A MODEL FOR PREDICTING THE JOINT CONDUCTANCE OF A WOVEN WIRE SCREEN CONTACTING TWO SOLIDS

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Abstract

An analysis is presented for predicting the local contact conductance of a woven metallic wire screen contacting two smooth solids under vacuum conditions. Hertzian theory is utilized to predict the shape and size of contact areas during first loading. The general constriction resistance theory of Yovanovich is used to predict the total resistance of a typical element. A model is presented for determining the effect of out-of-flatness, upon the average joint conductance. There is reasonably good agreement between this model and the available screen test data.

Nomenclature

A = geometric parameter
a = semimajor axis of elliptic contact area
B = geometric parameter
b = semiminor axis of elliptic contact area
C' = constant for a joint
c = wire spacing
CF = out-of-flatness correction factor, Eq. (45)
D = wire diameter
d = out-of-flatness
E = modulus of elasticity
E^*_{12} = $(1 + E_2/E_1)^{1/3}$
E^*_{23} = $(1 + E_2/E_3)^{1/3}$
F = force between contacting solids
h_j = local joint conductance
$\bar{h}_j$ = average joint conductance
K = complete elliptic integral of the first kind
k = thermal conductivity
k^*_{12} = $2(k_1/k_2)/(1 + k_1/k_2)$
k^*_{23} = $2(k_3/k_2)/(1 + k_3/k_2)$
m = semimajor axis parameter
n = semiminor axis parameter
P = contact pressure
P^* = $P(1 - \nu^2)/E$
Q = heat transfer rate
R = thermal constriction resistance
R^* = kcR
ΔT = temperature difference

Greek Symbols

α = wire spacing parameter
β = parameter defined by Eq. (35)
γ = contour area radius
Δ = parameter defined by Eq. (10)
δ = cylinder radius
κ = defined by Eq. (17)
ν = Poisson's ratio
π = pi
ρ = radius of curvature
τ = defined by Eq. (11)
ψ = defined by Eq. (16)

Subscripts

1,2,3 = solid 1, wire and solid 3
12 = solid 1/wire contact
23 = wire/solid 3 contact
a = apparent
c = circular
e = elliptic
j = joint
t = total

Superscript

T = isothermal

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$$a = m \left[\frac{3}{4} F \Delta\right]^{1/3} \quad (8)$$

$$b = n \left[\frac{3}{4} F \Delta\right]^{1/3} \quad (9)$$

where a and b are the semimajor and semiminor axes, respectively. The dimensionless parameters m and n depend only upon the radii of curvature of the contacting solids, F is the contact force and Δ is a geometric-physical parameter defined as follows:[8]

$$\Delta_{12} = \left[\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right]/(A + B) \quad (10)$$

In Eq. (10) E_1 and E_2 are the modulii of elasticity and ν_1 and ν_2 are the Poisson's ratios of the contacting solids. The parameters m and n are tabulated and depend upon τ where[8]

$$\tau = \cos^{-1} \left[\frac{B - A}{A + B}\right] \quad (11)$$

When $\tau = \pi/2$, $m = n = 1$ and the contact area is circular. For $\tau < \pi/2$, $m > n$ and the contact area is elliptical. For the woven screen wires commercially available, the parameter α ranges from 2.222 to 7.936. In this range τ has a maximum value of 60° and a minimum value of 20°, while the corresponding values of m/n or a/b are 2.07 and 9.28, respectively.

Thus for wire screens we can write the following approximate expressions:[5]

$$m = 0.830\ \alpha^{0.735} \quad (12)$$

and

$$\frac{m}{n} = 0.7905\ \alpha^{1.18} \quad (13)$$

which are valid in the range $2 < \alpha < 8$. It should be noted that the force on a typical node can be expressed in terms of the apparent contact pressure, the spacing between wires and the wire diameter:

$$F = Pc^2 = P\alpha^2 D^2 \quad (14)$$

For typical contact pressures and wire screens, it can be shown that the load per node is quite small, resulting in contact areas whose characteristic dimensions are very small relative to the wire diameter. This fact will be used in the subsequent thermal analysis.

Thermal Problem

Since the semimajor axis of the elliptic contact area is much smaller than the wire diameter, it will be assumed that the thermal constriction resistance within the wire due to the elliptic contact area can be approximated as the thermal constriction resistance within a half-space due to an elliptic contact area.

It has been shown by Yovanovich[7] that the general expression for the thermal constriction resistance due to an isothermal elliptic contact area supplying heat to a half-space can be determined by means of the following expression:

$$R = \psi_e^T / 4ka \quad (15)$$

where k is the thermal conductivity, a is the semimajor axis of the elliptic contact area, and ψ_e^T is a geometric constriction parameter defined as

$$\psi_e^T = \frac{2}{\pi} K(\kappa) \quad (16)$$

In Eq. (16), K is the complete elliptic integral of the first kind whose argument κ is dependent upon the ratio of the semimajor and semiminor axes in the following manner:

$$\kappa = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{n^2}{m^2}} \quad (17)$$

The constriction parameter, Eq. (16), has been tabulated by Yovanovich[8] for values of a/b running from 1 to 10.

An approximate expression for Eq. (16) valid for large values of m/n (or a/b) can be shown to be

$$\psi_e^T = \frac{2}{\pi} \ln\left(4\frac{m}{n}\right) \quad (18)$$

For $m/n > 3$, the difference between Eqs. (16) and (18) is less than 1.7%.

When $m = n$, $\psi_e^T = 1$ and Eq. (15) reduces to the well-known solution for an isothermal circular contact area of radius a.

The thermal constriction resistance within the flat whose thermal and mechanical properties are designated by the subscript 1 in contact with a wire designated by the subscript 2 can now be written as

$$R_{e1} = \frac{\psi_{e1}^T}{4k_1 m \left[\frac{3}{4} P\alpha^2 D^2 \Delta_{12}\right]^{1/3}} \quad (19)$$

Since we are assuming the constriction resistance within the wire due to the elliptic contact is equivalent to the constriction resistance within a half-space, we can write for this resistance:

$$R_{e2} = \frac{\psi_{e2}^T}{4k_2 m \left[\frac{3}{4} P\alpha^2 D^2 \Delta_{12}\right]^{1/3}} \quad (20)$$

Since ψ_e^T depends only upon the ratio m/n, we can put $\psi_{e2}^T = \psi_{e1}^T$. The total constriction resistance due to one elliptic contact area is the sum of Eqs. (19) and (20):

$$R_{e12} = \frac{\frac{2}{\pi} K(\kappa)\ [A + B]^{1/3}}{2k_{12}\ m\left[\frac{3}{4} P\alpha^2 D^2 \left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right)\right]^{1/3}} \quad (21)$$

where k_{12} is the harmonic mean thermal conductivity of the contact and is defined as

$$k_{12} = 2k_1 k_2/(k_1 + k_2) \quad (22)$$

Similar arguments will allow us to write directly the total constriction resistance due to the second elliptic contact area formed between the wire and a second flat whose thermal and physical properties are designated by the subscript 3. Thus the total resistance is

The circuits shown in Fig. (4) can be replaced by those shown in Fig. (5). An examination of these circuits leads to the conclusion that the total electrical resistance of the typical element can be expressed as

$$R_t = \frac{R_1 + R_3}{4} \qquad (24)$$

independent of the wire/wire contact. Replacing the electrical resistances R_1 and R_3 by the equivalent thermal constriction resistances we can show that the total constriction resistance of a typical element ABCD is simply the sum of Eqs. (21) and (23):

$$R_t = R_{e12} + R_{e23} \qquad (25)$$

Thermal Joint Conductance

Local Joint Conductance

It will be convenient for subsequent analyses to define a dimensionless total resistance of a typical element. Using the wire spacing c as a characteristic dimension of the joint and the thermal conductivity of the wire, it can be shown that

$$R_t^* = ck_2\, R_t \qquad (26)$$

where R_t, given by Eq. (25), can be expressed as

$$R_t^* = \frac{\frac{1}{\pi}\frac{K(\kappa)}{m}\left(\frac{4\alpha}{3}\right)^{1/3}\left[\frac{\alpha^2+3}{\alpha^2+1}\right]^{1/3}}{[P^*]^{1/3}}\left\{\frac{1}{k_{12}^*E_{12}^*} + \frac{1}{k_{23}^*E_{23}^*}\right\} \qquad (27)$$

where the following dimensionless parameters have been used:

$$P^* = P^*(1 - \nu_2^2)/E_2 \qquad (28)$$

$$k_{12}^* = 2(k_1/k_2)/[1 + k_1/k_2] \qquad (29)$$

$$k_{23}^* = 2(k_3/k_2)/[1 + k_3/k_2] \qquad (30)$$

$$E_{12}^* = [1 + E_2/E_1]^{1/3} \qquad (31)$$

$$E_{23}^* = [1 + E_2/E_3]^{1/3} \qquad (32)$$

It has been assumed that the ratios $(1 - \nu_3^2)/(1 - \nu_2^2)$ and $(1 - \nu_1^2)/(1 - \nu_2^2)$ can be replaced by unity with negligible error in the determination of the dimensionless total resistance.

The local joint conductance can be determined from the following relationship:

$$Q = h_j\, c^2\, \Delta T = \Delta T/R_t \qquad (33)$$

Eliminating ΔT from this relationship and defining a dimensionless joint conductance as $h_j c/k_2$ we note that this is equal to the reciprocal of the dimensionless total resistance.

Combining the dimensionless pressure P^* with the dimensionless joint conductance we can obtain the following expression

$$\left[\frac{P(1 - \nu_2^2)}{E_2}\right]^{1/3} \frac{k_2}{ch_j} = \beta \qquad (34)$$

where the geometric-physical-thermal parameter β is defined as

$$\beta = \left(\frac{4\alpha}{3}\right)^{1/3} \frac{K(\kappa)}{m\pi} \left[\frac{\alpha^2+3}{\alpha^2+1}\right]^{1/3} \left\{\frac{1}{k_{12}^*E_{12}^*} + \frac{1}{k_{23}^*E_{23}^*}\right\} \qquad (35)$$

For a joint consisting of identical materials, $E_1 = E_3 = E_2$, $k_1 = k_3 = k_2$, the parameter β reduces to

$$\beta = \left[\frac{16\alpha}{3}\right]^{1/3} \frac{K(\kappa)}{m\pi} \left[\frac{\alpha^2+3}{\alpha^2+1}\right]^{1/3} \qquad (36)$$

We note from Eq. (34) that the local joint conductance varies as $P^{1/3}$, everything else being independent of the load.

Overall Joint Conductance

If the local contact pressure is not constant over the apparent contact area, the local joint conductance will also vary over this area. It is convenient for such joints, to define an overall or average joint conductance. This can be done by integrating the product of the local conductance and the local apparent contact area over the entire contact area, then dividing by the total contact area:

$$h_j = \frac{1}{A_a}\iint_{A_a} h_j\, dA_a \qquad (37)$$

where h_j is given by Eqs. (34) and (35). It is obvious that whenever the contact pressure distribution is uniform, the overall joint conductance is equal to the local joint conductance.

Out-of-Flatness Correction for the Overall Conductance

To illustrate the effect of pressure distribution, consider the elastic contact of two right-circular cylinders having out-of-flatness d_1 and d_2 as shown in Fig. 6. The radii of curvature of the contacting surfaces can be expressed in terms of the out-of-flatness and the cylinder radius δ approximately as

$$\rho_1 = \frac{\delta^2}{2d_1} \quad \text{and} \quad \rho_2 = \frac{\delta^2}{2d_2} \qquad (38)$$

If the cylinders make elastic contact over a circular area $\pi\gamma^2$ where $\gamma < \delta$, the local pressure distribution will be a maximum in the center of the contact and zero everywhere outside the contact. The local pressure can be expressed as

$$P = \frac{1.5F}{\pi\gamma^2}\left[1 - \frac{r^2}{\gamma^2}\right]^{1/2}, \quad 0 \le r \le \gamma$$
$$P = 0, \qquad \gamma \le r \le \delta \qquad (39)$$

where F is the total load on the contact. The apparent contact pressure is $F/\pi\delta^2$.

If now we place a screen between these contacting cylinders, and assume the pressure distriterval, steady state conditions were assumed to prevail. The length of time required for steady state conditions ranged from 6 to 12 hours.

ta Reduction

The experimental thermal joint conductance values were determined from

$$h_j = \frac{Q/A}{T_1 - T_2} = \frac{Q/A}{\Delta T}$$

where T_1 and T_2 are the temperatures of the bounding surfaces of the joint, and Q/A is the heat flow per unit area. A linear least squares fit of the test specimen axial centerline temperatures was made for each set of temperature data in order to determine the interface temperatures. These interface temperatures were also checked by graphical extrapolation of the centerline temperature-axial location curves. The heat flux was then calculated from the axial temperature gradient of the test specimen temperatures and the thermal conductivity at the mean temperature of the test specimen. The heat flux value was then checked by subtracting the estimated heat losses from the measured heat input.

Results and Discussion

Tables I - III contain the screen wire characteristics, the surface characteristics of the contacting specimens as well as the predicted and experimental values of the overall joint conductance. The geometric and thermal characteristics of the screen wires used in the tests varied greatly. For example, the thermal conductivity ran from 10 to 220 Btu/hr.sq.ft.°F; α ran from 2.22 to 5.35, and the corresponding diameters ran from 0.0045 to 0.025 inches. The contacting specimens were limited to aluminum and stainless steel. Their out-of-flatness (OF) was determined to lie in a narrow range of 18 to 25 μin.

The test results given in Table III show the overall joint conductance varying over a very wide range, running from a low of 7.2 Btu/hr-sq ft to a high of 484 Btu/hr-sq ft. The lowest value was obtained with a 10 mesh stainless steel screen contacting stainless steel specimens at an apparent contact pressure of 103 psi at a mean joint temperature of 218°F. The highest value was obtained with a 30 mesh copper screen contacting aluminum specimens at a pressure of 233 psi at a temperature of 110°F. The last five columns of Table III contain the predicted and measured values of the overall joint conductance, and the corresponding percent difference based upon the predicted values. The values in the first column under h_j (theory) are based upon a thermal constriction parameter which is valid for the special case of isothermal elliptic contact areas, and a correction factor (CF) given by Eq. (45). The OF values used to determine the CF values are presented in Table II. The CF values ranged from 0.33 to 0.88. The corresponding column under percent difference is designated by a superscript (a). It can be seen that the predicted values are greater than those measured for all test conditions. The greatest difference was observed with the T3 test results and the least difference was observed with the T6 test results.

Several factors can influence the predicted values. It is our belief that the two most important factors are the constriction parameter and the OF. The constriction parameter enters directly into the thermal resistance and, therefore, can have a significant effect upon the predicted values of the conductance. It is a fact that the constriction parameter depends upon the boundary condition over the contact area. A constant flux or other related boundary condition will require a constriction parameter greater than the isothermal constriction parameter used here by as much as 12%.

It is also possible that the actual OF is up to 50% larger than the reported values in Table II.

If we use a constriction parameter 12% larger than the value given by Eq. (16) and an OF 50% larger than those used previously, the predicted values of the conductance are reduced by about 25%. These new values and the corresponding percent difference are reported in Table III.

It should also be noted that the CF is a function of the apparent contact pressure to the 4/9 power. From Eq. (34) we see that h_j is a function of the contact pressure to the 1/3 power. If these pressure effects are lumped together, Eq. (44), we conclude that the overall joint conductance should vary as apparent pressure to the 7/9 power. This is in good agreement with the conclusion of reference 4 that the joint conductance depends upon the contact pressure to the 0.72 power.

Conclusions

A model has been developed for predicting the overall joint conductance of screen wires placed between smooth solids under vacuum conditions. The model is based upon elasticity theory and the available thermal constriction resistance theory. A dimensionless joint conductance number, Eq. (34), has been obtained from a thermal analysis of a typical element. This dimensionless joint conductance shows clearly how the geometric, thermal, and physical characteristics as well as the contact pressure influence the local joint conductance. A simple model is also presented for taking into account the effect of out-of-flatness. There is qualitative agreement with the available test data.

It is recommended that further work be done to determine the constriction parameter for a hard (low conductivity), curved surface penetrating a soft (high conductivity), flat surface. This would correspond to a stainless steel screen contacting an aluminum surface.

Further work should be done to determine the effect of a screen placed between contacting smooth curved solids upon the contact area and the pressure distribution.

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Table III Comparison between theory and test results

Test	α	Tm	k_2	E_2	ν_2	$k_1 = k_3$	$E_1 = E_3$	Pa	h_j (Theory)		h_j (Test)	% Diff.	
		^oF	$Btu/Hr\text{-}Ft\text{-}^oF$	10^6 psi		$Btu/Hr\text{-}Ft\text{-}^oF$	10^6 psi	psi	$Btu/Hr\text{-}Ft^2$		$Btu/Hr\text{-}Ft^2$		
T11	3.13	218	91.0	56.0	0.28	108	10.1	99	185.2 (a)	138.8 (b)	118	-36.3 (a)	-15.0 (b)
T12	3.13	229	91.0	56.0	0.28	109	10.1	186	303.7	227.8	152	-50.0	-33.3
T13	3.13	246	91.0	56.0	0.28	109	10.1	278	415.1	311.4	186	-55.2	-40.3
T21	4.00	195	11.6	17.0	0.30	108	10.3	102	22.2	16.7	10.7	-51.8	-35.8
T22	4.00	192	11.6	17.0	0.30	108	10.3	186	35.4	26.6	16.3	-54.0	-38.6
T23	4.00	189	11.6	17.0	0.30	108	10.3	275	48.0	36.0	21.9	-54.4	-39.2
T31	2.22	216	10.0	27.5	0.28	108	10.1	96.0	172.5	129.3	64.6	-62.6	-50.0
T32	2.22	220	10.0	27.5	0.28	108	10.1	187.0	289.7	217.4	90.1	-68.9	-58.6
T33	2.22	229	10.0	27.5	0.28	108	10.1	277.0	393.3	294.8	118.0	-70.0	-60.0
T41	4.00	192	9.3	27.5	0.28	108	10.3	132.0	21.8	16.4	17.3	-20.6	+ 5.76
T42	4.00	190	9.3	27.5	0.28	108	10.3	192.0	29.2	21.9	19.5	-33.2	-10.9
T43	4.00	191	9.3	27.5	0.28	108	10.3	279.0	39.0	29.3	23.9	-38.7	-18.3
T51	2.22	249	10.0	27.4	0.28	10.0	27.4	104.0	53.7	40.2	32.1	-40.2	-20.2
T52	2.22	293	10.0	27.1	0.28	10.1	27.1	105.0	55.1	41.4	32.8	-40.8	-20.9
T53	2.22	236	10.0	27.6	0.28	10.0	27.6	100.0	51.8	38.8	32.8	-36.7	-15.4
T54	2.22	254	10.0	27.4	0.28	10.0	27.4	304.0	123.7	92.7	64.1	-48.2	-30.8
T55	2.22	-35	8.0	29.1	0.28	8.0	29.1	100.0	39.8	29.9	21.9	-44.9	-26.6
T56	2.22	-17	8.5	29.0	0.28	8.1	29.0	304.0	98.1	73.6	46.2	-52.9	-37.3
T61	4.0	218	10.0	27.6	0.28	10.0	27.6	103.0	7.32	5.49	7.2	- 1.63	+31.1
T62	4.0	219	10.0	27.6	0.28	10.0	27.6	307.0	17.2	12.9	12.8	-25.6	- 0.78
T71	3.33	104	224	17.3	0.33	104.5	10.8	120.0	519.3	389.2	305.0	-41.3	-21.6
T72	3.33	102	224	17.3	0.33	104.0	10.8	216.0	817.6	613.3	462.0	-43.4	-24.7
T73	3.33	110	224	17.3	0.33	105.0	10.8	233.0	872.9	654.7	484.0	-44.6	-26.1
T81	5.35	108	105	10.8	0.33	105.0	10.8	128.0	250.2	187.6	159.0	-36.4	-15.3
T82	5.35	108	105	10.8	0.33	105.0	10.8	312.0	500.3	375.4	283.0	-43.4	-24.6
T83	5.35	148	106	10.5	0.33	106.0	10.5	128.0	258.2	193.5	161.0	-37.6	-16.8
T84	5.35	145	106	10.5	0.33	106.0	10.5	312.0	516.2	387.4	302.0	-41.5	-22.0

(a) Theory based upon Eqs. (16) and (45).

(b) Theory based upon $\psi_e = 1.12 \ \psi_e^T$ and out-of-flatness 50% greater than Table II.