values of radius ratio R_1/R_2 and the optical thickness $R_2 - R_1$ are given in Table 2. These results appear in excellent agreement with Monte Carlo results of Perlmutter and Howell [3]. In fact in view of our results for parallel plates, we can assert that results reported in Table 2 are more accurate than results of Ref. [3].

We note that for this case, Heaslet and Baldwin [12] give an approximate relation,

$$q \sim \frac{1}{1 + \frac{3}{4}R_1 \log R_2/R_1} \tag{36}$$

The results corresponding to this expression are given in Table 3, and appear to compare somewhat less favorably with the results given in Table 2.

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OVERALL CONSTRICTION RESISTANCE BETWEEN CONTACTING ROUGH, WAVY SURFACES†

M. MICHAEL YOVANOVICH

Laboratoires d'Etudes Thermiques de l'Ecole Nationale Supérieure de Mécanique et d'Aérotechnique, Poitiers, France

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NOMENCLATURE

- a, contact spot radius;
- A, contour area radius;
- b, heat flux tube radius;
- B, heat channel radius;
- k, thermal conductivity, $k = 2k_1k_2/(k_1 + k_2)$;
- N, number of contact spots;
- R, overall thermal contact resistance, [°C/W].

Greek characters
- β, maldistribution factor $(1 < \beta < 1{\cdot}4)$;
- γ, ratio A/B;
- ϵ, ratio a/b;
- ψ, constriction factor defined by equations (2) and (3).

Subscripts
- 1, 2, metals 1 and 2;
- o, microscopic;
- c, macroscopic;

Superscript
- T, factor based upon uniform temperature.

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INTRODUCTION

In a recent article [1] the authors showed qualitatively that the heat-transfer anomalies associated with stainless steel/aluminium interfaces depend upon the roughness and waviness of the contacting surfaces. They further showed that the mathematical models based upon (a) nominally, flat rough surfaces [2] or (b) smooth spherical caps [3] were not adequate in themselves to predict the overall resistance. They concluded that their test data showed clearly that a model must be developed which includes the effects of both surface roughness and waviness. This communication presents the outlines of an overall constriction resistance theory developed by the author [4] and shows how it compares with the theories of Holm [5] and Kragelskii [6].

Contact between rough, wavy surfaces

Worked metallic surfaces, whether turned, ground, or sandblasted, exhibit a random distribution of asperity heights about some mean surface passing through the asperities. The distribution of the asperities over the apparent area usually is not random, but exhibits a lay. The lay or predominant direction of the asperities is called the surface waviness. It will depend upon the mechanical process: turning will produce a circular pattern, while grinding or milling will produce a linear pattern.

Let us now consider the physical interaction of two spherical caps possessing substantial roughness. Initially the contact will be made at the few highest asperities located at the highest part of the spherical caps. As the load increases, these initial contact spots (assumed circular), also called microcontacts, increase in size, and newer and smaller spots begin to form. Upon increasing the load further, the first spots grow even larger, the second group of spots also increase in size, and still newer and smaller spots appear. The process is repeated with each increase of the load.

We see from this simple description that the contact between rough, wavy surfaces will consist of a large number of discrete microcontacts which differ in size, frequency of occurrence and probably shape. The largest microcontacts are sparse, while the smallest are many, and also the largest can be an order of magnitude larger in size than the smallest. Furthermore, the contact spots are confined to a portion of the apparent area (projected area of solids), which is called the contour area, shown in Fig. 1. The contour area is the projected area determined by the outer limits of the microcontacts. In the region of the interface beyond the contour area, there is no physical contact between the two surfaces. The contour area lying wholly within the apparent area can occupy a fraction or the entire portion of the apparent area depending upon the surface characteristics, the material properties, and the load on the interface.

Overall constriction resistance

In the absence of an interstitial fluid and negligible radiation heat transfer across the gaps, any heat flow across the interface will be confined to the microcontacts which define the contour area. The overall constriction resistance

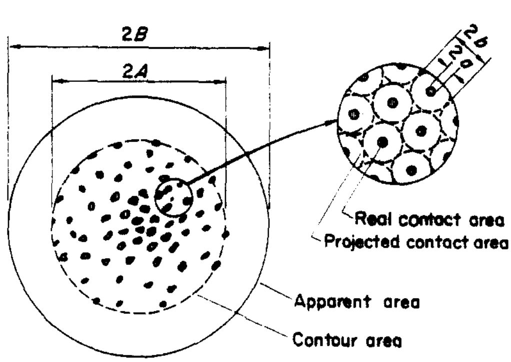

FIG. 1. Spherical contact model.

is postulated to be the sum of the roughness resistance (pinching of the heat flow lines due to the contact spots) and the waviness resistance (constriction of the heat flow lines due to the contour area). From symmetry about the contact plane we can express the overall constriction resistance as

$$R = \frac{\psi_o^T}{2k \sum_{i=1}^{N} \frac{a_i}{\beta_i}} + \frac{\psi_c^T}{2kA} \tag{1}$$

where the first term represents the cumulative microscopic or contact spot resistance, and the second term is the macroscopic or contour area resistance.

The microscopic resistance is based upon the following assumptions:

1. There are N circular microcontacts of radii a_i distributed rather uniformly over a plane which is perpendicular to the heat flux vector at large distances from the plane.
2. All the microcontacts are at the same uniform temperature.
3. The temperature perturbation due to the pinching of the flow lines as heat enters and leaves the microcontacts occurs in a very thin volume on either side of the contact plane.
4. There is associated with each microcontact a circular cylindrical heat flux tube defined by the outer limits of the heat which flows through the microcontact.

In particular, assumptions 2 and 4 lead to the microscopic constriction factor ψ_o^T, and a useful and accurate expression for it is [4, 7]

$$\psi_o^T = 1 - \frac{4}{\pi}\frac{a}{b} = 1 - 1{\cdot}28\epsilon \tag{2}$$

which is valid for each microcontact. A typical range of values of ϵ lies between 0 and 0·30.

The maldistribution factor β_i associated with each microcontact is a measure of its ability to conduct heat in the presence of a microcontact or an adiabatic wall. It has been shown experimentally [4, 8, 9] that this factor can be important when there is a great deal of crowding of microcontacts. Although an analytical expression for β_i has at present not been developed, it is sufficient to say that values of β_i lie between 1 and 1·4, and it seems to be independent of the contact spot size.

The macroscopic term of equation (1) is based upon the following assumptions:

1. All the microcontacts of radii a_i are distributed within a circular contour area of radius A.
2. The radius A is at least an order of magnitude larger than the average distance between microcontacts.
3. The contour area is concentric with the circular apparent area of radius B.
4. The macroscopic constriction resistance due to the contour area is based upon a very thin isothermal disc of radius A supplying (or receiving) heat to a circular cylinder or radius B.

These assumptions permit one to determine the macroscopic constriction factor ψ_c^T which can be written as [9].

$$\psi_c^T = (1 - \gamma)^{1\cdot 5} \tag{3}$$

for the entire range of the parameter γ.

Various aspects of equation (1) have been clearly demonstrated by means of electrolytic analog experiments [4, 8]. These experiments showed that equation (1) is valid for a very large range of the pertinent geometric parameters (a_i, N, A). It was also noted that the maldistribution factor β_i is practically unity for very small contact spots ($\epsilon \leqslant 0{\cdot}1$) and for distributions which appear uniform to the eye [9].

When all the contact spots are assumed to be of equal size ($a_1 = a_2 = \ldots = a_N$) and uniformly distributed ($\beta_i = 1$) over the contour area, equation (1) takes a simpler form

$$R = \frac{\psi_e^T}{2kNa} + \frac{\psi_c^T}{2kA}. \tag{4}$$

Very good correlation between equation (4) and electrolytic analog test data [10] has been obtained for a large range of values of the parameters a, A, ϵ, γ, N.

In the limit when $\epsilon \to 0$ and $\gamma \to 0$, equation (4) becomes

$$R = \frac{1}{2kNa} + \frac{1}{2kA} \tag{5}$$

which is the expression developed independently by Holm [5] and Kragelskii [6]. Equation (5) is restricted to a contact which consists of many very small ($\epsilon \leqslant 0{\cdot}01$) circular microcontacts (all the same size) uniformly distributed over a very small contour area ($\gamma \leqslant 0{\cdot}01$). A contact which satisfies these conditions is quite uncommon and would occur only when the contacting surfaces were very rough and very wavy, and the contact load very light.

CONCLUDING REMARKS

Equation (1) is a general expression for the overall constriction resistance when all the heat flows through only the microcontacts which lie in a plane comprising a fraction of the total apparent contact area. This equation has been verified by means of electrolytic analog experiments for a very large range of important geometric parameters for which agreement between theory and experiment was excellent.

The microscopic resistance term of equation (1) was recently used [9] to predict the thermal contact resistance between nominally, flat rough surfaces in vacuo. Agreement between theory and test was found to be quite good for a fairly wide range of surface roughness and applied load.

The macroscopic resistance term of equation (1) had been used by Clausing [3] in the study of the thermal resistance between contacting smooth spherical caps in vacuo. He found good agreement between the theory and test data as long as the surfaces were very smooth and applied loads less than a certain limit. The differences which Clausing observed can be attributed to the fact that all surfaces possess some roughness which becomes important under certain conditions of loading, and, therefore, cannot be ignored.

The prediction of the overall thermal constriction resistance as given by equation (1) has not been fully tested for real contacting solids possessing substantial surface roughness and waviness because, at present, the deformation of such surfaces is only partially understood. Thus one cannot predict the geometric parameters which enter into equation (1) from a knowledge of the geometric and physical characteristics of the contacting solids.

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THERMAL DIFFUSIVITY MEASUREMENTS FROM A STEP FUNCTION CHANGE IN FLUX INTO A DOUBLE LAYER INFINITE SLAB

E. K. HALTEMAN and R. W. GERRISH, JR.
Pittsburgh Corning Corporation, Pittsburgh, Pa. 15239

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NOMENCLATURE

- A, dimensionless argument;
- a, thermal diffusivity;
- C, specific heat;
- d, density;
- E, error in time integral;
- L, thickness;
- p, transformed time;
- q, parameter in transformed equation;
- t, time;
- TI, time integral;
- U, transformed temperature;
- x, distance.

Greek symbols
- α, root of transcendental equation;
- ζ, dimensionless ratio of conductances, $\lambda_2 L_1/\lambda_1 L_1$;
- θ, temperature;
- λ, thermal conductivity;
- Φ, flux;
- ρ, dimensionless ratio of heat capacity per unit area, $d_2 C_2 L_2/d_1 C_1 L_1$;
- σ, dimensionless ratio $[\lambda_2 d_2 C_2/\lambda_1 d_1 C_1]^{\frac{1}{2}}$;

Subscripts
- n, root index;
- x, differentiation with respect to distance;
- t, differentiation with respect to time;
- 1, 2, layer number.

1. INTRODUCTION

THE THERMAL diffusivity of ceramic and organic insulating materials is most readily obtained from transient linear heat flow through an infinite slab. The relative ease of fabricating the sample in the form of a slab makes this geometry attractive. Plummer, Campbell and Comstock [1] developed a method based on a constant flux into a thick slab of material which was treated as a semi-infinite solid. This method was further refined by Harmathy [2] who also developed a pulse heating scheme. Steere [3] used the constant flux method with samples of plastic assembled from multilayers of thin films. In all cases the finite samples were considered to be infinitely thick during the time when measurements were taken. Also, in each case the heat capacity of the heater was shown to be a small fraction of the heat capacity of the sample and was therefore not included in the analysis.

When the constant flux input method is used with a low density, low specific heat and low conductivity insulator such as foamed polyurethane, difficulties arise. The conductivities of many solid and foamed insulators are approximately proportional to their densities; hence, their diffusivities are similar. But the heat capacity per unit volume of the sample can vary widely since it depends upon density and specific heat. Thus, for low density organic insulators the heat capacity of the heater may represent an appreciable fraction of the heat capacity of the sample. In such cases it is necessary to treat the heater as a separate layer with its own thermal properties and to determine the diffusivity of the sample from an analysis of a double layer infinite slab model.

2. THEORY

The temperature distribution, $\theta(x, t)$, within an infinite slab of thickness, L, is given by the solution of the one dimensional equation of linear heat flow with specified boundary conditions.

$$a\theta_{xx}(x, t) = \theta_t(x, t) \text{ for } 0 < x < L. \tag{1}$$