Thermal Contact Correlations

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Abstract

The theoretical relationships for determining contact, gap, and joint conductances are developed for conforming rough surfaces for first loading. The dimensionless conductances are functions of the relative contact pressure, surface parameters, conductivity ratio, and a fluid parameter which depends upon several gas and surface characteristics. The proposed conductance correlations are supported quantitatively by some previous experimental results.

Nomenclature

A_a, A_r = apparent and real contact areas
a, a_i = mean and distributed contact spot radius
b, b_i = mean and distributed flux tube radius
C = dimensionless contact conductance, $\sigma h/k_s$
C_c, C_g, C_j = contact, gap, and joint dimensionless conductances
H = hardness
h_c, h_g, h_j = contact, gap, and joint conductances
K = conductivity ratio, k_{go}/k_s
k_1, k_2 = conductivity of the contacting solids
k_g, k_{go}, k_s = gap, gas, and harmonic mean conductivities

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M = gas parameter, $(\alpha\beta\Lambda)/\sigma$
m = mean absolute surface slope
n = contact spot density
P, P_g, P_{go} = relative contact pressure, gas pressure, and reference gas pressure
Pr = Prandtl number, $\mu C_p/k_g$
Q = heat flow rate
R_c, R_{ci} = contact and contact spot constriction resistance
T, T_o = gap and reference temperature
ΔT_c = contact temperature drop
x = dimensionless parameter, $Y/\sqrt{2}\sigma$
Y, Y_e = separation and effective separation between the surfaces
α, α_1, α_2 = accommodation parameter and accommodation coefficients for solids 1 and 2
β = gas parameter, $(2\gamma/\gamma+1)/Pr$
γ = specific heat ratio, C_p/C_v
Λ, Λ_o = molecular mean free path
σ, σ_1, σ_2 = effective surface roughness and surface roughnesses of solids 1 and 2
ψ_c, ψ_{ci} = mean and contact spot constriction parameters

Introduction

The problem of predicting and measuring contact, gap, and joint thermal conductances has received considerable attention during the past two decades because of the importance of the topic in many heat-transfer systems. Comprehensive surveys of literature on this subject can be found in several references.[1-8] Significant progress has been made in our understanding and ability to predict thermal contact conductance.[9-17] The study of thermal contact conductance in a vacuum is fundamental to our understanding of thermal gap conductance when interstitial fluids are present. The present state of knowledge has reached a point where simple, explicit correlations can be developed for the contact, gap, and joint conductances.

The purpose of this paper is to establish correlations for conforming rough surfaces when interstitial fluids such as greases and gases are present in the gap. The proposed correlations will be compared with existing empirical data to demonstrate quantitatively the validity of the assumptions used to develop the models.

Thermal Conductance Correlations: Geometric, Physical, and Thermal Assumptions

The contact and gap conductance correlations developed here are based upon the following assumptions:

1) The surfaces are microscopically rough and macroscopically conforming.

2) The asperity heights are Gaussian and the asperities are randomly distributed over the apparent contact area.

3) The contacting asperities deform plastically during the first loading.

4) The plastic deformation occurs in the softer solid and the flow pressure is constant and approximately equal to three times the elastic limit under tensile stress.

5) During the plastic deformation of the highest peaks, the substrate and the bulk of the solids undergo elastic deformation.

6) As a result of the deformation, there are N circular contact spots within the apparent area A_a.

7) The contact spot radii a_i are distributed.

8) An equivalent circular flux tube of radius b_i is associated with each contact spot.

9) The contact spots are isothermal.

10) The total heat flow rate through each flux tube can be separated into two independent heat flow rates: contact spot and gap flow rates.

11) The effective gap thickness is dependent upon the surface roughness and the relative contact pressure.

12) Noncontinuum gas effects must be taken into account.

13) The surfaces are clean free of oxides, films, etc.

14) Radiative heat transfer is negligible.

Contact Conductance Correlation

The total constriction resistance of the ith contact spot is

$$R_{ci} = \frac{\psi_{ci_1}}{4k_1 a_i} + \frac{\psi_{ci_2}}{4k_2 a_i} \quad (1)$$

where ψ_{ci_1} and ψ_{ci_2} are the thermal constriction (or spreading) parameters which depend upon the relative size of the contact spot. Because of geometric and thermal symmetry about the contact plane we can put

$$\psi_{ci_1} = \psi_{ci_2} = \psi_{ci} = [1 - (a_i/b_i)]^{1.5} \quad (2)$$

provided $0 < a_i/b_i \leq 0.3$.

If we let $k_s = 2k_1k_2/(k_1 + k_2)$, the harmonic mean thermal conductivity, then Eq. (1) can be written as

$$R_{ci} = \psi_{ci}/2k_s a_i \quad (3)$$

The total contact resistance of N contact spots thermally connected in parallel is therefore,

$$\frac{1}{R_c} = \sum_{i=1}^{N} \frac{1}{R_{ci}} = 2k_s \sum_{i=1}^{N} \frac{a_i}{\psi_{ci}} \quad (4)$$

The contact conductance can be derived by means of the following definition:

$$Q_c = h_c A_a \, \Delta T_c = \Delta T_c / R_c \quad (5)$$

Therefore,

$$h_c = \frac{2k_s}{A_a} \sum_{i=1}^{N} \frac{a_i}{\psi_{ci}} \quad (6)$$

Noting that $a_i/b_i \leq 0.3$ and $0.85 \leq \psi_{ci} < 1$, the specific constriction parameter ψ_{ci} appropriate to each contact spot can be replaced by the mean value of the constriction parameter ψ_c based upon the total set of contact spots.

Therefore we have

$$\psi_{ci} \simeq \psi_c = (1 - \varepsilon)^{1.5} \quad (7)$$

where $a/b = \varepsilon = \sqrt{A_r/A_a}$. The mean constriction parameter depends upon a and b, the contact spot and associated flux tube radii, respectively, determined by the total real and apparent areas.

A detailed geometric analysis of interacting conforming rough surfaces[12,15,16] yields the following important geometric results:

1) Contact conductance parameter

$$\sum_{i=1}^{N} \frac{a_i}{A_a} = \frac{1}{4\sqrt{2\pi}} \cdot \frac{m}{\sigma} \cdot \exp(-x^2) \quad (8)$$

2) Relative real contact area

$$\varepsilon^2 = \frac{A_r}{A_a} = \frac{1}{2} \operatorname{erfc}(x) \quad (9)$$

3) Contact spot density

$$n = \frac{1}{16}\left(\frac{m}{\sigma}\right)^2 \frac{\exp(-2x^2)}{\operatorname{erfc}(x)} \quad (10)$$

4) Mean contact spot radius

$$a = \sqrt{\frac{8}{\pi}}\, \frac{\sigma}{m} \exp(x^2) \operatorname{erfc}(x) \quad (11)$$

where $x = Y/\sqrt{2}\sigma$ and Y/σ is called the relative mean plane separation. The surface parameters σ and m are the effective rms surface roughness and the effective absolute surface slope, respectively. They are determined as follows:

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \quad (12)$$

and

$$m^2 = m_1^2 + m_2^2 \quad (13)$$

Assuming plastic deformation of the contacting asperities during the first loading leads to a relationship between the relative real contact area and the relative contact pressure. A force balance on the real and apparent contact areas gives

$$P/H = A_r/A_a = \varepsilon^2 = (1/2)\mathrm{erfc}\,(x) \tag{14}$$

This relationship between ε and x allows one to compute the other surface parameters. After substitution of the contact conductance parameter [Eq. (8)] into the contact conductance expression [Eq. (6)], we obtain after multiplying by σ/k_s the nondimensional contact conductance,

$$\frac{\sigma h_c}{k_s} = \frac{m}{2\sqrt{2\pi}} \frac{\exp\,(-x^2)}{(1-\varepsilon)^{1.5}} \tag{15}$$

with $x = \mathrm{erfc}^{-1}(2P/H)$ and $\varepsilon = \sqrt{P/H}$. The complex expression of Eq. (15) was correlated by the following simple expression:

$$\sigma h_c/k_s = 1.25\ m\ (P/H)^{0.95} \tag{16}$$

which agrees with the exact expression to within ±1.5% for $2 \le Y/\sigma \le 4.75$.

Alternate Development of Contact Conductance

Here we assume N microcontent spots all having the same mean radius a and associated circular flux tube of mean radius b. The total contact resistance is as above

$$R_c = \psi_c/2Nk_s a \tag{17}$$

and the contact conductance is

$$h_c = 2nak_s/\psi_c \tag{18}$$

where n is the contact spot density.

The product of Eqs. (10) and (11) yields

$$na = \frac{1}{4\sqrt{2\pi}} \frac{m}{\sigma} \exp(-x^2) \tag{19}$$

Substitution of Eq. (19) into Eq. (18) gives Eq. (15). To simplify further calculations, two additional correlations for Y/σ and a will be derived.

From Eq. (14) we obtain

$$\sqrt{2}x = Y/\sigma = 1.184[-\ln(3.132P/H)]^{0.547} \tag{20}$$

Multiplying Eq. (11) through by x gives

$$(m/\sigma)ax = \sqrt{(8/\pi)}\,x\exp(x^2)\mathrm{erfc}(x) \tag{21}$$

In the range of interest $1.41 \le x \le 3.16$, we can write

$$x\exp(x^2)\mathrm{erfc}(x) \approx 0.52 \tag{22}$$

Substitution of Eqs. (20) and (22) into Eq. (21) yields

$$ma/\sigma = 0.99[-\ln(3.132P/H)]^{-0.547} \tag{23}$$

Knowing the surface characteristics m and σ as well as the relative contact pressure P/H, one can compute the mean contact spot radius by means of Eq. (23).

Gap Conductance Correlation

Heat transfer across the gap is very difficult to analyze because of the complexity of the geometry of the gap which determines whether the local heat transfer can be modeled as continuum, slip, or rarefied. To overcome these difficulties it is necessary to model the heat transfer from an overall point of view. To this end it will be assumed that the gap heat transfer can be modeled as heat transfer between two isothermal parallel plates which are separated by the distance Y_e, the effective gap thickness. The effective gap thickness is some fraction of the distance between the mean planes Y.

The simplest model assumes $Y_e = Y$ for greases and liquids which completely wet the two surfaces, and for gases we take

$$Y_e = Y + \alpha\beta\Lambda \tag{24}$$

In Eq. (24) α is the accommodation coefficient defined as

$$\alpha = (2-\alpha_1)/\alpha_1 + (2-\alpha_2)/\alpha_2 \tag{25}$$

where α_1 and α_2 are the accommodation coefficients. These coefficients are complex parameters which depend upon the microgeometry; the gas and solid molecular weights; and surface contaminants such as oxides, films, etc. They are usually determined empirically; typical values for a range of gases are reported by Shlykov.[18]

The fluid parameter β is determined by

$$\beta = (2\gamma/\gamma + 1)(1/Pr) \tag{26}$$

where γ is the specific heat ratio and Pr is the Prandtl number.

The molecular mean free path Λ is dependent upon the gap mean temperature T and the gap pressure P_g,

$$\Lambda = \Lambda_o (T/T_o)(P_{go}/P_g) \tag{27}$$

In Eq. (27) the subscript refers to some reference level.

For the effective gap heat-transfer area, we have

$$A_g = A_a \ (1-\varepsilon^2) \simeq A_a \tag{28}$$

because $\varepsilon \leq 0.3$; therefore, for the gap conductance, we have

$$h_g = k_{go}/(Y + \alpha\beta\Lambda) \tag{29}$$

To nondimensionalize Eq. (29) multiply by (σ/k_s) to obtain the gap conductance correlation

$$\frac{\sigma h_g}{k_s} = \frac{(k_{go}/k_s)}{(Y/\sigma) + (\alpha\beta/\sigma)\Lambda_o(T/T_o)(P_{go}/P_g)} \tag{30}$$

or for convenience we write

$$\frac{\sigma h_g}{k_s} = \frac{K}{(Y/\sigma) + M} \tag{31}$$

The relative gap thickness (Y/σ) is determined by means of Eq. (20).

For most practical contact problems the various parameters appearing in Eq. (30) are limited to the ranges shown in Table 1.

Joint Conductance Correlation

The contact and gap heat-transfer rates are approximately independent for the practical contact problems of interest here. Therefore, the total or joint conductance h_j is equal to the sum of the contact and gap conductances,

$$h_j = h_c + h_g \tag{32}$$

Multiplying by (σ/k_s) gives

$$\frac{\sigma h_j}{k_s} = \frac{\sigma h_c}{k_s} + \frac{\sigma h_g}{k_s} \tag{33}$$

or for convenience as

$$C_j = C_c + C_g \tag{34}$$

Comparison of Theoretical and Experimental Results

The joint conductance correlation developed in this paper will be compared with the experimental results reported in Refs. 17 and 19. The pertinent geometric, physical, and thermal parameters are given in Tables 2 and 3.

Using Eqs. (16) and (30) with the data given in Table 2, we can develop the following joint conductance correlation for the contact described in Ref. 17:

$$C_j = 0.163\left(\frac{P}{H}\right)^{0.95} + \frac{1.18 \times 10^{-3}}{0.081 + 1.184\left[-\ln\left(3.132\frac{P}{H}\right)\right]^{0.547}} \tag{35}$$

The contact, gap, and joint conductances predicted by the theory and the measured joint conductances are presented in Table 4. It can be seen that there is good agreement between the theory and the measured values at three of the five relative contact pressures. The predicted

Table 1 Typical ranges of the parameters

$10^{-5} < P/H < 10^{-2}$	$1 < \beta < 2$
$4.26 > Y/\sigma > 2.34$	$0.04\mu m < \Lambda_o < 0.19\mu m$
$0.2\mu m < \sigma < 4\mu m$	$0.01 < \Lambda_o/\sigma < 1.0$
$0.03 < m < 0.3$	$10^{-4} < k_{go}/k_s < 2\times10^{-2}$
$2 < \alpha < 10$	

Table 2 Geometric, physical, and thermal parameters of Ref. 17

Solids: Stainless steel 416 pair	Fluid: Air at 377 K and 1 atm
$k_1 = k_2 = 25.26$ W/m·K	$k_{go} = 0.0298$ W/m·K
$k_s = 25.26$ W/m·K	$\therefore K = 1.18\times10^{-3}$
$H_1 = H_2 = 3.80\times10^{10}$ Pa	Assume $\alpha_1 = \alpha_2 = 0.9$
$H = 3.80\times10^{10}$ Pa	$\therefore \alpha = 2.24$
$\sigma_1 = 3.81\mu m$	$\beta = 1.64$
$\sigma_2 = 1.57\mu m$	$\Lambda_o = 6.40\times10^{-8}$ m
$\sigma = 4.11\mu m$	$\therefore \Lambda_o/\sigma = 1.50\times10^{-2}$
$m_1 = 0.092$	$T/T_o = 1.31$, $P_{go}/P_g = 1.0$
$m_2 = 0.092$	$M = \alpha\beta(\Lambda_o/\sigma)(T/T_o)(P_{go}/P_g)$
$m = 0.130$	$= 0.081$

Table 3 Geometric, physical, and thermal parameters of Ref. 17

Solids:	Molybdenum	Copper	Fluid: Air at 288 K and 1 atm
k, W/mK	140	380	$k_{go} = 0.0337$ W/mK
H, 10^{-8}Pa	18	5.0	$\alpha = 2.33$
k_s	204.6 W/m·K		$\beta = 1.64$
H	$5.0\times10^{+8}$ Pa		$\Lambda_o = 6.40\times10^{-8}$ m
P	6.90×10^{4}Pa		$\frac{\Lambda_o}{\sigma} = 7.24\times10^{-2}$
P/H	1.38×10^{-4}		$\frac{T}{T_o} = 1.00$, $\frac{P_{go}}{P_g} = 1$
$\sigma_1 = \sigma_2 = 0.625\mu m$			
$\therefore \sigma = 0.884\mu m$			
$m_1 = m_2 = 0.07$ (assumed)			$M = \alpha\beta(\Lambda_o/\sigma)(T/T_o)(P_{go}/P_g)$
$\therefore m = 0.10$			$= 0.277$

Table 4 Comparison between theory and experiment (Ref. 17)

$10^4 P/H$	8.92	21.6	40.5	108	216
$10^4 C_c$	2.07	4.79	8.70	22.1	42.7
$10^4 C_g$	3.71	4.05	4.35	4.98	5.63
$10^4 C_j$ (Theory)	5.78	8.84	13.05	27.08	48.33
$10^4 C_j$ (Exp.)	7.72	9.25	17.66	25.43	43.46
Difference, %	-33.6	-4.64	-35.3	+6.09	+10.1

values of C_j are approximately 35% below the measured values at the lowest load and the intermediate load. The author believes that the measured value of $C_j = 17.66\times10^{-4}$ is questionable because it is not consistent with the other measured values. It is difficult at this time to say anything about the discrepancy between the theory and the test result corresponding to the lowest value of P/H.

By means of Eqs. (16) and (30) and the data given in Table 3, one can determine the contact, gap, and joint conductances: $h_c = 6.23\times10^3$ W/m^2·K, $h_g = 9.75\times10^3$ W/m^2·K and $h_j = 15.98\times10^3$ W/m^2·K. The measured value reported in Ref. 19 is $h_j = 17.22\times10^3$ W/m^2·K. It is seen that the agreement is quite good.

In Ref. 19, a test result with silicon grease is also presented. The experimentally determined joint conductance was found to be: $h_j = 71.76\times10^3$ W/m^2·K. The present model predicts the gap conductance to be

$$h_g = \frac{k_g}{Y} = \frac{k_g}{(Y/\sigma)\sigma} \tag{36}$$

With $k_g = 0.20$ W/m·K for silicon grease[23] Eq. (36) predicts $h_g = 62.36\times10^3$ W/m^2·K. Adding this value to $h_c = 6.23\times10^3$ W/m^2·K yields $h_j = 68.59\times10^3$ W/m^2·K. The predicted and the measured values are in very good agreement.

Conclusions and Recommendations

The reasonably good agreement between the theoretical and measured values of the joint conductance is most encouraging. However, much more empirical data are required before one can conclude that the contact, gap and joint conductance correlations presented in this paper are valid for all contact conditions. It is recommended that an experimental program should be undertaken to obtain accurate test results for a range of the important contact parameters: P/H, σ/m, K, and M.

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