Apparent Conductivity of Glass Microspheres from Atmospheric Pressure Vacuum

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A theoretical study is presented for predicting the apparent thermal conductivity of beds of uniform diameter glass microspheres. The mathematical model is based upon thermal conduction resistance within the spheres and conduction resistance of an effective gas gap thickness. Both decoupled and coupled models are considered, and the latter is shown to be superior to the former. The theory is valid for gas pressures ranging from vacuum conditions to atmospheric. There is good agreement between the coupled model and available experimental data.


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ABSTRACT

An analytical study is presented for predicting the apparent thermal conductivity of beds of uniform diameter glass microspheres. The mathematical model is based upon thermal conduction resistance within the spheres and conduction resistance of an effective gas gap thickness. Both uncoupled and coupled models are considered, and the latter is shown to be superior to the former. The theory is valid for gas pressures ranging from vacuum conditions to atmospheric conditions. There is good to excellent agreement between the coupled model and available experimental data.

NOMENCLATURE

\( r \) radius of circular contact, \( m \)
\( r^* \) dimensionless contact radius \((r/D)\)
\( D \) diameter of spheres, \( m \)
\( E \) modulus of elasticity of spheres, \( N/m^2 \)
\( F \) contact force, \( N \)
\( \kappa_a \) apparent thermal conductivity of microspheres, \( W/m-K \)
\( \kappa_b \) apparent thermal conductivity of gas (air), \( W/m-K \)
\( \kappa_c \) apparent thermal conductivity of gas at atmospheric pressure, \( W/m-K \)
\( \kappa_e \) bulk thermal conductivity of glass spheres, \( W/m-K \)
\( \kappa_p \) dimensionless apparent conductivity \((\kappa_b/\kappa_c)\)
\( \kappa_p^* \) dimensionless apparent conductivity at vacuum conditions
\( \kappa_p^* \) dimensionless fluid conductivity \((\kappa_b/\kappa_c)\)
\( \kappa_e^* \) dimensionless parameter \((D/A)\)
\( g \) gas pressure, \( \text{kPa} \)
\( P \) heat flow rate, \( W \)
\( Q_0 \) heat flow rate via the contact area, \( W \)
\( Q_1 \) thermal resistance, \( \text{K/W} \)
\( Q_2 \) thermal conduction resistance, \( \text{K/W} \)
\( \kappa_p \) gas resistance, \( \text{K/W} \)
\( \kappa_p^* \) radiation resistance, \( \text{K/W} \)
\( \kappa_e \) temperature, \( \text{K} \)
\( \kappa_e^* \) contact area temperature, \( \text{K} \)
\( \kappa_e^* \) reduced absolute gas temperature \((\text{K}/\text{mol})\)
\( x \) dimensionless radial position \((r/a)\)
\( n^* \) mean free path of air at atmospheric pressure, \( \mu \)
\( \kappa_p \) dimensionless mean free path \((\kappa_p/\mu)\)
\( \alpha \) accommodation coefficient parameter,
\( \alpha = \frac{1}{\kappa_e^*} \cdot \frac{1}{\kappa_p^*} \)
\( \kappa_p^* \) equivalent gas gap thickness \((\kappa_p/\mu)\)
\( \kappa_p^* \) dimensionless equivalent gas gap thickness
\( \kappa_p^* \) equivalent dimensionless gas gap thickness
\( \kappa_p^* \) equivalent dimensionless gas gap thickness for close packed spheres
\( \kappa_p^* \) equivalent dimensionless gas gap thickness for loose packed spheres
\( \kappa_e^* \) thermal convection factor
\( \kappa_e^* \) Prandtl's ratio
\( \kappa_e^* \) lower limit of integration
\( \kappa_e^* \) porosity of the system
\( \kappa_e^* \) dimensionless parameter defined by Eq. (3)
\( \kappa_e^* \) dimensionless parameter defined by Eq. (10)

INTRODUCTION

Since the turn of this century, many scientists and engineers have become interested in heat transfer through porous substances [1]. In general, these porous substances have consisted of solid particles of various shapes and sizes in physical contact with each other, and voids of even more complicated shape. The two methods of predicting the apparent conductivity of these complex heterogeneous substances are (1) the method of generalized thermal conductivities and (2) the method based upon the geometry of a typical element. In the method of generalized conductivities it is postulated that the effective
thermal conductivity depends upon a, the thermal conductivities of the solid and fluid in the voids, b) the volume concentrations of the solid and fluid, and c) the distribution of the two phases within the porous substance. In the second method, one assumes a typical particle shape and size, a porous packing and calculates how much fluid can flow through such an element. The first method has been used and with some success to predict the effective or apparent thermal conductivity of very porous substances consisting of complete spherical solid particles and a fluid whose thermal conductivity is not greatly different from the solid. It cannot be used to predict the conductivity of closely packed substances when the thermal conductivities of the solids and fluid are very different. The second method has been employed in recent studies to predict fluid conductivity and has been successful in predicting the apparent thermal conductivity of very porous substances whose particles have well-defined geometries.

The study will be based upon the second method and will consider the effects of air pressure upon the apparent conductivity of glass microspheres. The analysis will be based upon the simplest cubic packing, and will consider unconfined and confined heat flows through the solid and gas phases.

Related studies

Heat transfer through composite materials such as foams, sand, rubber, ceramics, and many recently glass beads has been under experimental and analytical investigation during the past two decades [12-11, 12]. Inexpensive and lightweight but highly insulating, glass microspheres have attracted considerable interest as a thermal insulator because of their low density and large thermal conductivity of gas microspheres over a wide range of temperatures and gas pressures. They showed that radiation heat transfer is negligible for sphere diameters of 400 μm even when the temperature of the sphere approaches 400 °C. Glass microspheres have been used to correlate the data for low gas pressures, especially at vacuum conditions, because of their theory on point contact between spheres. Langer and Smith [12] examined, both empirically and analytically, the apparent thermal conductivity of powdered and solid porous materials over a wide range of temperatures, pressures, and rates of exchanging fluids. They compared their test results with theories proposed by other investigators as well as their own theory which included the effect of contact. This theory is much more complicated because it attempts to include the effect of surface roughness, the effect of various packing models as well as the effect of gas pressure. They made an attempt to correlate their test data and their theory over the entire range of gas pressure. They showed excellent agreement with some substances, moderate agreement with others, and some very poor agreement (errors > 200%) with steel balls in hydrogen. Hence at [15] determined the View factor between connecting hemispheres which can be used to predict the effect of radiation heat transfer when it is important. Meumann and Smith [15], obtained apparent conductivities of beds of spherical glass beads having uniform diameters less than 0.033 cm. These data are listed at normal air pressure levels ranging from 10^-6 to 760 mm Hg. All tests were conducted at an average bed temperature of 100 °C and were carried out essentially at extension of the Kim and Smith model. Their results indicate an attempt to include empirically the effect of contact.

Analysis

The analysis, both decoupled and coupled, will be based upon the geometry of Fig. 1 which shows two spherical particles in a void. The particles are insulated, and the particles are surrounded by a gaseous fluid. Mass transfer can occur between the spheres, and between the spheres and the fluid. The following assumptions are made: a) the spheres are identical in size, b) the spheres are smooth and clean, c) the contact area is circular and even, d) the spheres are insulated, e) there is no mass transfer between the spheres, and f) the spheres are in a vacuum.

The following calculations are made: a) the glass spheres are identical in size, b) the spheres are smooth and clean, c) the contact area is circular and even, d) the contact area is circular and even, e) the spheres are insulated, and f) there is no mass transfer between the particles. The analysis can be used to predict the contact radius as a function of sphere diameter, solidus, specific modulus and Poisson's ratio, 5) the simplest solid packing model will be assumed, and 6) the effects of different effective temperature models for microspheres at temperatures below 500 °C, and the total heat flow
rate $Q_g$ can be separated into two streams, $Q_{c}$ crossing via the contact and $Q_g$ crossing via the gas gap. For most porous substances $Q_c$ is equal to or greater than $Q_g$ and the two streams can be separated within the solid phase as shown in Fig. 2.

### Elastic Contact

![Diagram](image)

**Fig. 2 Solid and gas heat flow streams**

The elastic contact between identical spheres is given by [13]

$$D = R^3 \left[ \sqrt[3]{1 + \frac{1}{\pi \left[ 1 + \frac{1}{2} \left( 1 - \frac{1}{D} \right)^{1/3} \right]} - 1} \right]$$

where $R$ is the contact radius, $D$ is the sphere diameter, $f$ is the contact force normal to the contact plane, $v$ is Poisson’s ratio and $z$ is the module of elasticity.

The air gap thickness $\delta$ at any point outside the contact can be evaluated by means of

$$\delta = \frac{D}{4} \left[ 1 - \frac{1}{2} \left( 1 - \frac{1}{D} \right)^{1/3} \right]$$

which is based on elasticity theory, or by means of a simpler geometric expression:

$$\delta = \frac{D^2}{4} \left[ 1 - \frac{1}{\sqrt{2}} \right]$$

where $x = r/s$ is the dimensionless radial position relative to the contact radius. The geometric expression will be used in this analysis because it is easier to integrate, and it has been shown [12] that there is little difference in the final result between the two expressions of $\delta$.

### Constriction Resistance

If we assumed that $Q_c/Q_g > 1$, then the dividing flow line will be near the surface of the spherical particles and the resistance to heat flow within the solid phase is essentially the constriction resistance of a small circular contact area attached to a relatively large sphere. The total constriction resistance including both sides of a contact can be written as

$$G = \frac{L}{R_c}$$

where $L$ is the contact radius, $R$ is the solid conductivity and $G$ is the thermal constriction factor.

The constriction factor is complex depending upon the geometry of the contact areas, the geometry of the particle, and the boundary conditions. If the contact radius is very small relative to the diameter of the spherical particle, then the dependence of $G$ upon the particle geometry becomes less important, and $G$ is essentially a function of the boundary conditions alone. For constant temperature $T = T_0$ and for constant heat flux $q = 1.0$. Whichever the relative size of the contact area becomes large $(L/D > 0.01)$, or the ratio $Q_c/Q_g$ approaches unity, then $\eta = 1$ or 1.08 will not be correct. As an approximation we will use [15]

$$q = 1 - \frac{1}{(0.01)^2}$$

as the correction factor for $L/D$ large and $Q_c/Q_g$ near unity.

Equations (1), (3), (4) and (5) will be used in the decoupled and coupled models to follow.

### Decoupled Model

This model will be based upon the following assumptions: 1) $Q_c$ and $Q_g$ are independent of each other, 2) the boundary $\delta$ of the gas region, except very near the contact, is essentially isothermal, 3) the heat flow within the gas region is all parallel being perpendicular to the contact plane, 4) the thermal conductivity of the gas is uniform, and 5) the thermal resistance of an elemental volume of the gas, Fig. 3a, is given by

$$\delta = \frac{D}{4} \left( 1 - \frac{1}{D} \right)^{1/3}$$

where $L$ is the thermal conductivity of the gas under ambient conditions. The total conduction resistance, $G_c$, of the gas, after substitution of Eq. (3), is therefore

$$G_c = \int_0^L \frac{1}{k} \, dx$$

The lower limit of Eq. (7) cannot be unity because the integral is singular at this value of $x$, and also physically one cannot include the gas region adjacent...
to the contact in evaluating Eq. (7) because here the Knudsen number approaches a very large value, indicating that the gas no longer behaves like a continuum. A lower limit different from unity must be used. Hence, for \( \xi > 1 \), Eq. (7) becomes

\[
\frac{k_v}{k} = 1 - \frac{2}{\xi^2} \ln \left( \frac{\xi^2 - 1}{\xi^2 + 1} \right)
\]

(8)
The right-hand side of Eq. (8) will be defined as \( \theta_v(\xi) \), which is a dimensionless parameter depending upon the dimensionless contact and the lower limit \( \xi \). It has been shown empirically [12, 13] that for \( \xi \) small, \( \xi < 2 \) adequately correlates the resistance for two different sized sphere experiments.

The total resistance \( k_v \) of the typical element consists of the conduction and gas resistances thermally connected in parallel, therefore:

\[
\frac{1}{k_v} = \frac{1}{k} + \frac{1}{k_v}
\]

(9)

neglecting any radiative effects. If we now consider the typical element of cross-section \( S \), length \( L \), and apparent conductivity \( k_p \), its thermal resistance is given by

\[
k_p = \frac{1}{S/k_v}
\]

(10)

Equating Eqs. (9) and (10), solving for \( k_v \), and dividing by the bulk thermal conductivity of the sphere yields the following dimensionless expression:

\[
k_v = \frac{S}{k_p} = \frac{S}{k} + \frac{S}{k_v}
\]

(11)

Upon substitution of Eqs. (4), (5) and (8) into Eq. (11) we get

\[
k_v = \frac{S}{k_p} = \frac{S}{k} + \frac{S}{k_v}
\]

(12)

where \( k_v \) is the dimensionless apparent conductivity and \( k_v \) is the dimensionless gas conductivity. Eq. (12) is valid provided the gas behaves like a continuum.

To model the effect of reduced gas pressures, an effective gas gap thickness is defined as that volume of gas having a heat flow area \( A = \pi d^2 \), where \( d \) is the effective thickness \( \delta \), and whose total resistance is equal to the gas resistance given by Eq. (8). This leads directly to an expression for the dimensionless effective gas thickness:

\[
\delta^* = \frac{C_v}{C_v^*}
\]

(13)

where \( \delta^* = \delta / d \). It should be noted that \( \delta^* \) is independent of the thermal conductivities of the metal and gas, but does depend upon the relative contact size. The effect of gas pressure below the atmosphere will be taken into account by utilizing an effective gas thermal conductivity at any pressure \( P_g \) and any temperature \( T \) which is related to the gas conductivity under atmospheric conditions in the following manner [14]:

\[
C_v^* = C_v + \frac{1}{1 + 1.64 C_v^*}
\]

(14)

where \( \alpha \) is the accommodation parameter given by

\[
\alpha = \frac{(2 - 2.5C_v)}{1.5C_v + 0.5}
\]

(15)

in the dimensionless gas free path of the gas, \( \tau = 1/288 \) is the dimensionless average absolute temperature of the gas, and \( \tau = 1/760 \) is the dimensionless gas pressure. Eq. (15) is strictly valid for a diatomic gas layer bounded by parallel isothermal walls. It is assumed that Eq. (14) with \( \alpha \) (13) can be used to predict with reasonable accuracy the low gas pressure effects within the gas region shown in Fig. 3a.

For accommodation coefficients equal to unity, Eq. (14) with \( \alpha = 0 \) becomes

\[
C_v^* = \frac{C_v}{1 + 1.64 C_v^*}
\]

(16)

Copied Note: This model will be based upon the following assumptions: 1) \( \theta_v \) is the primary heat flow and is independent of \( \xi \); 2) \( \theta_v \) is the secondary heat flow and is dependent upon the temperature field set up by \( \theta_v \); 3) the temperature distribution over the boundary of the gas region can be determined by considering \( \theta_v \) and the conduction resistance between the contact and some arbitrary point.
within the sphere, 4. the ratio $Q_0 / Q_1$ is at least equal to unity, but is normally greater than unity, so that $Q_0$ flows through 700 or more of the cross-section of a sphere, 5. the thermal conductivity of the gas is uniform, and 6) the heat flow rate through an elemental volume of the gas, Fig. 1b, is given by the Fourier equation

$$
\dot{Q} = \frac{\alpha}{\delta} \left( T_0 - T_1 \right)
$$

(16)

Toschilich [14, 17] has shown that the local temperature drop can be related to the heat flow rate through the contact, the contact radius and the thermal conductivity of the solid in the following manner:

$$\frac{\alpha}{\delta} = \left( C_{\text{focal}} \right) \tan^{-1} \left( \frac{C_{\text{focal}}}{C_{\text{focal}}} \right)
$$

(17)

In the coupled model, it will be assumed that

Eq. (17) is a good approximation of the local temperature drop across the elemental volume of gas.

Upon substitution of Eqs. (6) and (17) into Eq. (16) and integrating over the effective gas region, one gets the following dimensionless expression:

$$\frac{C_{\text{focal}}}{C_{\text{focal}}} = \int_{a}^{b} g \left( \delta \right) \left( 1 - \delta \right) \left( 1 - \delta \right) \left( 1 - \delta \right) \left( 1 - \delta \right) \left( 1 - \delta \right)
$$

(18)

The lower limit is the integral cannot be unity.

It has been shown [12, 13] that a lower limit of about 2.5 adequately describes the heat flow through the gas, and this value will be used in this analysis.

The total heat flow rate $Q_0$ between adjacent spheres in the sum of the flows through the contact and the gas, therefore

$$Q_0 = Q_0 + Q_0 = Q_0 \left( 1 + \frac{C_{\text{focal}}}{C_{\text{focal}}} \right)
$$

(19)

where $Q_0$ is the integral of Eq. (12).

The total resistance is by definition the effective total temperature drop divided by the total heat flow rate. The effective total temperature drop is assumed to be the temperature drop associated with $Q_0$ and $Q_0$ without gas present.

Therefore

$$R_{\text{eff}} = \frac{1}{Q_0}
$$

(20)

The total resistance is also given by Eq. (10). Therefore equating Eqs. (10) and (20), solving for $R$, and rearranging leads to

$$R_{\text{eff}} = \frac{1 + \frac{C_{\text{focal}}}{C_{\text{focal}}}}{Q_0}
$$

(21)

as the expression for the dimensionless apparent conductivity.

In order to utilize Eq. (21) at reduced gas pressures, an effective gas gap thickness is required. Retarding the logic presented under the decoupled model, it can be shown that for the coupled model the dimensionless effective gas gap thickness can be determined by means of

$$\frac{\alpha}{\delta} = \frac{C_{\text{focal}}}{C_{\text{focal}}} = \frac{1 + \frac{C_{\text{focal}}}{C_{\text{focal}}}}{Q_0}
$$

(22)

using Eq. (16) for the effective gas conductivity at reduced gas pressures. Eq. (22), upon substitution of $a = \frac{C_{\text{focal}}}{C_{\text{focal}}}$ for accommodation coefficients of unity, becomes

$$\frac{\alpha}{\delta} = \frac{1 + \frac{C_{\text{focal}}}{C_{\text{focal}}}}{Q_0}
$$

(23)

**DISCUSSION AND COMPARISON OF MODELS**

Before comparing the results of the decoupled and coupled models with available test data the two models will be examined. It can be seen that both apparent conductivity expressions: Eq. (15), for the decoupled model, and Eq. (22), for the coupled model, yield the same value $R_{\text{eff}} = 1$ for vacuum conditions. If the contact force $F$ were known, Eq. (2) would yield $R$, Eq. (5) would be used to evaluate $a$ and the reciprocals of the product of $a$ and $R$, would yield the contribution to the apparent conductivity due to the contacts. The effective dimensionless gas gap thickness $a$ can be determined by means of Eq. (13) and Eq. (23) for the decoupled and coupled models, respectively, after having determined $R$ and $F$. Finally, either Eq. (5) for the decoupled model or Eq. (22) for the coupled model, can be used to determine the apparent conductivity at gas pressures from vacuum conditions to one atmosphere.

Since the contact force is not known for a particular system, the vacuum test conditions will be used to measure the parameter $F$. Both models under vacuum conditions lead to the same relationship between $L$ and $R_{\text{eff}}$.

$$\frac{R_{\text{eff}}}{L} = \frac{1}{L} = \frac{1}{L} \frac{C_{\text{focal}}}{C_{\text{focal}}}
$$

(24)

where $C_{\text{focal}}$ is the dimensionless apparent conductivity obtained from vacuum tests.

The validity of the two models will be compared with the test data of Dumes [8], and Macconnell and Smith [11]. Dumes obtained apparent conductivity data for clean glass microspheres- when dirty microspheres were used, the apparent conductivity data were the average of 300 to 500 tests.

The tests were conducted in a cylindrical system filled with clean water. The tests were conducted in a cylindrical system filled with clean water. The tests were conducted in a cylindrical system filled with clean water.
For convenience only, the two models, Eq. (15) and Eq. (23) have been rewritten in the following forms:

Decoupled Model

\[ K = \frac{K_0}{1 + \frac{K}{K_1}} \tag{25} \]

Coupled Model

\[ K = \frac{K_0}{1 + \frac{K}{K_1}} \tag{26} \]

where \( K \) is the gas pressure in mm Hg. The parameters \( K_0, K_2, K_3 \) and \( K_4 \) have been evaluated for a series of phase bands at the test conditions reported (4, 11). These parameters are presented for both sets of data in Table 1, 1, 3 and 4. The effective dimensionless gas gap thickness is also presented in Tables 1, 2, 3 and 4 for both sets of data. Tables 1 and 2 correspond to the decoupled model, while Tables 3 and 4 correspond to the coupled model. It should be noted that both models yield similar values of \( K \) for the two sets of data, namely \( K = 0.120 \) for the decoupled model and \( K = 0.130 \) for the coupled model. Tables 5 and 6 contain the data of Dumas (8), the predicted values of \( K \), the predicted values of \( K_2 \), and the predicted values of \( K_3 \). The agreement between the available test data (8, 11) and the two models, Eq. (25) and (26), can be seen in the last two columns. It is clear that the coupled model is superior to the decoupled model at gas pressures in excess of 5 mm Hg for both sets of data. The agreement between Eq. (26) which is based upon Eq. (25) and the coupled model, and the Dumas data (8) is excellent at low gas pressures (< 0.1 mm Hg) and reasonably good at high gas pressures (> 200 mm Hg). The agreement between Eq. (26) and the other test data (11) is excellent for all sphere diameters at low gas pressures (< 5.5 mm Hg) and high gas pressures (> 200 mm Hg). The largest difference (about 1%) between theory and test data (11) occurs at different gas pressures depending upon the sphere diameter: 50 mm Hg for D = 20 µ, 30 mm Hg for D = 40 µ, 10 mm Hg for D = 60 µ and 3 mm Hg for D = 470 µ. The largest difference (about 12%) between Eq. (26) and test (8) occurs at a gas pressure of 1 mm Hg consistent with their observed with the other test data (11).

Compared with Rumit and Smith model (11)

this analysis based upon contraction resistance due to heat flow through the contact area will be compared with the model of Rumit and Smith. In their model it was assumed that only mass contact existed between spheres with all the heat crossing the contact plane by conduction through the gas only. They also assumed that there was no bending of heat flow lines within the solid and gas phases. They modeled their typical element as two cylindrical solids of flow area 0.074, and thickness 20% separated by an effective gas gap thickness which can be calculated by means of the following expression:

\[ T = \frac{K_2}{K_0} \left( \frac{K_1}{K_2} \sin x \right) \]

\[ x = \tan^{-1} \left[ \frac{K_2}{K_1} \sin x \right] \]

where the parameter \( x \) depends upon the packing: for close packing \( x = 0.260 \), \( x = 22.3^\circ \) for loose packing \( x = 0.412 \), \( x = 92.9^\circ \). For an intermediate packing (0.160 0.450), they recommend a linear interpolation to determine the effective dimensionless gas gap thickness:

\[ x = x_1 + \left( x_2 - x_1 \right) \frac{0.160 - x}{0.410 - x} \]

Table 1 Parameters for decoupled model, Eq. (25), and Ref. [8].

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<tr>
<th>( K )</th>
<th>373</th>
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<td>( K_0 )</td>
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<td>( K_1 )</td>
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<td>( K_3 )</td>
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Table 2 Parameters for decoupled model, Eq. (25), and Ref. [11].

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Table 3 Parameters for coupled model, Eq. (26), and Ref. [8].

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Table 4 Parameters for coupled model, Eq. (26), and Ref. [11].

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</tr>
<tr>
<td>0.01</td>
<td>0.316</td>
<td>5.221</td>
<td>0.232</td>
<td>0.385</td>
</tr>
</tbody>
</table>

Table 5 Comparison of models and test data (8) at 373 K

<table>
<thead>
<tr>
<th>$P_a$</th>
<th>$V$</th>
<th>$V_0$</th>
<th>$V_{0(222)}$</th>
<th>$V_{0(222)}$</th>
<th>$V_1$</th>
<th>$V_1(221)$</th>
<th>$V_1(221)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.160</td>
<td>5.235</td>
<td>0.313</td>
<td>0.938</td>
<td>2.286</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.208</td>
<td>5.235</td>
<td>0.315</td>
<td>0.937</td>
<td>1.946</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.209</td>
<td>5.234</td>
<td>0.280</td>
<td>0.567</td>
<td>0.840</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.218</td>
<td>5.233</td>
<td>0.235</td>
<td>0.445</td>
<td>0.719</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.293</td>
<td>5.232</td>
<td>0.222</td>
<td>0.919</td>
<td>0.926</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.370</td>
<td>5.232</td>
<td>0.221</td>
<td>0.353</td>
<td>0.349</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6 Comparison of models and test data (8) at 473 K

<table>
<thead>
<tr>
<th>$P_a$</th>
<th>$V$</th>
<th>$V_0$</th>
<th>$V_{0(222)}$</th>
<th>$V_{0(222)}$</th>
<th>$V_1$</th>
<th>$V_1(221)$</th>
<th>$V_1(221)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.218</td>
<td>5.241</td>
<td>0.347</td>
<td>0.957</td>
<td>1.696</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.270</td>
<td>5.241</td>
<td>0.345</td>
<td>0.899</td>
<td>0.971</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.272</td>
<td>5.240</td>
<td>0.300</td>
<td>0.581</td>
<td>0.301</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.283</td>
<td>5.240</td>
<td>0.255</td>
<td>0.494</td>
<td>0.443</td>
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<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.366</td>
<td>5.240</td>
<td>0.236</td>
<td>0.348</td>
<td>0.275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.401</td>
<td>5.240</td>
<td>0.230</td>
<td>0.216</td>
<td>0.180</td>
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<td></td>
</tr>
</tbody>
</table>

Table 7 Comparison of models and test data (11) at $D = 150$

<table>
<thead>
<tr>
<th>$P_a$</th>
<th>$V$</th>
<th>$V_0$</th>
<th>$V_{0(222)}$</th>
<th>$V_{0(222)}$</th>
<th>$V_1$</th>
<th>$V_1(221)$</th>
<th>$V_1(221)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.232</td>
<td>5.719</td>
<td>0.259</td>
<td>0.914</td>
<td>1.030</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.254</td>
<td>5.717</td>
<td>0.267</td>
<td>0.928</td>
<td>0.988</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.257</td>
<td>5.716</td>
<td>0.223</td>
<td>0.402</td>
<td>0.329</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.263</td>
<td>5.716</td>
<td>0.177</td>
<td>0.357</td>
<td>0.347</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.315</td>
<td>5.716</td>
<td>0.127</td>
<td>0.408</td>
<td>0.473</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.383</td>
<td>5.716</td>
<td>0.125</td>
<td>0.364</td>
<td>0.378</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8 Comparison of models and test data (11) at $D = 80$

<table>
<thead>
<tr>
<th>$P_a$</th>
<th>$V$</th>
<th>$V_0$</th>
<th>$V_{0(222)}$</th>
<th>$V_{0(222)}$</th>
<th>$V_1$</th>
<th>$V_1(221)$</th>
<th>$V_1(221)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>0.231</td>
<td>5.707</td>
<td>0.248</td>
<td>0.939</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>0.254</td>
<td>5.707</td>
<td>0.258</td>
<td>0.955</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.256</td>
<td>5.707</td>
<td>0.214</td>
<td>0.423</td>
<td>0.405</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.262</td>
<td>5.707</td>
<td>0.167</td>
<td>0.382</td>
<td>0.367</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>0.314</td>
<td>5.707</td>
<td>0.107</td>
<td>0.417</td>
<td>0.494</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.382</td>
<td>5.707</td>
<td>0.106</td>
<td>0.374</td>
<td>0.397</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9 Comparison of models and test data (11) at $D = 200$


Otherwise they recommend that one put
\[ \delta^E = \delta^1 \]
for \( \theta < 0.256 \)
and
\[ \delta^E = \delta^2 \]
for \( \theta > 0.476 \)
where \( \delta^1 \) corresponds to close packed beds and \( \delta^2 \)
corresponds to loose packed beds.

For negligible radiation effects their model leads to the following expression for the dimensionless apparent conductivity:
\[ \delta^E = 1 + (1 - y) (\delta^1 - \delta^2) \quad (30) \]

It will be noted that the Hunt-Smith model depends upon the albedo conductivities and the packing arrangement, but is independent of sphere size and any loading parameter. This model cannot be used to predict low gas pressure effects on apparent conductivities under vacuum conditions.

The comparison between test data [8, 11] and the Hunt-Smith model (4) is summarized in Table 11. The comparison can only be made in atmospheric conditions because Hunt and Smith did not take into account the effect of contact areas. It can be seen in Table 11 that there is good agreement between theory and test at 573 K and moderate agreement at 573 K when the porosity is 0.33. The 10% difference between theory and test data [11] is also observed when the porosity is 0.38 and the temperature is 315 K for all sphere diameters.

The coupled model proposed in this study based upon the single cubic packing is superior to the Hunt and Smith model for the glass bead/air system at atmospheric pressure, and for reduced air pressures as well.

CONCLUDING REMARKS

A method has been proposed to predict the apparent conductivity of glass microspheres in terms of the system physical and thermal properties, the system geometry as well as the system temperature and pressure. It has been demonstrated that both the decoupled and coupled models agree quite well with available test data at low and high air pressures, and reasonably well at moderate gas pressures. It was shown that test data under vacuum conditions could be used to determine the effective radiation between the contacting spheres, as well as the contribution of the contact areas to the apparent conductivity of the system over the entire pressure range. For the glass bead/air system the model proposed here is superior to the Hunt-Smith model.

ACKNOWLEDGMENTS

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REFERENCES


