$$S_{\text{gen}} = \dot{m}(s_{\text{out}} - s_{\text{in}}) - \frac{Q}{T_b} \tag{5}$$

where T_b represents the absolute temperature of the heat sink base.

Integrating Gibb's equation from inlet to the outlet gives

$$h_{\text{out}} - h_{\text{in}} = T_a(s_{\text{out}} - s_{\text{in}}) + \frac{1}{\rho}(P_{\text{out}} - P_{\text{in}}). \tag{6}$$

Combining (3), (5), and (6), the entropy generation rate can be written as

$$S_{\text{gen}} = Q\left[\frac{1}{T_a} - \frac{1}{T_b}\right] + \frac{\dot{m}\Delta P}{\rho T_a}. \tag{7}$$

Rearranging the terms and writing $\theta_b = T_b - T_a$, we have

$$S_{\text{gen}} = \frac{Q\theta_b}{T_a T_b} + \frac{\dot{m}\Delta P}{\rho T_a}. \tag{8}$$

As $\theta_b = QR_{\text{th}}$, the entropy generation rate can be written as

$$S_{\text{gen}} = \frac{Q^2 R_{\text{th}}}{T_a T_b} + \frac{\dot{m}\Delta P}{\rho T_a} \tag{9}$$

where R_{th} is the total thermal resistance and ΔP is the total pressure drop across the channel. This expression describes the entropy generation rate model completely and it shows that the entropy generation rate depends on the total thermal resistance R_{th} and the pressure drop across the channel, provided that the heat load and ambient conditions are specified. If G is the volume flow rate, then the total mass flow rate can be written as

$$\dot{m} = \rho G. \tag{10}$$

The average velocity in the channel U_{av} is given by

$$U_{av} = \frac{\dot{m}}{N\rho(2w_c)H_c} \tag{11}$$

where N is the number of channels given by

$$N = \frac{W - w_w}{w_c + w_w}. \tag{12}$$

IV. Thermal Resistance

The total thermal resistance is defined as

$$\begin{aligned} R_{\text{th}} &= \frac{\theta_b}{Q} \\ &= \frac{T_b - T_f}{Q} + \frac{T_f - T_a}{Q} \\ &= R_{\text{conv}} + R_{\text{cap}} \end{aligned} \tag{13}$$

where T_b is the heat sink base temperature and T_f is the bulk fluid temperature. These temperatures are given by

$$T_b = T_a + QR_{\text{th}} \tag{14}$$

$$T_f = T_a + \frac{Q}{2\dot{m}C_p}. \tag{15}$$

Assuming $h_{\text{fin}} = h_{\text{base}} = h_{av}$, the convective and fluid thermal resistances can be written as

$$R_{\text{conv}} = \frac{1}{h_{av}A} \tag{16}$$

$$R_{\text{fluid}} = \frac{1}{\dot{m}C_p} \tag{17}$$

where A is the total heat surface area and is given by

$$A = 2NL(w_c + H_c\eta_{\text{fin}}) \tag{18}$$

with

$$\eta_{\text{fin}} = \frac{\tanh(mH_c)}{mH_c} \tag{19}$$

$$m = \sqrt{\frac{2h_{av}}{kw_w}}. \tag{20}$$

Using slip flow velocity and temperature jump boundary conditions, Khan [29] solved the energy equation and developed the following theoretical model for the dimensionless heat transfer coefficient for a parallel plate microchannel:

$$Nu_{D_h} = \frac{h_{av}D_h}{k_f} = \frac{140}{17(1+\alpha_c)K_s} \tag{21}$$

where

$$K_s = 1 - \frac{6}{17}\left(\frac{U_s}{U_{av}}\right) + \frac{2}{51}\left(\frac{U_s}{U_{av}}\right)^2 - \frac{140}{17}\zeta_t \tag{22}$$

$$\alpha_c = \frac{2w_c}{H_c} < 1 \tag{23}$$

with

$$\frac{U_s}{U_{av}} = \frac{6\alpha}{1+6\alpha} \tag{24}$$

$$\alpha = \frac{2\zeta_u}{1+\alpha_c} \tag{25}$$

$$\zeta_u = \left(\frac{2-\sigma}{\sigma}\right)Kn \tag{26}$$

$$\zeta_t = \left(\frac{2-\sigma_t}{\sigma_t}\right) \cdot \frac{2\gamma}{\gamma+1} \cdot \frac{Kn}{Pr}. \tag{27}$$

Using (14)–(21), (13) can be written as

$$R_{\text{th}} = \frac{2C_3\alpha_{hs}}{k_f L C_1 C_2} \tag{28}$$

where

$$C_1 = N\alpha_{hs}(2\eta_{\text{fin}} + \alpha_c) \tag{29}$$

$$C_2 = \frac{(1+\alpha_c)}{\alpha_c} \tag{30}$$

$$C_3 = \frac{1}{Nu_{D_h}} + \frac{C_1}{Pe_{D_h}} \tag{31}$$

with

$$\alpha_{hs} = \frac{L}{2w_c} \tag{32}$$

$$\eta_{\text{fin}} = \frac{\tanh(mH_c)}{mH_c} \tag{33}$$

$$mH_c = \frac{1}{\alpha_c}\sqrt{2\beta k_{eq} Nu_{D_h}(1+\alpha_c)} \tag{34}$$

$$k_{eq} = \frac{k_f}{k} \tag{35}$$

$$\beta = \frac{w_c}{w_w}. \tag{36}$$

V. Pressure Drop

The pressure drop associated with flow across the channel is given by

$$\Delta P = \frac{\rho U_{av}^2}{2}\left[k_{ce} + \left(f\frac{L}{D_h}\right)\right] \tag{37}$$

where the friction factor f depends on the Reynolds number, channel aspect ratio, and slip velocity coefficient [29], and can be written as

$$f = \frac{24}{\text{Re}_{D_h}}\left(\frac{1}{1+6\alpha}\right)\left(\frac{1}{1+\alpha_c}\right). \tag{38}$$

Kleiner *et al.* [13] used experimental data from Kays and London [30] and derived the following empirical correlation for the entrance and exit losses k_{ce} in terms of channel width and fin thickness:

$$k_{ce} = 1.79 - 2.32\left(\frac{w_c}{w_c+w_w}\right) + 0.53\left(\frac{w_c}{w_c+w_w}\right)^2. \tag{39}$$

VI. Entropy Generation Rate

Substituting (28) and (37) into (9), we get

$$S_{\text{gen}} = \frac{Q^2}{T_aT_b}\cdot\frac{2\alpha_{hs}C_3}{Lk_fC_1C_2} + \frac{\rho w_cH_cU_{av}^3}{T_a}C_4 \tag{40}$$

$$= S_{\text{gen},h} + S_{\text{gen},f} \tag{41}$$

where $S_{\text{gen},h}$ and $S_{\text{gen},f}$ show the entropy generation rates due to heat transfer and fluid friction, respectively, and C_4 is given by

$$C_4 = \left[k_{ce} + \left(f\frac{L}{D_h}\right)\right]. \tag{42}$$

VII. Optimization Procedure

The problem considered in this study is to minimize the entropy generation rate, given by (9) or (40), for the optimal overall performance of microchannel heat sinks. If $f(\mathbf{x})$ represents the entropy generation rate that is to be minimized subject to equality constraints $g_j(x_1, x_2, \ldots, x_n) = 0$ and inequality constraints $l_k(x_1, x_2, \ldots, x_n) \geq 0$, then the complete mathematical formulation of the optimization problem may be written in the following form:

$$\text{minimize } f(\mathbf{x}) = S_{\text{gen}}(\mathbf{x}) \tag{43}$$

TABLE I
Assumed Parameter Values

Parameter	Assumed Values
Channel or heat sink length, L (mm)	51
Width of heat sink, W (mm)	51
Channel height, H_c (mm)	1.7
Channel width, $2w_c$ (mm)	0.25
Fin thickness, $2w_w$ (mm)	0.14
Thermal conductivity of solid $(W/m \cdot K)$	148
Thermal conductivity of air $(W/m \cdot K)$	0.0261
Density of air (kg/m^3)	1.1614
Specific heat of air $(J/kg \cdot K)$	1007
Kinematic viscosity (m^2/s)	1.58×10^{-5}
Prandtl number (air)	0.71
Heat flux (W/cm^2)	15
Volume flow rate (m^3/s)	0.007
Ambient temperature $(°C)$	27
Tangential momentum accommodation coefficient	0.85
Thermal energy coefficient	0.85

subject to the equality constraints

$$g_j(\mathbf{x}) = 0, \qquad j = 1, 2, \ldots, m \tag{44}$$

and inequality constraints

$$l_j(\mathbf{x}) \geq 0, \qquad j = m+1, \ldots, n \tag{45}$$

where g_j and l_j are the imposed equality and inequality constraints and $\mathbf{x}$ denotes the vector of the design variables $(x_1, x_2, x_3, \ldots, x_n)^T$. The objective function can be redefined by using the Lagrangian function as follows:

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\chi}) = f(\mathbf{x}) + \sum_{j=1}^{m}\lambda_j g_j(\mathbf{x}) - \sum_{j=m+1}^{n}\chi_j l_j(\mathbf{x}) \tag{46}$$

where λ_j and χ_j are the Lagrange multipliers. The λ_j can be positive or negative but the χ_j must be ≥ 0. The necessary condition for $\mathbf{x}^*$ to be a local minimum of the problem, under consideration, is that the Hessian matrix of $\mathcal{L}$ should be positive semidefinite, i.e.,

$$\mathbf{v}^T\nabla^2[(\mathbf{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\chi}^*)]\mathbf{v} \geq 0. \tag{47}$$

For a local minimum to be a global minimum, all the eigenvalues of the Hessian matrix should be ≥ 0.

A system of nonlinear equations is obtained, which can be solved using numerical methods such as a multivariable

TABLE II
RESULTS OF OPTIMIZATION

Kn	G	Optimized Design Variables		Performance Parameters		
	m^3/s	Channel Aspect Ratio α_c	Fin Spacing Ratio β	R_{hs} K/W	ΔP Pa	S_{gen} W/K
0.1	0.005	0.164	3.06	0.201	924	0.463
	0.007	0.194	4.99	0.160	902	0.398
	0.009	0.222	7.31	0.142	895	0.370
0.01	0.005	0.172	3.13	0.224	1549	0.511
	0.007	0.203	5.16	0.189	1485	0.462
	0.009	0.233	7.59	0.177	1441	0.451
0.001	0.005	0.175	3.16	0.229	1673	0.521
	0.007	0.207	5.20	0.195	1591	0.476
	0.009	0.237	7.67	0.185	1537	0.468

Newton–Raphson method. This method has been described in [31] and applied by Culham and Muzychka [32] and Culham *et al.* [33] to study the optimization of plate fin heat sinks, and by Khan *et al.* [34], [35] for pin fin heat sinks and tube banks. In this paper, the same approach is used to optimize the overall performance of a microchannel heat sink in such a manner that all relevant design conditions combine to produce the best possible heat sink for the given constraints. The optimized results are then presented in graphical form.

VIII. Case Studies and Discussion

The assumed parameter values [13], given in Table I, are used as the default case to determine the overall performance of microchannel heat sinks. The fluid properties are evaluated at the ambient temperature. The results obtained are shown in Table II.

Microchannel width, height, and fin thickness are optimized in terms of channel aspect ratio and fin spacing ratio for three different volume flow rates and Knudsen numbers in the slip flow region. The corresponding values of the total thermal resistance, pressure drop, and entropy generation rate are tabulated. It is observed that both channel aspect and fin spacing ratios increase with the increase in volume flow rate and Knudsen numbers in the slip flow region. The total thermal resistance and the pressure drop are found to decrease with the increase in volume flow rates and increase with the decrease in Knudsen numbers.

A parametric study is carried out to investigate the dependence of the optimal dimensions of the microchannel heat sink on the thermal and hydraulic performance. The current study attempts to find an optimal flow rate, channel width, heat sink material, and the effect of tangential momentum and energy coefficients. Fig. 2 shows the variation of the total entropy generation rate as a function of volume flow rate for three different values of Knudsen numbers. This figure shows that as the Knudsen number increases the optimal entropy generation rate decreases due to the increase in velocity slip and temperature jump at the wall that lead to reduced heat transfer and momentum transfer from the wall to the fluid. It also shows that the optimal flow rate increases with the increase in Knudsen number.

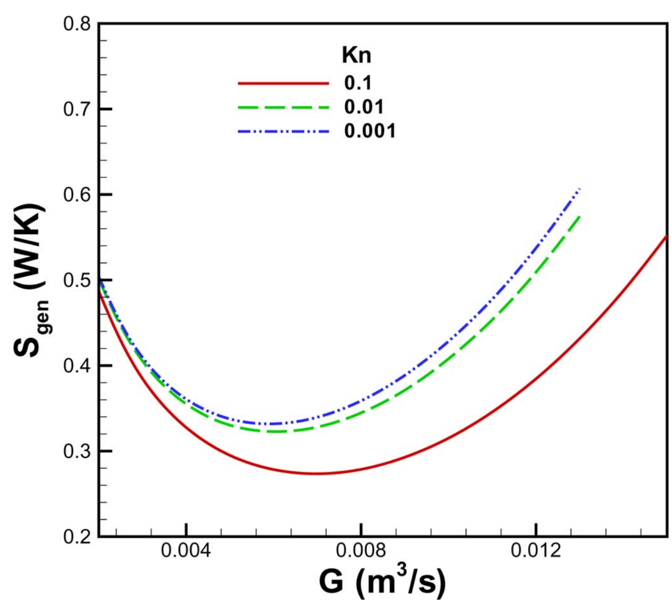


Fig. 2. Effect of volume flow rate on S_{gen}.

The friction factor and the Nusselt number, in the laminar flow region, depend on the channel aspect ratio α_c. The effect of this aspect ratio is shown in Fig. 3 for three different Knudsen numbers in the slip flow region. The increase in aspect ratio increases the cross-sectional area available for flow and the total surface area available for convective heat transfer, which reduces thermal and hydraulic resistances. This figure also shows the decrease in the optimum entropy generation rate with an increase in Knudsen number.

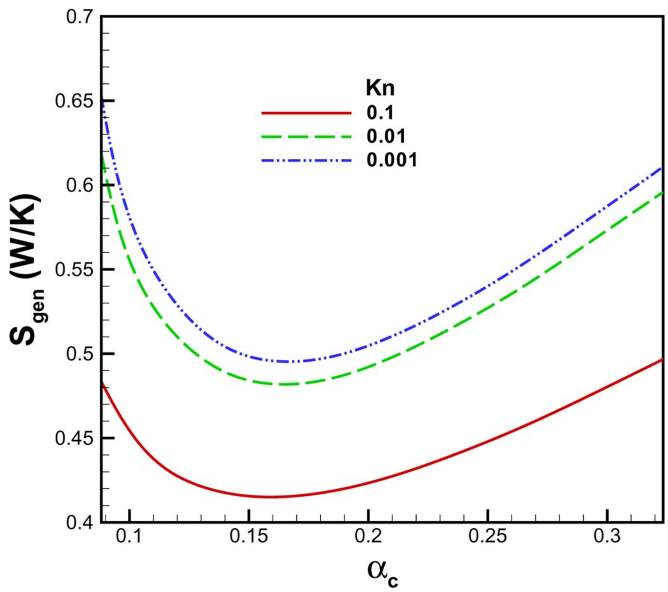


Fig. 3. Effect of channel aspect ratio on S_{gen}.

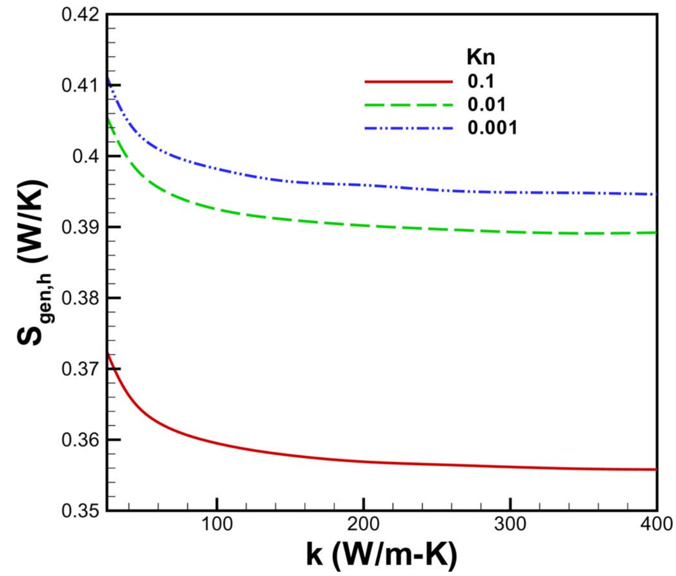


Fig. 5. Effect of heat sink material on S_{gen}.

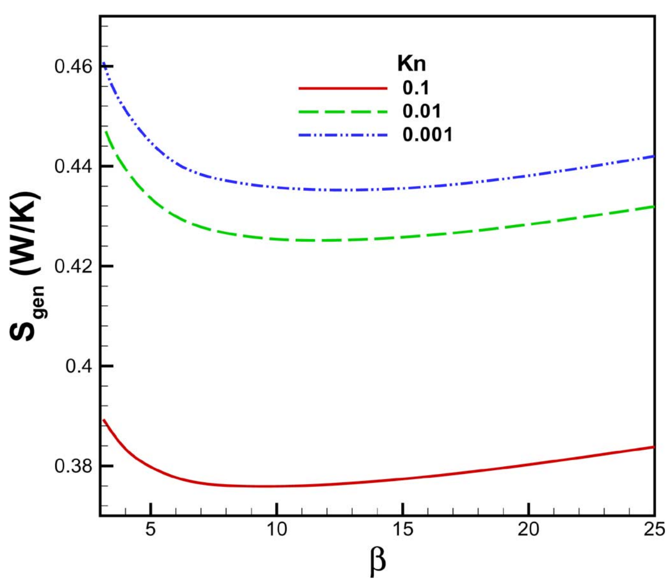


Fig. 4. Effect of fin spacing ratio on S_{gen}.

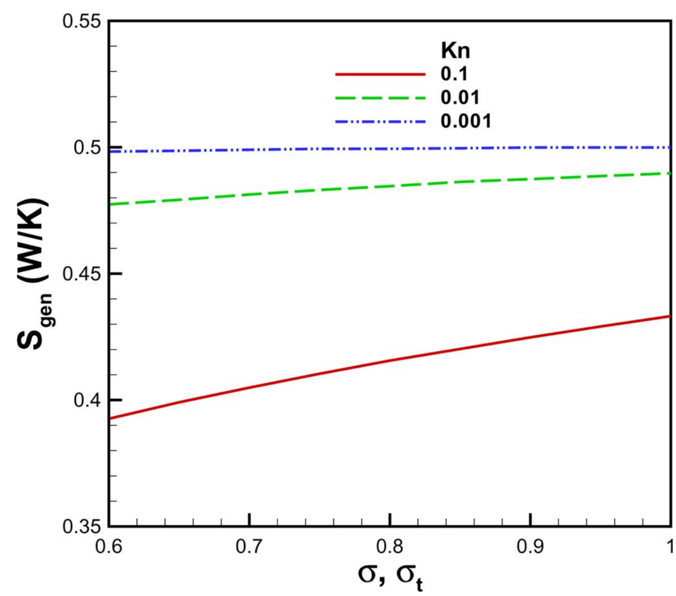


Fig. 6. Effect of accommodation coefficients on S_{gen}.

The fin spacing ratio β plays an important role in the heat transfer analysis. It should be greater than 1 to ensure that there is flow in the microchannel. The effect of this ratio is shown in Fig. 4 for the same Knudsen numbers in the slip flow region. For each Knudsen number, the fin spacing ratio is optimized to get a minimum entropy generation rate. It can be observed that the optimum fin spacing ratio is a very weak function of the Knudsen number, however, optimal entropy generation rate depends upon the Knudsen number and decreases with an increase in the Knudsen number. It shows that lower fin spacing ratios are appropriate in the case of microchannel heat sinks.

The total entropy generation rate S_{gen} includes the contributions due to heat transfer and viscous dissipation. The thermal conductivity of the heat sink material affects only the contribution due to heat transfer $S_{\text{gen},h}$, whereas the contribution due to viscous dissipation $S_{\text{gen},f}$ remains unchanged. Fig. 5 shows the effect of thermal conductivity on the $S_{\text{gen},h}$. Again the entropy generation rate decreases with the increase in Knudsen numbers. This figure shows that the entropy generation rate due to heat transfer decreases sharply from 25 to 180 W/m·K and then becomes almost constant.

The accommodation coefficients model the momentum and energy exchange of the gas molecules impinging on the walls. They are dependent on the specific gas and the surface quality and are tabulated in [36]. Very low values of σ and σ_t will increase the slip on the walls considerably even for small Knudsen number flows, due to the 2-σ/σ factor in (26) and (27). The effect of these accommodation coefficients is shown in Fig. 6. This figure shows that for high Knudsen numbers, in the slip flow region, there is considerable change in the total entropy generation rate but as the Knudsen number decreases, the losses due to heat transfer and fluid friction become negligible.

IX. Conclusion

Based on the results of case studies and parametric optimization, we have the following conclusions.

1) Thermal resistance and pressure drop across the microchannel decrease with an increase in volume flow rate and increase with a decrease in Knudsen numbers in the slip flow region.
2) The optimum channel aspect and fin spacing ratios increase with the volume flow rate to allow the decrease in thermal resistance and pressure drop.
3) The optimum entropy generation rate decreases with the increase in Knudsen numbers in the slip flow region.
4) Due to slip flow and temperature jump boundary conditions, fluid friction decreases and heat transfer increases in the microchannels which decreases the total entropy generation rate.
5) A low thermal conductivity heat sink with a large number of microchannels gives acceptable performance in terms of entropy generation rate.
6) Lower tangential momentum and energy accommodation coefficients results in higher entropy generation rates.

References

[1] D. B. Tuckerman and R. F. W. Pease, "High-performance heat sinking for VLSI," *IEEE Electron Device Lett.*, vol. EDL-2, no. 5, pp. 126–129, May 1981.

[2] M. E. Steinke and S. G. Kandlikar, "Single-phase heat transfer enhancement techniques in microchannel and minichannel flows," in *Proc. 2nd Int. Conf. Microchannels Minichannels*, Rochester, NY, Jun. 17–19, 2004, pp. 141–148.

[3] S. G. Kandlikar and W. J. Grande, "Evaluation of single phase flow in microchannels for high flux chip cooling—Thermohydraulic performance evaluation and fabrication technology," *Heat Transfer Eng.*, vol. 25, no. 8, pp. 5–16, 2004.

[4] R. W. Knight, J. S. Goodling, and D. J. Hall, "Optimal thermal design of forced convection heat sinks-analytical," *J. Electron. Packaging*, vol. 113, no. 3, pp. 313–321, 1991.

[5] R. W. Knight, D. J. Hall, J. S. Goodling, and R. C. Jaeger, "Heat sink optimization with application to micro-channels," *IEEE Trans. Compon. Hybrids Manuf. Technol.*, vol. 15, no. 5, pp. 832–842, Aug. 1992.

[6] C. Perret, C. Schaeffer, and J. Boussey, "Microchannel integrated heat sinks in silicon technology," in *Proc. IEEE Ind. Appl. Conf.*, St.Louis, MO, Oct. 12–15, 1998, vol. 2, pp. 1051–1055.

[7] C. Perret, J. Boussey, C. Schaeffer, and M. Coyaud, "Analytic modeling, optimization, and realization of cooling devices in silicon technology," *IEEE Trans. Compon. Packag. Technol.*, vol. 23, no. 4, pp. 665–672, Dec. 2000.

[8] S. J. Kim, "Methods for thermal optimization of microchannel heat sinks," *Heat Transfer Eng.*, vol. 25, no. 1, pp. 37–49, 2004.

[9] H. R. Upadhye and S. G. Kandlikar, "Optimization of microchannel geometry for direct chip cooling using single phase heat transfer," in *Proc. 2nd Int. Conf. Microchannels Minichannels*, 2004, pp. 679–685.

[10] D. Liu and S. V. Garimella, "Analysis and optimization of the thermal performance of microchannel heat sinks," *Int. J. Numer. Methods Heat Fluid Flow*, vol. 15, no. 1, pp. 7–26, 2005.

[11] V. Singhal, D. Liu, and S. V. Garimella, "Analysis of pumping requirements for microchannel cooling systems," in *Proc. Int. Electron. Packag. Tech. Conf. Exhib.*, Maui, HI, Jul. 6–11, 2003, vol. 2, Advances in Electronic Packaging, pp. 473–479.

[12] S. G. Kandlikar and H. R. Upadhye, "Extending the heat flux limit with enhanced microchannels in direct single-phase cooling of computer chips," in *Proc. IEEE 21st Annu. Symp. Semiconductor Thermal Meas. Manage.*, Mar. 15–17, 2005, pp. 8–15.

[13] M. B. Kleiner, S. A. Kuhn, and K. Haberger, "High performance forced air cooling scheme employing microchannel heat exchangers," *IEEE Trans. Compon. Packag. Manuf. Technol. A*, vol. 18, no. 4, pp. 795–804, Dec. 1995.

[14] A. Aranyosi, L. M. R. Bolle, and H. A. Buyse, "Compact air-cooled heat sinks for power packages," *IEEE Trans. Compon. Packag. Manuf. Technol. A*, vol. 20, no. 4, pp. 442–451, Dec. 1997.

[15] C. Harris, M. Despa, and K. Kelly, "Design and fabrication of a cross flow micro heat exchanger," *J. Microelectromech. Syst.*, vol. 9, pp. 502–508, 2000.

[16] S. V. Garimella and V. Singhal, "Single-phase flow and heat transport and pumping considerations in microchannel heat sinks," *Heat Transfer Eng.*, vol. 25, no. 1, pp. 15–25, 2004.

[17] S. P. Jang and S. J. Kim, "Fluid flow and thermal characteristics of a microchannel heat sink subject to an impinging air jet," *J. Heat Transfer*, vol. 127, no. 7, pp. 770–779, 2005.

[18] S. F. Choquette, M. Faghri, M. Charmchi, and Y. Asako, "Optimum design of microchannel heat sinks," *ASME Dyn. Syst. Control Div.*, vol. 59, pp. 115–126, 1996.

[19] W. Zhimin and C. K. Fah, "Optimum thermal design of microchannel heat sinks," in *Proc. 1st Electron. Packag. Technol. Conf.*, Singapore, Oct. 8–10, 1997, pp. 123–129.

[20] L. Meysenc, L. Saludjian, A. Bricard, S. Rael, and C. Schaeffer, "A high heat flux IGBT micro exchanger setup," *IEEE Trans. Compon. Packag. Manuf. Technol. A*, vol. 20, no. 3, pp. 334–341, Sep. 1997.

[21] S. H. Chong, K. T. Ooi, and W. T. Wong, "Optimization of single and double layer counter flow microchannel heat sinks," *Appl. Thermal Eng.*, vol. 22, no. 14, pp. 1569–1585, 2002.

[22] X. S. Liao, Y. Liu, Y. Q. Ning, D. M. Cheng, L. Wang, and L. J. Wang, "The optimal design of structure parameters for microchannel heat sink," *Proc. SPIE—Int. Soc. Opt. Eng.*, vol. 4914, pp. 181–186, 2002.

[23] J. H. Ryu, D. H. Choi, and S. J. Kim, "Numerical optimization of the thermal performance of a microchannel heat sink," *Int. J. Heat Mass Transfer*, vol. 45, no. 13, pp. 2823–2827, 2002.

[24] X. Wei and Y. Joshi, "Optimization study of stacked micro-channel heat sinks for micro-electronic cooling," *IEEE Trans. Compon. Packag. Technol.*, vol. 26, no. 1, pp. 55–61, Mar. 2003.

[25] G. Jean-Antoine, B. Christophe, and T. Bernard, "Extruded microchannel-structured heat exchangers," *Heat Transfer Eng.*, vol. 26, no. 3, pp. 56–63, 2005.

[26] J. Y. Min, S. P. Jang, and S. J. Kim, "Effect of tip clearance on the cooling performance of a microchannel heat sink," *Int. J. Heat Mass Transfer*, vol. 47, no. 5, pp. 1099–1103, 2004.

[27] E. R. Delsman, A. Pierik, M. H. J. M. De Croon, G. J. Kramer, and J. C. Schouten, "Microchannel plate geometry optimization for even flow distribution at high flow rates," *Chem. Eng. Res. Design*, vol. 82, no. 2, pp. 267–273, 2004.

[28] O. Haddad, M. Abuzaid, and M. Al-Nimr, "Entropy generation due to laminar incompressible forced convection flow through parallel-plates microchannel," *Entropy*, vol. 6, no. 5, pp. 413–426.

[29] W. A. Khan, M. M. Yovanovich, and J. R. Culham, "Analytical modeling of fluid flow and heat transfer in microchannel heat sinks," *Int. J. Heat Mass Transfer*, submitted for publication.

[30] W. M. Kays and A. L. London, *Compact Heat Exchangers*. New York: McGraw-Hill, 1964.

[31] W. F. Stoecker, *Design of Thermal Systems*. New York: McGraw-Hill, 1989.

[32] R. J. Culham and Y. S. Muzychka, "Optimization of plate fin heat sinks using entropy generation minimization," *IEEE Trans. Compon. Packag. Technol.*, vol. 24, no. 2, pp. 159–165, Jun. 2001.

[33] R. J. Culham, W. A. Khan, M. M. Yovanovich, and Y. S. Muzychka, "The influence of material properties and spreading resistance in the thermal design of plate fin heat sinks," in *Proc. 35th Nat. Heat Transfer Conf.*, Anaheim, CA, Jun. 10–12, 2001, pp. 240–246.

[34] W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Optimization of pin-fin heat sinks using entropy generation minimization," *IEEE Trans. Compon. Packag. Technol.*, vol. 28, no. 2, pp. 247–254, Jun. 2005.

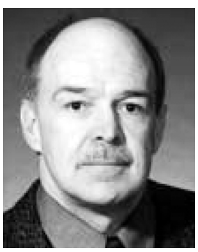
[35] W. A. Khan, J. R. Culham, and M. M. Yovanovich, "Optimal design of tube banks in crossflow using entropy generation minimization method," *AIAA J. Thermophys. Heat Transfer*, vol. 21, no. 2, pp. 372–378, Apr./Jun. 2007.

[36] S. A. Schaaf and P. L. Chambre, *Flow of Rarefied Gases*. Princeton, NJ: Princeton Univ. Press, 1961.

Waqar Ahmed Khan received the Ph.D. degree from the University of Waterloo, Waterloo, ON, Canada.

Currently, he is an Associate Professor of Mechanical Engineering at the National University of Sciences and Technology, Karachi, Pakistan. He has developed several unique analytical models for the fluid flow and heat transfer across single cylinders (circular/elliptical), tube banks and pin-fin heat sinks to Newtonian and non-Newtonian fluids. His research interests include modeling of forced convection heat transfer from complex geometries, microchannel heat sinks, thermal system optimization using entropy generation minimization, forced and mixed convection, and conjugate heat transfer in air and liquid cooled applications. He has more than 28 publications in refereed journals and international conferences.

He is a member of the American Society of Mechanical Engineers (ASME), the American Institute of Aeronautics and Astronautics (AIAA), and the Pakistan engineering council.



J. Richard Culham (M'98) received the Ph.D. degree from the University of Waterloo, Waterloo, ON, Canada.

Currently, he is a Professor of Mechanical and Mechatronics Engineering and the Associate Dean of Engineering for Research and External Partnerships at the University of Waterloo. He is the Director and a Founding Member of the Microelectronics Heat Transfer Laboratory. Research interests include modeling and characterization of contacting interfaces and thermal interface materials, development of compact analytical and empirical models at micro- and nanoscales, natural and forced convection cooling, optimization of electronics systems using entropy generation minimization, and the characterization of thermophysical properties in electronics and optoelectronics materials. He has more than 100 publications in refereed journals and conferences in addition to numerous technical reports related to microelectronics cooling.

Prof. Culham is a member of the Professional Engineers Ontario.

M. Michael Yovanovich received the Sc.D. degree from the Massachusetts Institute of Technology, Cambridge.

Currently, he is a Distinguished Professor Emeritus of Mechanical Engineering at the University of Waterloo, Waterloo, ON, Canada, and is the Principal Scientific Advisor to the Microelectronics Heat Transfer Laboratory. His research in the field of thermal modeling includes analysis of complex heat conduction problems, external and internal natural and forced convection heat transfer from and in complex geometries, and contact resistance theory and applications. He has published more than 350 journal and conference papers, and numerous technical reports, as well as three chapters in handbooks on conduction and thermal contact resistance. He has been a consultant to several North American nuclear, aerospace and microelectronics industries and national laboratories.

Dr. Yovanovich has received numerous awards for his teaching and his significant contributions to science and engineering. He is the recipient of the American Institute of Aeronautics and Astronautics (AIAA) Thermophysics Award and the American Society of Mechanical Engineers (ASME) Heat Transfer Award. He is a Fellow of AAAS, AIAA, and ASME.