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CONSTRICTION RESISTANCE OF CIRCULAR FLUX TUBES WITH MIXED BOUNDARY CONDITIONS BY LINEAR SUPERPOSITION OF NEUMANN SOLUTIONS

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ABSTRACT

An approximate technique for the solution of mixed boundary value problems has been developed. The methodology consists of superposing Neumann-specified solutions to Laplace's equation according to a least squares criterion such that the true mixed boundary conditions are approximated. In this work an isothermal circular disk supplying heat to an otherwise insulated semi-infinite coaxial cylinder has been investigated. Superposition of two to four different Neumann-specified solutions gives the thermal constriction resistance parameter for a wide range of relative contact sizes. In addition a simple but accurate correlation of the final results has been provided.

NOMENCLATURE

a - contact radius
A_c - total contact area
b - flux tube radius
C_j - linear scaling factors of Neumann solutions
D_n - series coefficients
E - total error squared
f_j - constriction temperature rise of each Neumann solution
F_j - function relating contact flux to series coefficients
g_j - form of specified flux profiles
$J_0(\cdot)$ - Bessel function of first kind, zero order
$J_1(\cdot)$ - Bessel function of first kind, first order
k - thermal conductivity
M - number of linearly superposed Neumann solutions
N - number of test points
q_j - contact flux distribution
q_o - center-line contact flux
[illegible] - uniform flux in tube at large depth
[illegible] - total flux entering tube
[illegible] - total flux due to each Neumann solution
r - polar coordinate
R_c - constriction resistance
T - temperature
T_c - constriction temperature rise
$\bar{T}_c$ - average constriction temperature rise
T_s - specified isothermal contact constriction temperature rise
u - dimensionless contact position
z - coordinate

Greek symbols

δ_n - solutions to $J_1(\delta_n)=0$
ϵ - dimensionless contact size
λ_n - eigenvalues for Neumann solutions
ϕ - general function in all Neumann solutions
ψ - dimensionless thermal constriction resistance parameter

Subscripts

i - refers to a test point
j,ℓ - refers to a Neumann solution
n - refers to a term in the infinite series of the Neumann solutions

INTRODUCTION

In the past several investigators [1-7] have examined the problem of the thermal constriction resistance of a circular contact area on an insulated semi-infinite, coaxial cylinder as shown in Fig. 1. This situation is commonly called a thermal constriction resistance flux tube.

It can be shown by symmetry arguments that for a real interface between two contacting solids the circular contact area must be isothermal. Since the area outside the contact area on the top of the flux tube is modelled as adiabatic, a mixed boundary value problem results. In most real contact problems, usually based on the theory of conforming rough surfaces [8], the ratio of contact radius 'a' to flux tube radius 'b', $\epsilon \equiv a/b$, is small ($0 < \epsilon \leq .2$). In this range a prescribed flux distribution proportional to $(1-(r/a)^2)^{-1/2}$ will closely approximate the true

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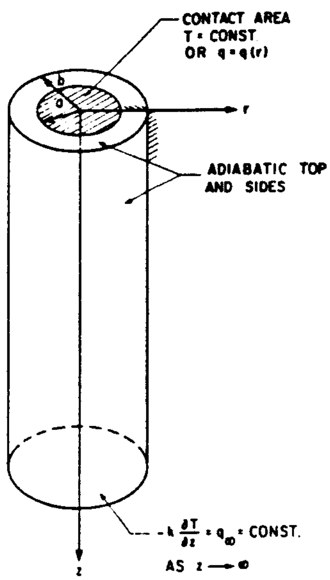

Fig. 1 Circular contact on an insulated, semi-infinite, coaxial cylinder

isothermal boundary condition over the contact.

The problem can also be formulated in terms of dual integral equations [9] which can then be solved numerically [5] or approximately by assuming ε is small [4]. An attempt to solve this problem numerically by finite difference was also limited to small ε [7].

In this work the mixed boundary value problem will be solved by linear superposition of two to four different prescribed flux distributions using a linear least squares criterion. There are two important goals in this work. The first goal is to obtain accurate values of the constriction resistance over the large range of $0 < \varepsilon \leq .9$. These values are necessary for models which describe the contact resistance between rough, wavy surfaces [10] and are also applicable to other analogous field problems such as electrical current flow [6]. The second goal is to demonstrate the versatility of this technique which could be used to solve mixed boundary value problems where other techniques are not readily applicable.

Problem Statement and Solutions for Various Flux Distributions

Consider the steady conduction of heat through the right cylinder shown in Fig. 1. The governing differential equation and boundary conditions are

$$\nabla^2 T = 0 \tag{1}$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

$$r = 0, \quad 0 \leq z \leq L \quad \frac{\partial T}{\partial r} = 0 \tag{2}$$

$$r = b, \quad 0 \leq z \leq L \quad \frac{\partial T}{\partial r} = 0 \tag{3}$$

$$0 \leq r \leq a, \quad z = 0 \quad -k\frac{\partial T}{\partial z}(r,0) = q_j(r) \tag{4}$$

where $q_j(r)$ is some prescribed flux distribution which is applied to the contact area.

$$a < r \leq b, \quad z = 0 \quad \frac{\partial T}{\partial z} = 0 \tag{5}$$

$$0 \leq r \leq b, \quad z \to \infty \quad -k\frac{\partial T}{\partial z} = q_\infty \tag{6}$$

where $q_\infty = Q/\pi b^2$ (7)

and Q is the total heat flux entering the tube.

From consideration of (1), (2), (3), (6) and (7) superposition of two solutions gives

$$T(r,z) = D_o - \frac{q_\infty z}{k} + \sum_{n=1}^{\infty} D_n J_o(\lambda_n r) e^{-\lambda_n z} \tag{8}$$

where $\delta_n \equiv \lambda_n b$ are the roots of

$$J_1(\delta_n) = 0 \tag{9}$$

The prescribed flux over the contact area is then related to the temperature field by

$$\frac{\partial T}{\partial z}(r,0) = -\frac{q_\infty}{k} - \sum_{n=1}^{\infty} D_n \lambda_n J_o(\lambda_n r) \tag{10}$$

To evaluate the unknown coefficients D_n, both sides of (10) are multiplied by $rJ_o(\lambda_m r)$ and integrated from r=0 to r=b.

$$\int_0^b \frac{\partial T}{\partial z}(r,0) r J_o(\lambda_m r) = -\frac{q_\infty}{k}\int_0^b r J_o(\lambda_m r)dr - \sum_{n=1}^{\infty} D_n \lambda_n \int_0^b r J_o(\lambda_m r) J_o(\lambda_n r)dr \tag{11}$$

The first term on the right-hand side of (11) is zero identically since both $J_1(\lambda_m b)$ and $J_1(0)$ are zero. The second term of the right-hand side is zero when $\lambda_m \neq \lambda_n$ by the orthogonality property of Bessel functions. Thus

$$D_n \lambda_n \int_0^b r J_o^2(\lambda_n r)dr = -\int_0^b \frac{\partial T}{\partial z}(r,0) r J_o(\lambda_n r)dr \tag{12}$$

which after integrating the left-hand side [11] becomes

$$\frac{D_n \delta_n^2 J_o^2(\delta_n)}{2\lambda_n} = -\int_0^b \frac{\partial T}{\partial z}(r,0) r J_o(\lambda_n r)dr \tag{13}$$

Note that conditions (4) and (5) require that

$$\frac{\partial T}{\partial z}(r,0) = -\frac{q_j(r)}{k} \quad \text{for} \quad 0 \leq r \leq a \tag{14}$$

$$\frac{\partial T}{\partial z}(r,0) = 0 \quad \text{for} \quad a < r \leq b \tag{15}$$

For the constriction resistance of a flux tube only the surface temperature over the contact area is of immediate interest. Thus a dimensionless radial distance, $u \equiv r/a$, shall be used to give

$$T(u) = D_o + \frac{2}{k}\sum_{n=1}^{\infty} \frac{J_o(\delta_n \varepsilon u)\lambda_n a^2}{\delta_n^2 J_o^2(\delta_n)} \int_0^1 q_j(u) u J_o(\lambda_n \varepsilon u)du \tag{16}$$

However the thermal constriction resistance of the flux tube, defined as the average contact temperature minus the average temperature in the z=0 plane divided by the total heat flux in the tube, is the quantity which will ultimately be determined. Since it can be easily shown as in [1] that the average temperature in the z=0 plane is simply D_o, then a constriction temperature rise on the contact surface is simply $T_c(u) = T(u) - D_o$ or thus

$$T_c(u) = \frac{2a}{k}\sum_{n=1}^{\infty} \frac{J_o(\delta_n \varepsilon u)\delta_n \varepsilon}{\delta_n^2 J_o^2(\delta_n)} \int_0^1 q_j(u) u J_o(\delta_n \varepsilon u)du \tag{17}$$

The applied flux distribution $q_j(u)$ will however given in terms of some function of position $g_j(u)$ and a linear scaling factor C_j such that

$$q_j(u) = C_j \, g_j(u) \tag{18}$$

Thus, if there are 'M' different flux distributions applied simultaneously to the contact area, the constriction temperature rise on the contact will be given by

$$\frac{kT_c(u)}{a} = 2 \sum_{j=1}^{M} C_j \sum_{n=1}^{\infty} \phi(\delta_n \varepsilon, u) \, F_j(\delta_n \varepsilon) \tag{19}$$

where
$$\phi(\delta_n \varepsilon, u) \equiv \frac{J_o(\delta_n \varepsilon u)}{\delta_n^2 J_o^2(\delta_n)} \tag{20}$$

and
$$F_j(\delta_n \varepsilon) \equiv \delta_n \varepsilon \int_0^1 g_j(u) u \, J_o(\delta_n \varepsilon u) du \tag{21}$$

In this work four different applied flux distributions will be considered. The corresponding F_j as defined by (21) are given in Table 1 [11]. The first flux distribution is often called the equivalent isothermal because it gives a true isothermal contact temperature on a half-space and a close approximation for small $\varepsilon \equiv a/b$. The second is the uniform flux distribution which would give a true isothermal contact temperature if $\varepsilon = 1$ (although the constriction resistance is zero for $\varepsilon = 1$). The third and fourth distributions were added to help in obtaining accurate results for large ε.

Superposition of Applied Flux Distributions by Linear Least Squares Criterion

If M of the previously described flux profiles are linearly superposed, then together they form a hybrid flux distribution such that the constriction temperature rise on the contact, Eq. (19), can be written as

$$\frac{kT_c(u)}{a} = \sum_{j=1}^{M} C_j \, f_j(\varepsilon, u) \tag{22}$$

where
$$f_j(\varepsilon, u) = 2 \sum_{n=1}^{\infty} F_j(\delta_n \varepsilon) \phi(\delta_n \varepsilon, u) \tag{23}$$

and $\phi(\delta_n \varepsilon, u)$ is given by (20) and $F_j(\delta_n \varepsilon)$ is tabulated in Table 1.

Since it is desired to combine the flux distributions in such a manner that the local constriction temperature rise on the contact is uniform, then the C_j will be determined such that

$$\frac{kT_c(u)}{a} \doteq T_s = \text{const.} \tag{24}$$

Table 1. Function F_j as defined in Eq. (21) for Four Different Flux Distributions

j	$g_j(u)$	$F_j(\delta_n \varepsilon)$
1	$(1-u^2)^{-1/2}$	$\sin(\delta_n \varepsilon)$
2	1	$J_1(\delta_n \varepsilon)$
3	u^2	$J_1(\delta_n \varepsilon)[1-4/(\delta_n \varepsilon)^2] + (2/\delta_n \varepsilon) J_o(\delta_n \varepsilon)$
4	$(1-u^2)^{1/2}$	$[1/(\delta_n \varepsilon)^2][\sin(\delta_n \varepsilon) - (\delta_n \varepsilon) \cos(\delta_n \varepsilon)]$

where T_s represents a specified constant constriction temperature rise for the contact.

However it can be readily shown that if the constants C_j are determined by a linear least squares analysis, then each C_j will be directly proportional to T_s. Thus for convenience T_s is set to unity (or $T_c(u) \doteq a/k$). If 'N' collocation or test points, u_i, are considered then the total error squared is

$$E = \sum_{i=1}^{N} [1 - \sum_{j=1}^{M} C_j \, f_j(\varepsilon, u_i)]^2 \tag{25}$$

By the method of linear least squares the unknown constants C_j are chosen such that E is a minimum, or

$$\frac{\partial E}{\partial C_\ell} = 0 \qquad (\ell = 1, 2, \ldots, M) \tag{26}$$

This criterion creates a system of M simultaneous linear equations of the form

$$\sum_{j=1}^{M} C_j \sum_{\ell=1}^{M} \sum_{i=1}^{N} [f_\ell(\varepsilon, u_i) f_j(\varepsilon, u_i)] = \sum_{\ell=1}^{M} \sum_{i=1}^{M} f_\ell(\varepsilon, u_i) \tag{27}$$

or in matrix format

$$\left[\sum_{i=1}^{N} f_\ell(\varepsilon, u_i) f_j(\varepsilon, u_i) \right] \left\{ C_j \right\} = \left\{ \sum_{i=1}^{N} f_\ell(\varepsilon, u_i) \right\} \tag{28}$$

Definition of Thermal Constriction Resistance and Selection of Discretization Points

By definition the thermal constriction resistance for a flux tube is

$$R_c = \frac{\bar{T}_c}{Q} \tag{29}$$

where $\bar{T}_c$ is the average constriction temperature rise on the contact (= average contact temperature rise minus average temperature rise over the plane z=0), or

$$\bar{T}_c = \frac{1}{A_c} \iint_{A_c} T_c(u) dA_c \tag{30}$$

$$\doteq \frac{1}{A_c} \sum_{i=1}^{N} T_c(u_i) A_{ci} \tag{31}$$

where $A_c = \pi a^2$ is the total contact area.

Then for

$$\bar{T}_c \doteq \frac{1}{N} \sum_{i=1}^{N} T_c(u_i) \tag{32}$$

all the A_{ci} are equal at $A_{ci} = A_c/N$ and the u_i are chosen as the center of equal area annuli. This technique insures that the least squares analysis will be performed to give maximum accuracy in the calculation of thermal constriction resistance.

The total flux Q consists of the sum of the total fluxes of each prescribed flux distribution, or

$$Q = \sum_{j=1}^{M} Q_j \tag{33}$$

where it can easily be shown that for the four flux profiles considered in this work

$$Q_1 = 2 C_1 \pi a^2 , \quad Q_2 = C_2 \pi a^2$$
$$Q_3 = \frac{1}{2} C_3 \pi a^2 , \quad Q_4 = \frac{2}{3} C_4 \pi a^2$$

And thus the thermal constriction resistance is

$$R_c = \frac{(k\bar{T}_c/a)(a/k)}{\pi a^2(2C_1 + C_2 + \frac{1}{2}C + \frac{2}{3}C_4)} \tag{34}$$

or by defining a dimensionless thermal constriction resistance parameter

$$\psi \equiv 4ka\ R_c \tag{35}$$

and noting that $k\bar{T}_c/a = T_s = 1$, then

$$\psi = \frac{4}{\pi(2C_1 + C_2 + \frac{1}{2}C_3 + \frac{2}{3}C_4)} \tag{36}$$

where if fewer than all four flux distributions are being combined, then the unused constant(s) C_j should be set to zero in Eq. (36).

Presentation and Discussion of Results

As mentioned previously, one major goal of this work is to obtain accurate results for the constriction parameter over the complete range of $0 < \varepsilon \le .9$. To achieve this goal with accuracy, up to four flux profiles and 35 test points were used to ensure the validity of the results.

All computations were made on an IBM-PC using double precision. Note that after developing the analytic work for each flux distribution, it is of little consequence to either the computer programming or execution time whether two, three or four different flux distributions are combined.

To assemble the system of linear equations described by (27) requires repeated calculations of the roots of $J_1(\delta_n) = 0$ and evaluations of the Bessel functions $J_0(\cdot)$ and $J_1(\cdot)$. The roots are easily determined to 10 decimal accuracy by the Stokes' approximation [12]. The Bessel functions $J_0(\cdot)$ and $J_1(\cdot)$ are evaluated to an absolute error less than 10^{-7} by simple and efficient polynomial approximations [12].

Table 2 shows how closely the resultant calculated temperatures are to the specified temperature of $T_s = 1$ for combinations of two, three and four flux distributions. Note that for $\varepsilon \le .2$ the first flux distribution f_1 alone will yield extremely uniform contact temperatures. Also when the number of test points is increased, the relative accuracy remains roughly as seen in Table 2 for any particular combination of flux distributions at any given ε.

It must also be emphasized that the slight relative errors in the calculated temperatures of Table 2 cause considerably lower relative errors in the subsequent calculations of the dimensionless constriction parameter. This results because the constriction parameter represents an integrated value of the calculated temperature distribution and thus minor deviations above and below the specified temperature tend to have cancelling effects.

Figures 2, 3 and 4 show respectively for $\varepsilon = .5$, .7 and .9 how the optimised linear combination of the four f_j produces a virtually uniform resultant temperature distribution. Note that the plots shown of each f_j in Figures 2, 3 and 4 represent the form of the temperature distribution which would result from that particular prescribed flux distribution.

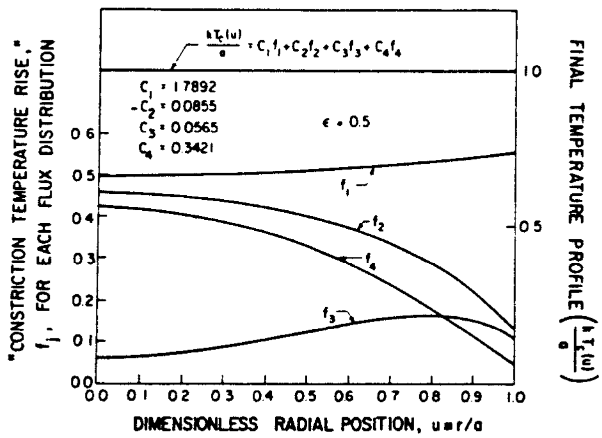

Fig. 2 Constriction temperature rise on contact for each component flux case and for final optimised combination: $\varepsilon \equiv a/b = 0.5$.

Table 2. Resultant Temperature Profiles for Different ε and Number of Flux Profiles Used, M.

u_i	$\varepsilon = .3$		$\varepsilon = .5$			$\varepsilon = .7$			$\varepsilon = .9$	
	M=2	M=3	M=2	M=3	M=4	M=2	M=3	M=4	M=3	M=4
0.1291	0.9994	1.0001	0.9950	1.0005	0.9999	0.9691	1.0041	1.0001	1.0816	0.9933
0.3117	0.9996	1.0000	0.9970	1.0002	1.0000	0.9804	1.0011	1.0000	1.0219	1.0002
0.4062	0.9998	1.0000	0.9983	0.9999	1.0000	0.9887	0.9992	0.9999	0.9847	1.0037
0.4818	0.9999	1.0000	0.9997	0.9997	1.0001	0.9969	0.9982	1.0000	0.9591	1.0050
0.5469	0.9998	1.0000	1.0007	0.9996	1.0001	1.0042	0.9974	0.9999	0.9442	1.0039
0.6050	1.0000	1.0000	1.0015	0.9996	1.0000	1.0100	0.9974	1.0000	0.9413	1.0031
0.6578	1.0001	0.9999	1.0023	0.9997	1.0001	1.0144	0.9978	1.0000	0.9468	0.9996
0.7067	1.0004	0.9999	1.0029	0.9998	1.0000	1.0188	0.9986	1.0000	0.9623	0.9967
0.7525	1.0005	1.0000	1.0034	0.9999	0.9998	1.0198	0.9997	1.0000	0.9874	0.9965
0.7956	1.0005	1.0000	1.0053	1.0001	1.0002	1.0201	1.0010	1.0000	1.0155	0.9946
0.8364	1.0008	1.0000	1.0026	1.0003	0.9998	1.0185	1.0028	1.0005	1.0457	0.9952
0.8754	1.0009	1.0000	1.0020	1.0004	1.0002	1.0128	1.0032	0.9999	1.0729	1.0006
0.9127	1.0006	1.0001	1.0001	1.0006	0.9998	1.0005	1.0030	0.9995	1.0775	1.0026
0.9485	1.0005	1.0001	0.9970	1.0001	1.0001	0.9825	1.0018	1.0002	1.0423	1.01[illegible]
0.9831	0.9975	0.9998	0.9935	0.9993	0.9998	0.9576	0.9946	1.0000	0.8623	0.99[illegible]

Note: Ideal result would be $T = T_s = 1.0000$.

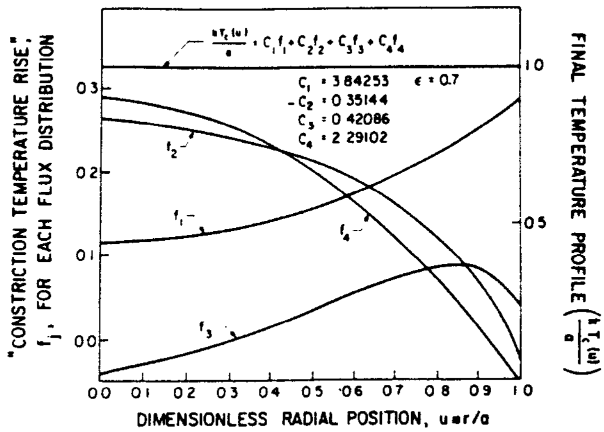

Fig. 3 Constriction temperature rise on contact for each component flux case and for final optimised combination: ε ≡ a/b = 0.7.

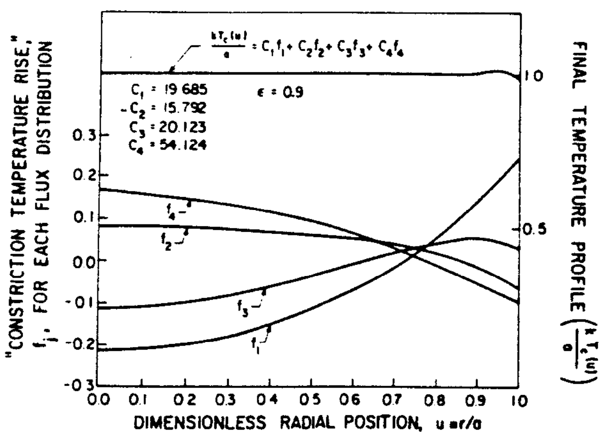

Fig. 4 Constriction temperature rise on contact for each component flux case and for final optimised combination: ε ≡ a/b = 0.9.

Figure 5 shows the hybrid flux distributions which result from the linear combination of the four flux distributions for ε = .5, .7 and .9. Note that the results are plotted as a dimensionless quantity relative to the centerline (u=0) flux. The hybrid flux distribution falls between the bounds of the equivalent isothermal flux distribution and the uniform flux distribution. As expected, for small ε the hybrid flux distribution resembles the equivalent isothermal and as ε becomes close to unity, the hybrid flux distribution tends toward a uniform value.

Table 3 shows the slight "convergence" of the constriction resistance as the number of test points is increased. Since the computer execution time is directly proportional to the number of test points, it is advantageous to use as few points as necessary.

Table 4 is a comparison of the constriction parameter calculated by the technique of this paper with those of other researchers. The result of Roess is really just the use of the first flux distribution, the equivalent isothermal. Smythe's results reflect an attempt to correlate the resultant resistances of the first two flux distributions presented in this work. Gibson's solution is the true mixed boundary problem solved by using a series solution of dual integral equations by assuming that ε is small. Despite this limitation there is excellent agreement with the accurate results presented here.

Figure 6 shows graphically the relationship between the dimensionless thermal constriction parameter, $\psi \equiv 4kaR_c$, and the relative flux tube contact radius, $\epsilon \equiv a/b$.

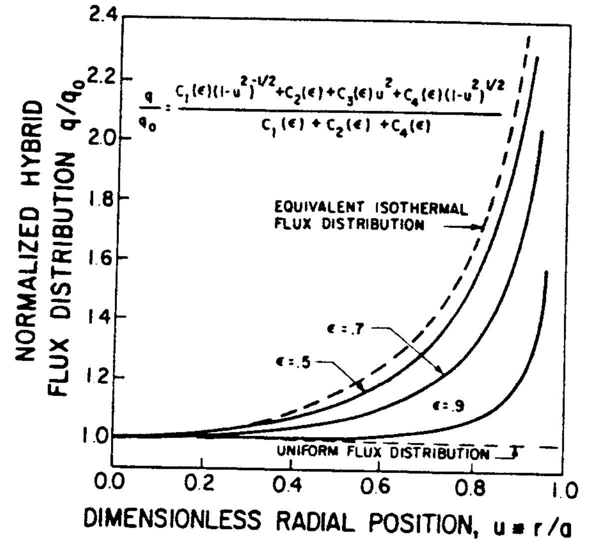

Fig. 5 Resultant flux distribution on contact for final optimised combination.

Table 3. Constriction Parameter, ψ, Using all Four Flux Profiles for all with Different Numbers of Test Points.

N \ ε	0.3	0.4	0.5	0.6	0.7	0.8	0.9
15	.5866	.4585	.3396	.2324	.1404	.0672	.0183
20	.5866	.4586	.3396	.2324	.1403	.0672	.0183
25	.5865	.4586	.3396	.2324	.1403	.0672	.0183
35	.5865	.4586	.3396	.2324	.1403	.0672	.0183

Table 4. Comparison of Constriction Parameter ψ as Calculated by Four Different Techniques

Researcher	ε=0.3	ε=0.4	ε=0.5	ε=0.6	ε=0.7	ε=0.8
Negus, Yovanovich	.5865	.4586	.3396	.2324	.1403	.0672
Roess [2]	.5853	.4558	.3342	.2232	.1262	.0483
Smythe [6]	.5865	.4588	.3408	.2350	.1444	.0723
Gibson [4]	.5865	.4586	.3398	.2328	.1409	.0680

Correlation of Results

Since the series solution of Gibson gave excellent agreement with the results presented here, a correlation was made using basically the same functional form. The resultant correlation for the dimensionless thermal constriction parameter is

$$\psi = 1 - 1.40978\epsilon + .34406\epsilon^3 + .0435\epsilon^5 + .02271\epsilon^7 \qquad (37)$$

alid over the range $0 \leq \epsilon \leq .9$. This correlation has maximum relative error of .02% at $\epsilon = .7$ but with round-off gives complete four decimal place agreement with all the correlated data.

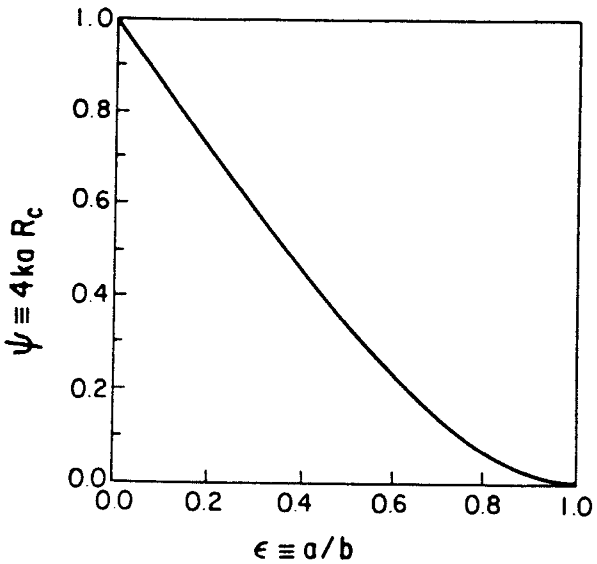

Fig. 6 Constriction resistance parameter ψ for any relative contact size ϵ.

Conclusions

The thermal constriction resistance parameter for circular flux tube with a true isothermal contact has been calculated accurately over a wide range of the relative contact size. From this data a correlation (Eq. (37)) has been developed which is more than adequate for design purposes and is valid over a larger range than necessary for any anticipated physical problem. In addition this correlation also gives the electrical constriction resistance parameter for an analogous problem which occurs generally with the same mixed boundary conditions as the thermal problem.

The unique solution method presented in this work has proved to be very successful. This technique of linearly combining different flux distributions so as to satisfy imposed mixed boundary conditions can also solve other problems of this type where other methods such as integral equations or finite difference are either impractical or too expensive or both. For example, two problems which are presently under consideration are the circular contact on a square flux tube [13] and the elliptical contact on a rectangular flux tube [14]. The solutions to the flux specified problems for these two examples are also complex.

Finally, this method of creating hybrid flux distributions is especially useful for problems where an integrated quantity such as constriction resistance or electrostatic capacitance is the desired end result. For such problems slight deviations from the desired temperature distribution tend to "cancel out" in the calculation of the integrated quantity.

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