Electrical Measurement of Joint Resistance at Perfect Contact Interfaces: Application to Joint Conductance

By

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ELECTRICAL MEASUREMENT OF JOINT RESISTANCE AT PERFECT CONTACT INTERFACES: APPLICATION TO JOINT CONDUCTANCE

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Abstract

This paper presents an analytical and experimental investigation on the electrical resistance and by analogy the thermal joint conductance across rough but perfect (continuous) - contact interfaces. Two types of joints were examined: single interfaces produced by electroplating, soldering or explosion bonding, and double interfaces produced by soldering. Continuous potential distribution plots across an interface were obtained by passing an a.c. current through the sample of interest and monitoring the voltage difference between a stationary and moving probe. The pseudo-voltage drop and thereby interface resistance was measured by extrapolating from the undisturbed regions to the interface(s). Numerous samples were tested with electrical resistivity ratios of 1.6 to 63 and surface roughness, produced by sanding and glass-peening, of 0.5μm to 4.5μm rms. The electrical tests showed that the analogous thermal joint conductance at perfect interfaces would range from 160 to 2400 w/cm^2 - ^{o}C (per interface).

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Nomenclature

A	=	sample cross-sectional area
h_j	=	joint conductance, per interface, Eq. (4)
I	=	current
J	=	current density
k_i	=	thermal conductivity of ith material
N_i	=	number of interfaces with ρ_i on either side
q	=	heat flux
R_c	=	constriction resistance; total
T	=	temperature
t	=	average thickness of sandwich material
t'	=	effective constriction resistance length; per interface, Eq. (3)
V	=	voltage potential
ΔV_m	=	measured voltage drop; total
δ	=	specific resistance; total, Eq. (1)
ρ_i	=	electrical resistivity of ith material
σ	=	surface roughness; rms

Subscripts

1,2,3	=	materials forming a joint
c	=	constriction
j	=	joint
t	=	thermal

Introduction

The problem of electrical or thermal resistance at continuous contact interfaces is currently of great interest[2-7] because many electrical and thermal systems have components with interfaces between the source and sink. The resistance at these interfaces may be substantial.

The main purpose of this study was to measure the electrical resistance across nominally flat but rough, continuous contact, interfaces. For this reason, electroplated, explosion bonded and soldered samples were considered. Although the project involved measurement of electrical resistance, its analogous thermal resistance was of primary interest. Since both modes of transport obey Laplace's equation $\nabla^2 V=0$ and $\nabla^2 T=0$ with similar boundary conditions, the electrical resistance results can readily be converted to thermal resistance or conductance.

In thermal resistance measurements, thermocouples are placed at finite distances from the interface (usually located in the undisturbed region), and the temperature drop at the interface is found by extrapolating from both sides[1]. The pseudo-temperature drop is a direct measure of the thermal interface resistance. Such a method is limited to relatively large samples and high resistances occurring with mechanical contacts. An alternative method is to measure the temperature drop along a vertical column of liquid metal intersected by many thin metal plates[2,3]; in this way the interface resistance is multiplied by the number of interfaces. Such experimental results are applicable for comparison since a liquid metal-solid interface under complete wettability and no flow condition is equivalent to a continuous contact metal-metal interface.

A literature survey was conducted, and it was found that Schmidt and Jung[2], of Germany, had measured the thermal resistance across stainless steel-liquid sodium interfaces. They stacked vertically numerous stainless steel plates separated by liquid sodium and measured the over-all temperature drop. From the total resistance they subtracted the resistance of the plates and liquid sodium, and calculated the total constriction resistance. For the best tests with clean liquid sodium and stainless steel surfaces with excellent wettability, they measured (from an average of 70 tests) a resistance, per interface, equivalent to a stainless steel thickness of 0.015 mm. This gives an effective constriction resistance length $t' = 0.013$ mm and a joint conductance of $h_j = 110\ w/cm^2 - {}^oC$.

Bleunven et al.[3], of France, measured by a similar method, the thermal resistance across stainless steel-liquid NaK interfaces. Their best results yielded an equivalent resistance of $t' = 0.015$ mm, slightly decreasing with increasing pressure.

Yovanovich[4] performed thermal resistance tests with tin soldered joints. His results with brass/brass, brass/stainless steel and stainless steel/stainless steel joints yielded a minimum resistance of $t' = 0.05$ mm.

An electrical resistance test was performed by Vandenberg[5] using a method similar to the thermal experiments. He measured the electrical resistance across 347 SS diaphragms, of decreasing thickness, immersed in liquid sodium. His results yielded a $t' = 0.025$ mm.

Mengali and Seiler[6] measured the electrical resistance across a number of metal-thermoelectric material contacts prepared by electroplating and metal spraying. Large and varying electrical resistances were measured, indicating an imperfect joint. Scans at lateral displacements of 0.25 mm showed variations of interface resistance by a factor as large as 12. However, some samples had a low interface resistance with the smallest specific resistance observed being 1.86×10^{-7} ohm-cm^2,

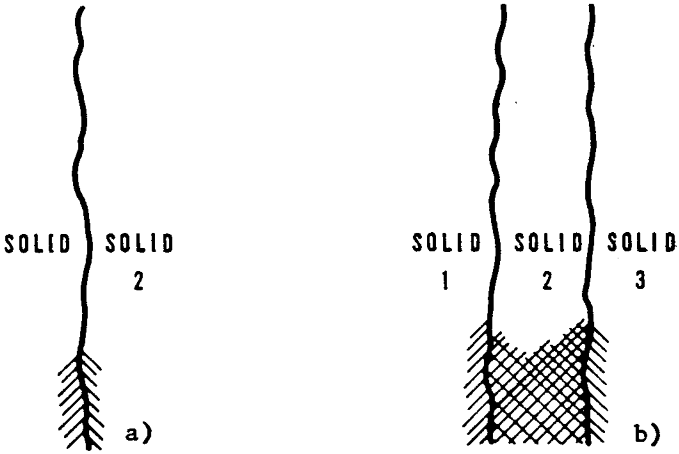

Fig. 1 Schematic of single and double continuous contact interfaces.

corresponding to an effective constriction resistance length of t' = 0.005 mm.

In the present study, the potential-probing technique was used to measure the electrical resistance across single interfaces produced by electroplating, soldering or explosion bonding, Fig. 1a, and double interfaces obtained by soldering, Fig. 1b. In the conventional method[6,7] voltage readings are taken at discrete locations and extrapolated to the interface to obtain the pseudo-voltage drop. In this research, a continuous recording of voltage vs distance across the interface was obtained. This permitted a continuous display of the potential and electrical resistance distribution right up to the interface. The constriction voltage drop across the interface could therefore be measured with higher accuracy.

Theory

Current Flow Across a Continuous Contact Interface

If the two contacting solids make continuous contact, as in electroplated surfaces, the flow lines refract at the interface due to the different component resistivities. The incident and refracted angles are determined from the following equation[8]:

$$\rho_1 \tan\alpha_1 = \rho_2 \tan\alpha_2$$

where $\alpha_{1,2}$ ≡ angle of flow lines to the normal.

Since the current is forced to follow a nonparallel flow path, an additional resistance exists, and it is called the constriction resistance. It is expected that the constriction resistance of a continuous contact interface is much smaller than that of a mechanical contact. When the interface is flat and perfectly smooth, the flow lines are parallel to each other and normal to the interface, and the constriction resistance should be zero.

Measurement of Constriction Resistance

In the conventional method, voltage readings are taken at discrete locations and extrapolated to the interface. A pseudo-voltage drop (ΔV_m) will be observed due to the resistance to current flow in the region of the interface, Fig. 2a. The voltage drop and thereby constriction resistance is measured by extrapolating from the undisturbed regions to the interface. From the measured voltage drop a specific constriction resistance can be calculated by the following relationship:

$$\delta = R_c A = \Delta V_m / J \tag{1}$$

where J is the current density normal to the plane of the interface defined as

$$J = - \partial V / \rho \partial n$$

which is equal to I/A for constant voltage gradients.

For multiple interfaces the expression for the resistance is defined in the following manner:

$$\delta = \sum_{i=2}^{n-1} t_i(\rho_1 - \rho_i) - \frac{\Delta V_m}{J} \tag{2}$$

for n components with (n-1) interfaces, Fig. 2b. (The dotted line in this figure represents the potential distribution with an interface resistance.)

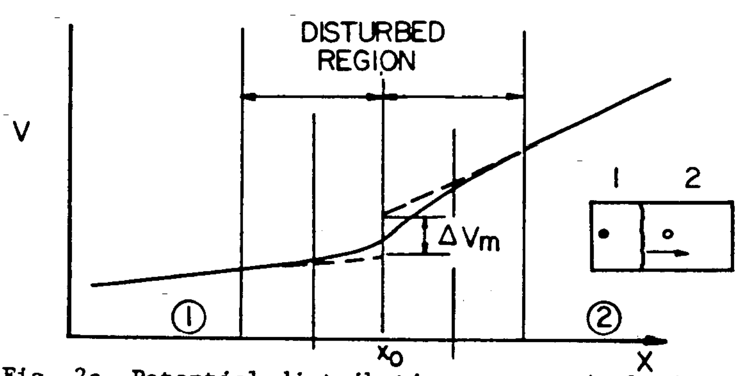

Fig. 2a Potential distribution across single interface.

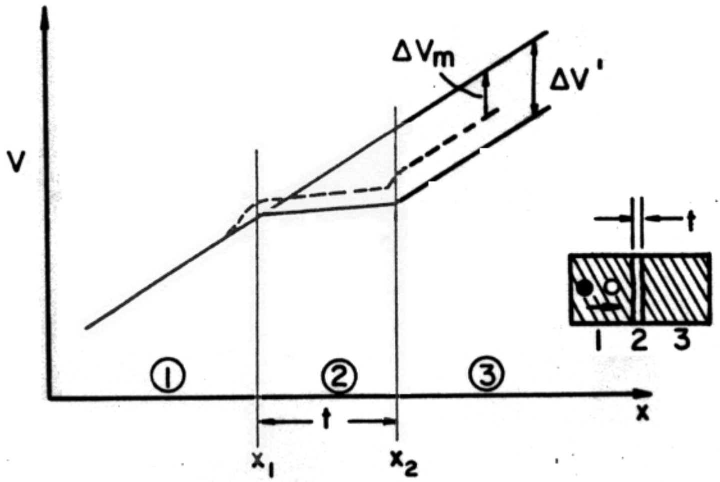

Fig. 2b Potential distribution across double interface.

To normalize the test results an effective constriction resistance length is introduced to represent the electrical and thermal resistances

$$t' = \delta \Big/ \sum_{i=1}^{n} N_i \rho_i = \delta_t \Big/ \sum_{i=1}^{n} N_i / k_i \tag{3}$$

which reduces to

$$t' = \delta / (\rho_1 + \rho_2)$$

for a single interface sample and

$$t' = \delta / (\rho_1 + 2\rho_2 + \rho_3)$$

for a double interface.

The equivalent thermal conductance for a single interface, by analogy, will be (Appendix B)

$$h_j = \frac{\rho_1 + \rho_2}{\delta} \left[1/k_1 + 1/k_2 \right]^{-1}$$

For multiple interfaces the expression becomes

$$h_j = \frac{\sum_{i=1}^{n} N_i \rho_i}{\delta} \left[\sum_{i=1}^{n} N_i / k_i \right]^{-1}$$

Description of Equipment

In this investigation an a.c. potential-probing method was used. An a.c. current having a frequency range of 10 - 300 Hz was passed through the sample of interest, typically about 0.25 x 0.25 x 1.3 cm, with most measurements performed at 25 and 105 Hz. A d.c. current could not be used because it would introduce errors due to the Seebeck voltage (thermocouple effect) at the probe-sample contact. The electrical resistivity of the material creates a voltage difference between the probes. One probe is kept stationary while the other moves at constant speed (approximately 0.025 mm/min.). The signal is amplified with a differential amplifier and monitored with a wave analyzer, which acts as a narrow band filter and rms meter. A continuous trace is registered on a chart recorder. The moving probe has a radius of about 0.0025 mm and is used to measure local potential distributions as it traverses the lateral surfaces of two contacting solids. The probe begins in a region of uniform potential or constant potential gradient

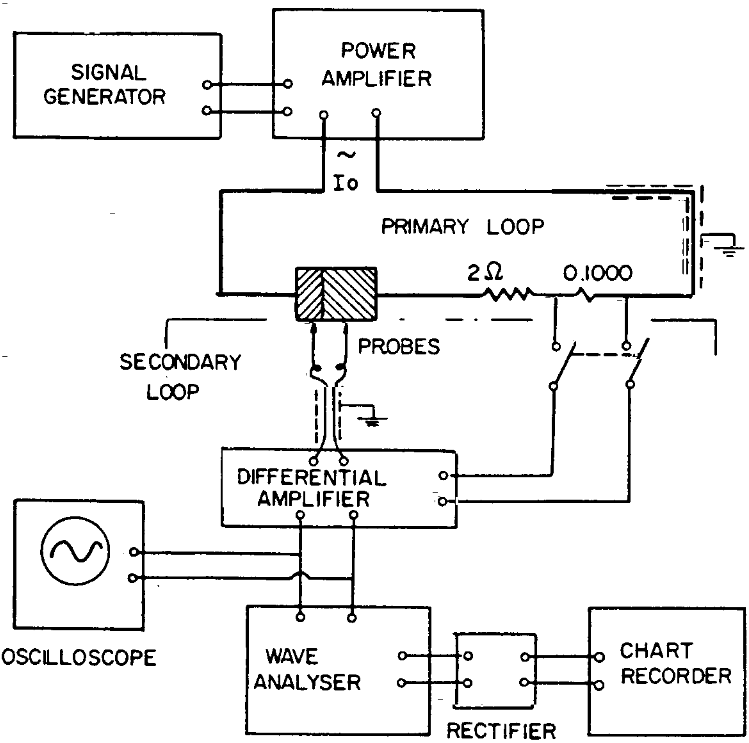

Fig. 3 Potential-probe apparatus: a.c. current.

located about 1.3 mm from the interface, crosses a region of disturbed potential located on either side of the interface, and finally enters another constant gradient region. The thickness of the disturbed region ranges from 0.025 to 0.25 mm; the entire probed region does not usually exceed 2.5 mm. A direct measure of the joint resistance is made by extrapolating from the undisturbed regions to the interface.

The circuit is made up of two parts (Fig. 3): the primary loop and secondary pick-up loop. The primary loop consists of a signal generator, power amplifier, load and precision calibration resistor and the sample. It is the high current loop. The secondary loop comprises the stationary or reference probe, the movable probe and low-voltage signal amplification and monitoring equipment.

Calibration of the current is made by measuring the voltage drop across the 0.1 ohm precision resistor. This is spot monitored with the wave analyzer, at least once per scan. The resistivity of oxygen free high conductivity copper was measured at 24°C using a 0.245 x 0.387 x 1.9 cm sample. The measurement was 1.71×10^{-6} ohm-cm compared to 1.69×10^{-6} ohm-cm found in standard resistivity tables with correction for temperature. They agree within 1.2%.

Sample Preparation and Interface Examination

The samples were first prepared by machining on a lathe or on a milling machine. This produced a roughness of approximately 0.25 to 0.5 μm rms. These samples were then ready for glass-peening or sanding. Glass-peening with various diameter glass beads, 0.05 mm to 1.3 mm, and nozzle air pressure 7 to 52 N/cm^2 produced roughness ranging from 0.75 to 7.5 μm rms, depending on the material hardness. For 304 SS samples, the asperity slope had an equivalent angle of 1.4 to 5° rms.

Sanding of the sample (only the interface of interest) was performed in two steps: it was first sanded with decreasing coarseness to achieve a smooth and flat surface and then the required roughness was obtained by sanding with increasing coarseness paper. Three grades of carbide paper were usually sufficient to achieve the desired results. The roughness ranged from 0.5 to 2.5 μm rms depending on the material hardness. For 304 SS the maximum roughness achieved was 1.3 μm rms and the slope rms, had an equivalent angle of 2.9 to 6.8° rms. This meant that the asperity angle ranged from 174.2 to 166.4° at the 68% confidence level.

The samples were then electroplated or soldered and finally milled or cut to final size. The critical parameters were the interface plane relative position and the cross-sectional area. The interface plane had to be flat and normal to the side of the sample to within 0.2° for a single interface sample and 2° for a double interface. The cross-sectional area had to be constant to within 1%.

The surface of the sample to be scanned had to be polished with metallographic polishing cloth to a roughness less than 0.25 μm. This was necessary to insure continuous probe to sample contact even with light loads of 1 g.

The quality of the interface contact was checked by polishing the top or side surface of the sample and observing it under the microscope. The light microscope was used for low magnification (up to 1000x) and whenever color contrast was significant and the scanning electron microscope for higher resolution and greater magnification. The electron beam microprobe was also used for specimen current photographs and for the characteristic x-ray line scans. These three techniques of interface examination allow accurate analysis of the quality of the bond. (No mechanical tests were performed to check for bond strength.) From a close inspection of the photographs taken, it was concluded that the samples tested formed continuous contact at the interfaces.

Experimental Data

Single Interface

Samples with single interfaces were produced by copper electroplating, silver soldering and explosion bonding. The interfaces were prepared by sanding, glass-peening and lathe turning to various roughness.

Figure 4 shows a trace of the potential distribution across the interface of a Zn-Cu sample, with the following characteristics:

Size:	0.452 x 0.399 x 0.77 cm; area = 0.181 cm^2
Cu plating:	0.10 cm thick
Roughness:	1.3 μm rms
Sanded	

The test conducted with I = 3 amp and f = 25 Hz, gave the following resistance results:

$$Rc = 2 \times 10^{-8} \text{ ohms}$$
$$\delta = 3.6 \times 10^{-9} \text{ ohm-cm}^2$$

$$t' = 0.49 \times 10^{-3}\ \text{cm}$$
$$h_j = 1800\ \text{w/cm}^2 - {}^{\circ}\text{C}$$

The average resistance, from a number of tests, are presented in Table 1.

Double Interface

Samples with a double interface were prepared by silver soldering (with Silvalloy-45) two metal pieces together. The interface surfaces were prepared by sanding, glass-peening and lathe turning to produce the required surface roughness.

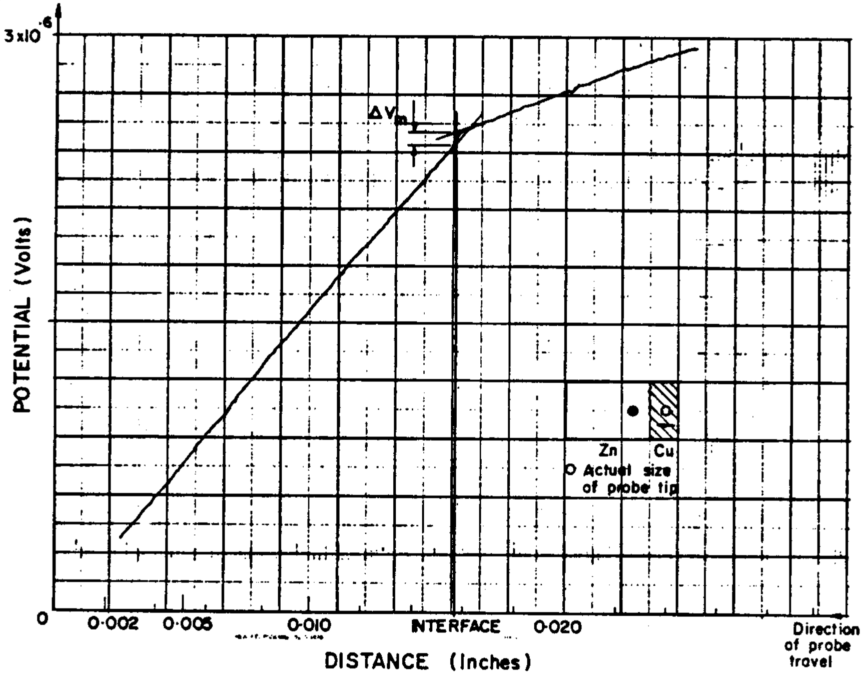

Fig. 4 Potential plot of a Zn-Cu sample (electrodeposited).

The resistance was determined by extrapolating one constant voltage gradient line and then measuring the voltage difference between the two parallel voltage gradient lines. This voltage drop was converted to thermal constriction resistance values by means of Eqs. 3 and 5.

Figure 5 illustrates the potential distribution of a scan across the solder joint of a 304 stainless steel-silver soldered sample. The sample dimensions and interface surface preparation characteristics are the following:

Size: $0.224 \times 0.142 \times 1.5$ cm; area = 0.032 cm^2
Ag_s thickness: $t = 0.0043$ cm
Roughness: 1.3 μm rms
Glass peened

The test was performed with I = 1 amp and f = 105 Hz. The measured constriction resistance, per interface, was

$$R_c = 2.2 \times 10^{-7} \text{ ohms}$$
$$\delta = 0.7 \times 10^{-8} \text{ ohm-cm}^2$$
$$t' = 0.39 \times 10^{-4} \text{ cm}$$
$$h_j = 3200 \text{ w/cm}^2 - {}^{\circ}\text{C}$$

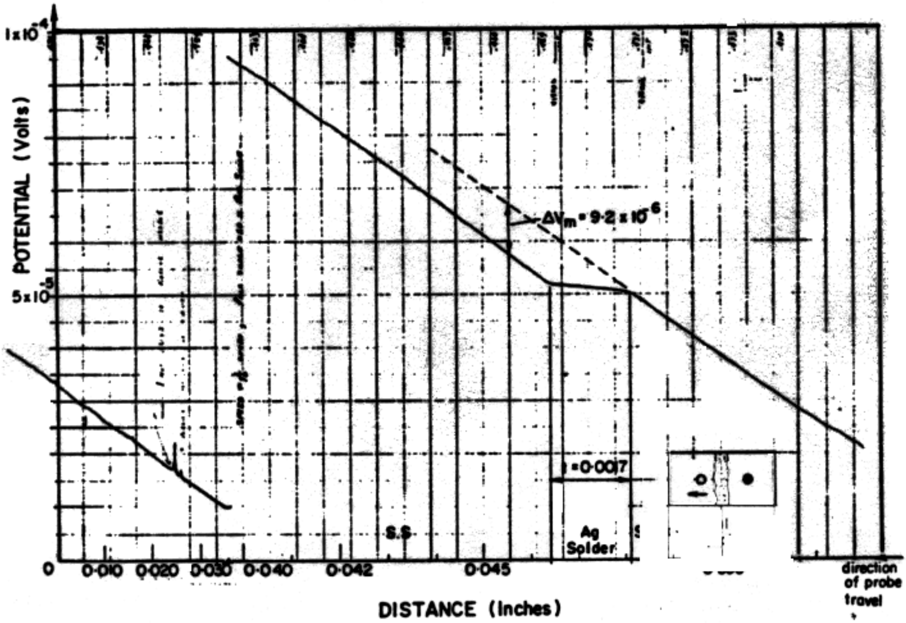

Fig. 5 Potential plot of a stainless steel-silver solder joint.

The average results for various samples with surface roughness created by sanding, glass-peening, and lathe turning are shown in Table 1.

Error Analysis

Single Interface

Since the accuracy of the interfacial resistance for a single interface depends critically on the location of the interface (Fig. 2a), it is important to examine the magnitude of the error. The effective constriction resistance length is

Table 1 Joint Conductance for Single and Double Interface Samples

Metal/Metal	Type of Interface	$t'_{(avg)} \times 10^3$ [cm]	h_j [Watt/cm - °C]
Ni-Cu	Electroplated	.95	750
Bi-Cu	Electroplated	.51	160
Zn-Cu	Electroplated	.61	1400
Fe-Ag_s	Soldered	.46	620
Al-Cu	Explosion Bonded	.54	2400
Fe-Ag_s-Fe	Soldered	.22	1200
SS-Ag_s-SS	Soldered	.20	620

given by

$$t' = \frac{\Delta V_m}{J(\rho_1+\rho_2)} = \frac{\Delta V_o + J(\rho_1 - \rho_2)(x_o - x_m)}{J(\rho_1 + \rho_2)}$$

where "o" denotes the exact interface location and "m" the measured location.

On the assumption that the deviations of the independent variables from the mean are random, and that their co-variance is negligible (the error of the measured quantities are independent of each other), the equation for the propagation of errors can be used:

$$S_{t'} = \left[\sum_{i=1}^{n}\left(\frac{\partial t'}{\partial x_i}\right)^2 \left(S_{x_i}\right)^2\right]^{1/2}$$

for n independent variables.

A typical test of a Ni-Cu sample was analyzed. Even with the optimal test conditions the error of t' was estimated to be 44% for t' = 0.0013 cm. Because of such a large error it was decided to test multiple interface samples, and as shown in the next section, the error is greatly reduced being only 23% for t' = 0.00048 cm.

A resistance of t' = 0.0013 cm for a Ni-Cu sample is equivalent to a thermal joint conductance of h_j = 550 w/cm^2 - °C.

Multiple Interfaces

For multiple interfaces (Fig. 2b), it is not necessary to know the location of the interfaces since only the thickness of the sandwiched material is important (Eq. 2). Since the material can be photographed at high magnification, the thickness can be measured with high accuracy. The error would only be significant for very rough (>2.5 μm rms) interfaces or wavy interfaces.

The equation relating the effective constriction resistance length per interface (from Eq. 2 and 3) is

$$t' = \frac{\sum_{i=2}^{n-1} t_i(\rho_1 - \rho_i) - \Delta V_m/J}{\sum_{i=1}^{n} N_i\rho_i} \tag{8}$$

and for a single soldered joint it reduces to

$$t' = \frac{t(\rho_1 - \rho_2) - \Delta V_m/J}{2(\rho_1 + \rho_2)} \tag{9}$$

For a typical 304 SS silver solder sample, a constriction resistance with an equivalent $t' = 0.00048$ cm, and assuming a large error of 10% on all the measured quantities, the error in t' is 51%. For the best controlled test, the error would reduce to 23%. The accuracy for a soldered joint appears to be 3 to 5 times better than that for a single interface.

A resistance of $t' = 0.00048$ cm, for the same sample, is equivalent to a thermal joint conductance of $h_j = 300$ w/cm^2-$^\circ$C.

Discussion and Conclusions

The primary purpose of the study was to determine if there was a constriction resistance across a rough but continuous contact interface. Electrical resistance was measured for the purpose of predicting thermal constriction resistance.

Samples were prepared with interfaces by electroplating, silver soldering and explosion bonding. The interface surface was given varying roughness by sanding, glass-peening or lathe turning.

It was felt that, if an interfacial resistance existed across a continuous contact interface, it should have been a function of the sample's component resistivity and interface roughness. A range of samples was therefore tested with $\rho_2/\rho_1 = 1.6$ to 63, and surface roughness $\sigma = 0.5$ μm to 4.5 μm rms.

The electrical resistance measurements were performed with an a.c. potential probing method. Continuous voltage scans were taken across the interface and the pseudo-voltage drop was measured by extrapolating the constant voltage gradient lines to the interface. Scans at various lateral locations yielded consistent results. New terms and definitions were introduced to calculate the constriction resistance across multiple interfaces. To normalize the test results an effective constriction resistance length, t', was introduced to represent the electrical and thermal interfacial resistance.

Test results for single and double interface samples demonstrated that the constriction resistance was extremely small and independent of the surface roughness ($\sigma < 4$ μm rms) and the method of surface preparation. The effective constriction resistance length for the single interface samples was found to be t' < 0.0010 cm and for the soldered joint samples to be t' < 0.00035 cm. The equivalent joint conductance of perfect interfaces (from Table 1) ranges from 160 to 2400 $w/cm^2 - {}^oC$. The maximum values of t' for single and double interface samples, are at the limit of sensitivity of the test equipment; the error analysis showed that they could have an error of approximately 50%. These values, therefore, represent the upper limit of the interface resistance. Testing of multiple interface samples would establish the resistance with higher accuracy.

The results from this study demonstrated an interfacial resistance approximately five times lower than that of other investigators[2,3,5] who investigated stainless steel-liquid metal interfaces under conditions of excellent wettability. This lower resistance can be attributed to the better contact interfaces achieved with small samples and to the higher resolution of the continuous potential probing equipment.

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Appendix A

Derivation of the Effective Constriction Resistance Length

The effective constriction length t' is a geometric factor having units of length. For mechanical contacts the constriction resistance on either side of the interface is directly proportional to the material resistivity, and therefore t' is the same on both sides. For a continuous contact the voltage distribution close to the interface depends on ρ_2/ρ_1. This creates a coupling at the interface. Analysis of the flow distribution across an interface, using Teledeltos paper[9], indicates that the disturbed region extends nearly equal distance on both sides of the interface. The constriction resistance should therefore be, at least, approximately proportional to the material resistivity, making t' approximately constant on both sides.

The total resistance of a sample can be determined as follows. For a single interface, (Fig. 6).

$$\begin{aligned} AR_T &= (\ell_1 + t')\rho_1 + (\ell_2 + t')\rho_2 \\ &= \ell_1\rho_1 + \ell_2\rho_2 + t'(\rho_1 + \rho_2) \\ &= AR_s + \delta \end{aligned}$$

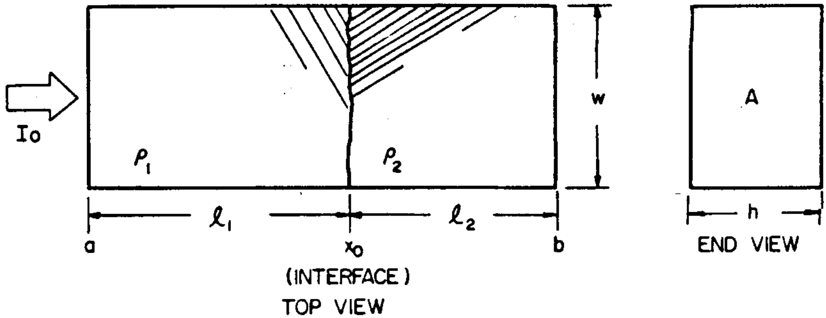

Fig. 6 Current flow through a sample .

where R_T and R_s represent the total and series resistance, respectively, and ℓ_1 and ℓ_2 the length (thickness) of material 1 and 2, respectively. Therefore $\delta = t'(\rho_1 + \rho_2)$ and thus $t' = \delta/(\rho_1 + \rho_2)$. For multiple interfaces

$$AR_T = \sum_{i=1}^{n} (\ell_i + N_i\, t')\rho_i$$

$$= \sum_{i=1}^{n} \ell_i \rho_i + \sum_{i=1}^{n} N_i\, t'\, \rho_i$$

$$= AR_s + \delta$$

Therefore,

$$\delta = \sum_{i=1}^{n} N_i\, t'\, \rho_i = t' \sum_{i=1}^{n} N_i\, \rho_i \quad \text{and} \quad t' = \delta \Big/ \sum_{i=1}^{n} N_i \rho_i$$

For the multiple interface samples, if all the interfaces do not have an equal t', each interface must be considered separately.

Appendix B

Conversion from Electrical to Thermal Resistance

Both the electrical and thermal modes of transport (electrostatic and temperature potentials, respectively) obey Laplace's equation

$$\nabla^2 V = 0$$

with the current flow proportional to the voltage gradient

$$J = -\,\partial V/\rho\,\partial n$$

which is Ohm's Law for current flow, and

$$\nabla^2 T = 0$$

with the heat flow proportional to the temperature gradient

$$q = -k\partial T/\partial n$$

which is Fourier's Law of Heat Conduction. Therefore, for the electrical case (single interface)

$$\delta = \frac{\Delta V_m}{J} \quad \text{and} \quad t' = \frac{\delta}{\rho_1 + \rho_2}$$

and for the thermal analog we have

$$\delta_t = \frac{\Delta T}{q} \quad \text{and} \quad t' = \frac{\delta_t}{1/k_1 + 1/k_2}$$

Therefore, $$\delta_t = \delta(1/k_1 + 1/k_2)/(\rho_1 + \rho_2)$$

and for multiple interfaces

$$\delta_t = \delta\left(\sum_{i=1}^{n} N_i/k_i\right)\left[\sum_{i=1}^{n} N_i\rho_i\right]^{-1}$$

The thermal joint conductance, per interface (or contact conductance), is given by

$$h_j = 1/\delta_t$$

or

$$h_j = \frac{1}{t'(1/k_1 + 1/k_2)}$$

It should be noted that, for a continuous contact interface, the constriction resistance might be a function of ρ_2/ρ_1, ($R_c = f(\rho_2/\rho_1)$) and, therefore, for the above conversions to hold, $\rho_2/\rho_1 = k_1/k_2$.