MODIFIED EFFECTIVE CONDUCTIVITY MODELS FOR BASIC CELLS OF SIMPLE CUBIC PACKED BEDS

P.J. Turyk, Graduate Student and M.M. Yovanovich, Professor
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada

ABSTRACT

A new modification to the Yovanovich-Ogniewicz model for the thermal conductivity of a packed bed with uniformly sized spheres is presented. The modification includes the effect of sphere surface roughness in improving the model for gap resistance. Despite the simple application of contacting conforming rough surface theory, the model visibly improves the accuracy of the prediction of total resistance when compared with experimental results in macrosphere basic cells.

NOMENCLATURE

a	contact radius (m)
A	area (m^2)
D	sphere diameter (m)
E	Young's modulus (N/m^2)
f	the attenuation factor of the modified Ogniewicz model
g	function defined by Eq. (14a)
H	hardness
I	integral defined by Eqs. (11), (15), (16), (18), and (22)
k	conductivity (W/mK)
K	conductivity ratio $\frac{k_o}{k_s}$
l	heat transfer length (m)
L	dimensionless diameter $\frac{D}{2a}$
M	$= 2\frac{\alpha\beta\Lambda}{D}$, dimensionless gas parameter
N	load (N)
p	mechanical pressure (N/m^2)
P	gas pressure (N/m^2)
Pr	Prandtl number
Q	heat flow rate (W)
r	radial distance from axis of symmetry (m)
R	thermal resistance (K/W)
T	temperature (K)
x	dimensionless radial distance $\frac{r}{a}$
Y	separation of the mean planes of contacting rough surfaces (m)
z	coordinate in the direction of the heat flow (m)

Greek Symbols

α	total accomodation coefficient at gas-solid interface $= \frac{2-\alpha_1}{\alpha_1} + \frac{2-\alpha_2}{\alpha_2}$
β	a gas property $= \frac{2\gamma}{Pr(\gamma+1)}$
γ	ratio of specific heats
δ	gap width (m)
ϵ	size ratio of spheres; radiative surface emissivity
Λ	gas mean free path (m)
ν	Poisson's ratio
σ	surface roughness (m); Stefan-Boltzmann constant $(\frac{W}{m^2K^4})$

Subscripts

a	apparent area
c	contact

e	effective
g	gap, gas
ht	heat transfer
mod	Ogniewicz modified model
o	gas under continuum conditions; reference conditions
r	radiation; real area
R_m	roughness modified model
s	harmonic mean conductivity; solid; subtended angle
t	total
$1D$	unidirectional model
1	sphere 1
2	sphere 2

Superscripts

*	dimensionless with respect to k_o

INTRODUCTION

The determination of the effective thermal conductivity k_{te} of a packed bed consisting of spherical particles has been extensively studied by various workers. One of the approaches used for k_{te} is the basic cell method (1-4), in which a single unit of the packed bed is analyzed, and taken as representing the entire bed. Yovanovich (2) and Ogniewicz (3) calculated the effective conductivity of the basic cell, and included the effect of the thermal resistance through the contact between the spheres. This resistance, coupled with the resistance of heat flow through the interstitial gap, yields the apparent conductivity of the cell. The model was compared against some experimental data (2,3) and predicted k_{te} accurately.

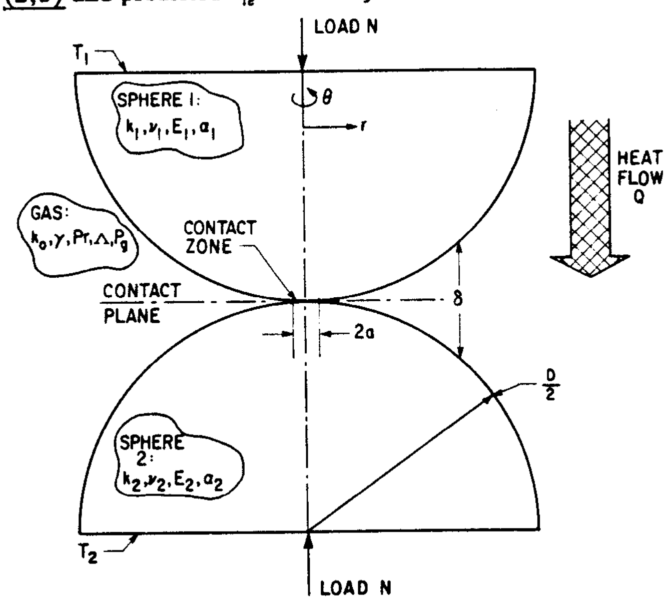

Figure 1
Simple Cubic Packing Basic Cell

The accuracy of the model, however, diminishes at high gas pressures: it overpredicts the apparent conductivity (underpredicts the total resistance). According to the experimental results of (3,5-7,10-12), the conductivity should approach a constant upper bound asymptotically. This paper presents a new modification to the Yovanovich-Ogniewicz (YO) model (2,3), which is different from the modification proposed in (3). The present modification includes the effect of the surface roughness of the particles (8,9). The roughness parameter Y, the separation between mean planes of the surfaces, alters the form of the integrated heat transfer expression, and improves the accuracy of the model at high gas pressures. The results of the modified models will also be compared with the experimental results of (3) and (10).

REVIEW OF YO MODEL

The simplest basic cell is a single unit of a simple cubic packed bed of spheres which comprises two smooth spheres in contact with each other as shown in Figure 1. A mechanical load N creates material deformation at the point of contact which results in a finite area contact spot of radius a. Gas occupies the space or gap between the spheres. A temperature gradient normal to the contact plane is imposed across the cell; heat flows through the cell via conduction in the solid spheres through the contact spot and via conduction through the gas in the gap, Figure 2.

In terms of the thermal resistance approach to heat transfer, the total heat flow through the basic cell is

$$Q_t = \frac{\Delta T_{12}}{R_t}, \tag{1}$$

where $\Delta T_{12} = T_1 - T_2$ is the temperature difference between the midplanes of the spheres. The resistance to heat flow in the gap is R_g, and R_c is the resistance at the contact; these are combined in the following relation for R_t:

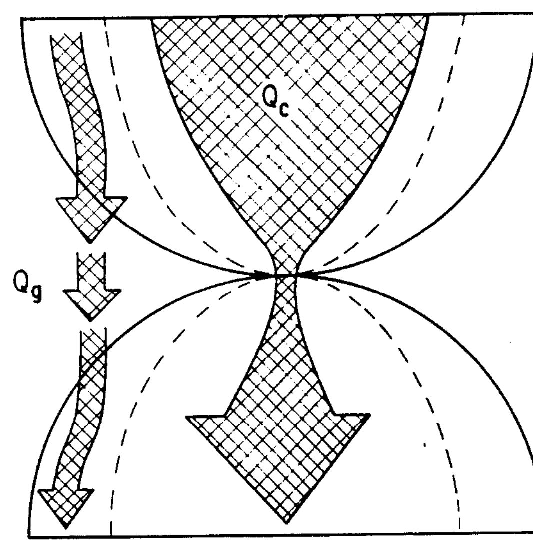

Figure 2
Heat Conduction in the Basic Cell

$$R_t = \left[\frac{1}{R_c} + \frac{1}{R_g}\right]^{-1} \tag{2}$$

The theory of heat transfer through the contact which connects the spheres is well documented, e.g. (15). The contact is assumed to be isothermal, in which case the resistance is

$$R_c = \frac{1}{2k_s a}, \tag{3}$$

where k_s is the harmonic mean conductivity of spheres 1 and 2:

$$k_s = \frac{2k_1k_2}{k_1 + k_2}. \tag{4}$$

It is assumed that $k_s = k_1 = k_2$. The contact radius a is found by applying the Hertz elastic contact theory, and depends on the normal load (see (14)):

$$a^3 = \frac{3}{4(1+\epsilon)} ND_1 \frac{1-\nu^2}{E}, \tag{5}$$

where $\epsilon = \frac{D_1}{D_2}$ is the size ratio of the spheres, and ν and E are material properties of the spheres. For equal sized particles, $\epsilon = 1$, and for sphere-flat contacts, $\epsilon = 0$. The resistance R_c is caused by the constriction or spreading of the heat flow lines through the contact, as well as the resistance to heat flow through the material of the spheres themselves.

The local rate of heat transfer through the gas in the gap is defined by the Fourier heat equation:

$$dQ_g = -k_g \frac{\partial T}{\partial z} dA. \tag{6}$$

where the z coordinate is in the direction of heat flow. The thermal conductivity of the gas is k_g. The gradient $-\partial T/\partial z$ is the temperature slope within the gas. It is assumed linear and is idealized as $\Delta T/\delta$, where δ is the gap width, and ΔT is the temperature drop between the surfaces of the spheres. The differential area dA is taken in circular polar co-ordinates where the line joining the centres of the two spheres is the central axis: $dA = 2\pi r dr$. The quantities k_g, ΔT and δ are each dependent on the radial location in the gap, and the total heat flow is an integral quantity:

$$Q_g = \int_A dQ_g = \int_A k_g(r) \frac{\Delta T(r)}{\delta(r)} dA. \tag{7}$$

The gas conductivity within the gap is

$$k_g = \frac{k_o}{1 + \frac{\alpha\beta\Lambda}{\delta}}, \tag{8}$$

where k_o is the gas conductivity at continuum conditions, α is the thermal accomodation coefficient between the gas and the sphere surface, β is a thermophysical parameter of the gas, and Λ is the mean free path of the gas. The mean free path varies with temperature and gas pressure according to the relation

$$\Lambda = \Lambda_o \left(\frac{T}{T_o}\right)\left(\frac{P_o}{P}\right),$$

where the subscript o denotes reference conditions. It is assumed that within the differential area dA, the opposing surfaces of the spheres may be considered parallel, and Eq. (8) is the expression for gas conductivity between parallel plates (see Kennard (13) and Reference (3)).

The temperature distribution is

$$\Delta T(r) = (T_1 - T_2)\frac{2}{\pi}\tan^{-1}\left(\left(\frac{r}{a}\right)^2 - 1\right)^{\frac{1}{2}}. \tag{9}$$

This is the temperature drop between the surfaces of two large bodies connected by a common contact spot, which is derived from the temperature distribution on the surface of a halfspace heated by an isothermal circular spot (see (14)). Since it is assumed that the contact spot is very small compared to the size of the spheres, the halfspace solution may be used. The nondimensional gap width $\delta(x)$ is

$$\delta\left(\frac{r}{a}\right) = \delta(x) = (L^2-1)^{\frac{1}{2}} - (L^2-x^2)^{\frac{1}{2}} + \frac{(L^2-\epsilon^2)^{\frac{1}{2}} - (L^2-\epsilon^2x^2)^{\frac{1}{2}}}{\epsilon} + \frac{(\epsilon+1)}{\pi L}\left[(2-x^2)\sin^{-1}\left(\frac{1}{x}\right) + \left(x^2-1\right)^{\frac{1}{2}} - \frac{\pi}{2}\right] \tag{10}$$

Substituting Eqs. (8) to (10) into (7), nondimensionalizing with respect to a, integrating from $x=1$ to $x=L$, and simplifying gives

$$Q_g = 2k_o a\Delta T_{12}\int_1^L \frac{2x\tan^{-1}(x^2-1)^{\frac{1}{2}}dx}{\delta + ML} = 2k_o a \Delta T_{12} I, \tag{11}$$

where $L = \frac{D}{2a}$, and ranges typically from $L=50$ (very heavy mechanical load) to $L=1000$ (light load). The parameter $M = \frac{2\alpha\beta\Lambda}{D}$ is called the gas parameter, which increases with temperature, and decreases with increasing gas pressure. The integral, which is represented by I, is evaluated by an adaptive numerical quadrature. The gap resistance is

$$\frac{1}{R_g} = \frac{Q_g}{\Delta T_{12}} = 2k_o a I. \tag{12}$$

Substituting Eqs. (3) and (12) into (2) gives the total resistance

$$\frac{1}{R_t} = 2k_o a\left(\frac{k_s}{k_o} + I\right). \qquad (13)$$

The total effective conductivity of the cell may be defined as follows:

$$k_{te} = \frac{l}{R_t A_{ht}},$$

where l and A_{ht} are representative heat transfer length and area. These are taken as D and D^2, respectively, so that

$$k_{te} = \frac{k_o}{L}\left(\frac{1}{K} + I\right), \qquad (14)$$

where $K = \frac{k_o}{k_s}$, the conductivity ratio.

Figure 3 displays the variation of dimensionless total effective conductivity $k^*_{te} = \frac{k_{te}}{k_o}$ versus the gas parameter M for various conductivity ratios K and for $L = 1000$. Figure 3 shows that the total conductivity increases with gas pressure, decreasing M. At low gas pressures, high M, the gas plays an insignificant role in the conductivity, and k^*_{te} reaches the vacuum value asymptotically, $k_{te,vac} = \frac{k_s}{L}$.

The variation of k_{te} with pressure is due to I, which varies with M. As M increases, I approaches 0. However, at high pressures, the model predicts that the conductivity should continue to increase almost linearly. Experimental results of (5-7) show that the conductivity levels off and approaches a constant value asymptotically.

The reason that the Ogniewicz model overpredicts I at high pressures is the assumption of perfectly smooth spheres in the derivation of $\delta(x)$, the gap width. In Eq. (11), the assumption of smooth spheres allows $\delta(x)$ to be equal to zero at the edge of the contact. Hence, there is a radial distance $x = \frac{r}{a}$ at which $\delta(x) \approx ML$. If M is small (high pressures), the denominator of the integrand is also small. As M becomes smaller (higher pressures), the denominator becomes equally small because $\delta(x)$ may be zero. The result is that the integrand becomes very large locally where $\delta(x) \approx ML$. In turn, the integral I grows as M decreases, causing k^*_{te} to increase proportionally according to Eq. (14).

The next two sections deal with two modifications to the original YO gap resistance model, which attempt to attempt to reduce the linear increase in I at high gas pressure. The first uses the concept of upper and lower bounds, and the combination of these bounds in some manner to obtain a more exact model. The second applies knowledge about the surfaces of the spheres to obtain a model based on further physical reasoning without resorting to model blending.

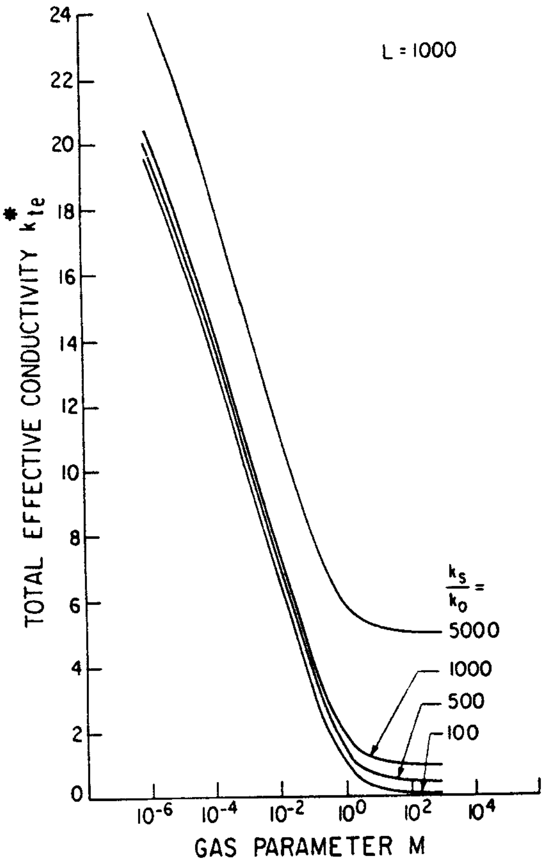

Figure 3
Total Effective Conductivity of the YO Model for $L = 1000$

MODIFICATION DUE TO OGNIEWICZ

A simplification made by YO was the assumption that the vacuum temperature distribution within the gap was valid in the case of a pressurized environment. It was reasoned that the presence of the interstitial gas would not alter the temperature field in the spheres because of the low gas conductivity. Initially, this assumption may seem to be an oversimplification, but a more physically realistic view may be less practical: consider Figure 2. The more realistic view is one in which the heat flow that eventually passes from the solid to the gas may be thought of as flowing through a thin region near the surface of the sphere. The dashed lines delineate this region from the region in which heat flows through the solid only, and are assumed to be adiabats. The depth of this skin layer is not known and would be difficult to determine. The size varies with such factors as the type of gas, the gas pressure, and the contact size: a higher gas pressure would tend to increase the layer depth because heat flow through the gas is made less resistive; a smaller contact zone would decrease the amount of heat flow through that path and increase the layer depth. The skin layer size has a direct effect on the temperature distribution since the vacuum temperature field would only be valid along the adiabats (same condition). However, it is not an oversimplification to assume that whatever the gas conditions, the depth of

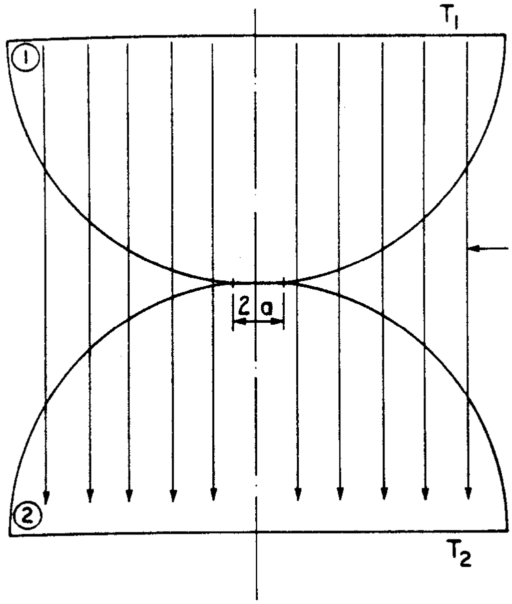

Figure 4
One Dimensional Heat Flow Model

this layer would be small. This is especially true very near the contact.

Ogniewicz attempted to modify the model for R_g by assuming that the true temperature distribution lies between the temperature distributions of the original YO model and the unidirectional flow model. The unidirectional heat flow model assumes that the heat flow within the basic cell is linear, Figure 4, and the gap resistance is

$$\frac{1}{R_g} = 2k_o a\pi \int_1^L \frac{x dx}{g(x)} = 2k_o a I_{1D} \tag{15a}$$

where

$$g(x) = (1-K)\delta(x) + K(2(L^2-1)^{\frac{1}{2}} - \frac{1}{L}) + ML. \tag{15b}$$

He reasoned that the two temperatures distributions were upper and lower bounds on the actual situation, and took the arithmetic mean temperature of the bounds to represent the true distribution. This leads to

$$I_{mod} = \frac{I + I_{1D}}{2} \tag{16}$$

The total effective conductivity becomes

$$k_{te}^{*} = \frac{1}{L}\left(\frac{k_s}{k_o} + \frac{I + I_{1D}}{2}\right). \tag{17}$$

Essentially, the modification attempts to include the effect of the skin layer depth which was described above. The temperature distribution will lie somewhere between the two bounds, but there is no justification from a physical standpoint to choose the arithmetic mean. This fixes the depth of the skin layer for all gas conditions. As explained above, the layer depth will change with varying conditions. If this approach is to be used, a better approximation is to attenuate each I by a factor f:

$$I_{modf} = fI + (1-f)I_{1D} \tag{18}$$

The attenuation factor f will be a function of the gas conditions, the load, and the conductivity ratio:

$$f = f(L, M, K). \tag{19}$$

Exactly how f varies with L, M, and K is not known nor investigated here. With the factor f, the total effective conductivity becomes

$$k_{te}^{*} = \frac{1}{L}\left(fI + (1-f)I_{1D} + \frac{k_s}{k_o}\right) \tag{20}$$

where Ogniewicz used $f = \frac{1}{2}$. Comparison of Ogniewicz's modification with the experimental data will be shown below.

INCORPORATION OF SPHERE SURFACE ROUGHNESS

A different modification to the model for k_{ge}^{*} assumes that the skin layer depth is small compared to the diameter of the particles. The vacuum temperature distribution is thus not altered significantly due to the presence of the gas. The modification involves adding another physical parameter to the analysis of particle geometry: the surface roughness. Yovanovich, DeVaal and Hegazy (9) summarize the surface geometry aspects of two rough conforming surfaces in contact. From their analysis, two parameters are important here: Y, the separation of the mean planes of the two contacting surfaces; and σ, the root mean squared effective roughness of the surfaces ($\sigma = (\sigma_1^2 + \sigma_2^2)^{\frac{1}{2}}$, where σ_1 and σ_2 are the roughnesses of the individual surfaces). These are shown in Figure 5.

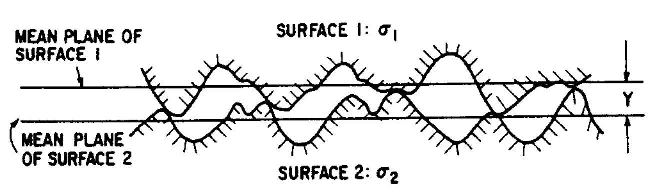

Figure 5
Profile View of Two Conforming Rough Surfaces in Contact

In the context of the work in (9), conforming means two flat surfaces in contact. Clearly, two spheres in contact are not conforming, however, the concepts of (9) may still be used, as illustrated in Figure 6. It is assumed that each surface has a roughness σ_1 and σ_2. When the two spheres touch, it is further assumed that the region within the contact is flat, i.e. the contact region is modelled as two rough conforming surfaces. The mean planes of the spheres (within the contact) are separated by a distance Y. Hence, at the edge of the contact, the gap width is Y, not zero as in the case of smooth spheres. The assumption of Hertzian contact is still maintained, except that it is applied to the mean planes of the spheres: the contact radius is a at the mean plane. Outside of the contact, the spheres are assumed smooth, and the expression for the gap width remains the same except for the addition of the distance Y. Thus, the total gap width at any radial distance r is

$$\text{gap width} = \delta(r) + Y \tag{21}$$

It should be cautioned that the assumption of flatness within the contact region is a simplification. Burde and Yovanovich (8) investigated the contact between a smooth sphere and a rough flat surface. The separation between the mean planes is not a constant as it appears here, but a function of the radial distance within the contact. However, it is assumed that $Y(r)$ at the edge of the contact as found by Burde and Yovanovich will not differ from Y used here by very much. Actually, the edge of the contact is a hazy zone for two rough spheres in contact; and despite the fact that the assumption of constant Y is not exact, it serves the purpose of reducing the near singularity in I around $x=1$.

The work of Yovanovich, DeVaal and Hegazy (9) dealt primarily with the heat transfer within the gap between the contacting surfaces. The work of Burde and Yovanovich (8) showed that resistance of the contact spot for the sphere-rough flat cell is the same as for the sphere-smooth flat contact. This is extended to the rough sphere-sphere cell, in which R_c is given by Eq. (3).

Returning to the gap resistance, substitution of Eq. (21) into (7) yields

$$\frac{1}{R_g} = 2k_o a \int_1^L \frac{2x\tan^{-1}(x^2-1)^{\frac{1}{2}}}{\delta(x) + \frac{Y}{a} + ML} = 2k_o a I_{R_m}. \tag{22}$$

The parameter Y is a dimensional quantity, and is often difficult to determine. In conforming rough surface theory, Y is usually obtained from the expression

$$\frac{p}{H} = \frac{A_r}{A_a} = \frac{1}{2}\text{erfc}\left(\frac{Y}{\sqrt{2}\sigma}\right) \tag{23}$$

where p/H is the ratio of the contact pressure to the material surface hardness, A_r is the total contact area of the deformed asperities, and A_a is the apparent contact area. The quantity $\frac{Y}{\sigma}$ is thus dependent on the mechanical load and is in the range $2 \le \frac{Y}{\sigma} \le 4$ for nominal loads where the lower limit corresponds to very heavy loads. The quantity $\frac{Y}{a}$ in Eq. (22) can be taken as a combination of other geometric parameters:

$$\frac{Y}{a} = \frac{Y}{\sigma} \cdot \frac{\sigma}{D} \cdot 2L. \tag{24}$$

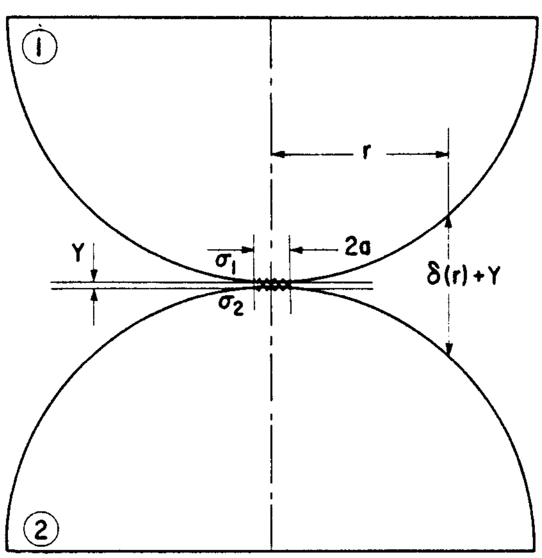

Figure 6
The Use of Conforming Surface Concepts
for Two Contacting Spheres

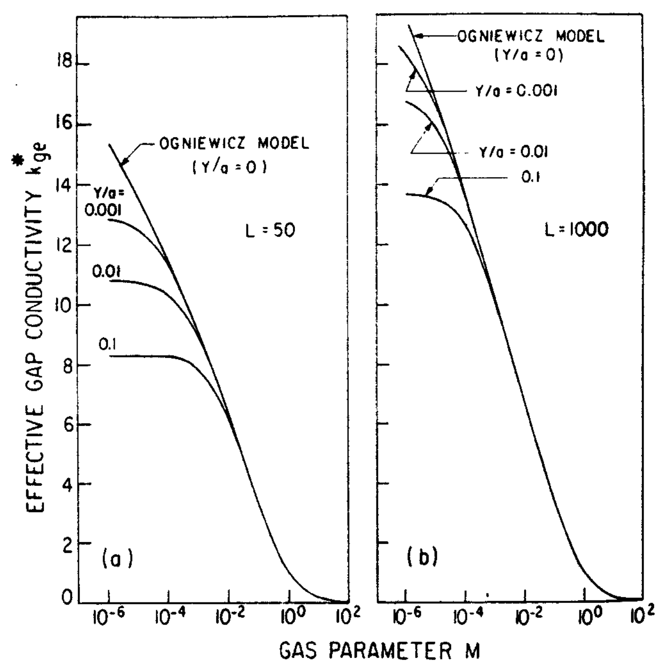

Figure 7
Effective Gap Conductivity for the Roughness Modified Model

Values of $\frac{Y}{a}$ for smooth spheres may be calculated using Eq. (24). Typically, for smooth spheres, $\sigma \approx 0.3 \mu m = 10 \mu in$. Light mechanical loads give $L=1000$ and $\frac{Y}{\sigma}=4$; and for heavy loads $L=50$ and $\frac{Y}{\sigma}=2$. Rougher surfaces tend to increase the limits on $\frac{Y}{a}$. The variation of $\frac{Y}{\sigma}$ with the load L is difficult to determine from Eq. (23), plus (23) includes another parameter: H, the hardness. It is difficult to determine the parametric range of $\frac{Y}{a}$, but, $0.001 \leq \frac{Y}{a} \leq 0.1$ will be used to show the behaviour of k_{ge}^{*} with the present modification.

Figure 7 displays $k_{ge}^{*}=\frac{I}{L}$ for the Ogniewicz model and the roughness modified model for $\frac{Y}{a}=0.1$, 0.01, and 0.001, and $L=50$ and 1000. Table 1 illustrates the data of Figure 7. Also included in the figure are the predictions of the original YO model. Notice that the modified model tends to decrease the effective gap conductivity at high pressures, small M. The degree to which k_{ge}^{*} is lowered depends on $\frac{Y}{a}$: as roughness increases, larger $\frac{Y}{a}$, the effective conductivity decreases for a given pressure. Also, k_{ge}^{*} here is not dependent on the conductivity ratio K, and thus the model cannot be directly compared with the blended model discussed in the previous section. The present modification has the desired effect of levelling k_{ge}^{*} at high pressures in agreement with the experimental results. An in depth analysis which investigates all the effects of the parameters is not presented here, but the results show that further attention is warranted.

COMPARISON WITH EXPERIMENTAL DATA

The models for resistance of a basic cell were compared with the experimental results of Kitscha (10) and Ogniewicz (3), who describe measurements of the total resistance of a specific basic cell. A complete description of the experimental apparati, as well as the material properties, may be found in the references. This section describes theoretical calculations of total resistance for the systems of (3) and (10) and compares them to the experimental measurements. The models used for comparisons were the original YO, Ogniewicz modified, and roughness modified models. The results were plotted for constant L against the gas parameter M. The experiments were performed on large diameter sphere cells, with sizes varying from 25.4 mm (1 inch) to 28.575 mm (1.125 inches). These are termed macrosphere cells.

The total resistance is

$$\frac{1}{R_t} = \frac{1}{R_c} + \frac{1}{R_g} + \frac{1}{R_r} \tag{25}$$

where R_r is the radiation resistance. For the purposes of this study, the resistances were nondimensionalized as follows:

$$R^{*} = k_s D R. \tag{26}$$

Using this definition

$$R_c^{*} = L, \tag{27a}$$

and

$$R_g^{*} = \frac{L}{KI}, \tag{27b}$$

Table 1
Effective Gap Conductivity for the Roughness Modified Model

Table 1a

M	k_{ge}^{*} ($L=50$), $\frac{Y}{a}=0$	0.001	0.01	0.1
10^{-6}	15.3326	12.8483	10.8043	8.3288
10^{-5}	13.4706	12.5426	10.7614	8.3235
10^{-4}	11.4678	11.2978	10.4070	8.2715
10^{-3}	9.1277	9.1049	8.9222	7.8417
10^{-2}	6.3174	6.3148	6.2916	6.0789
10^{-1}	3.3060	3.3057	3.3035	3.2813
10^{0}	0.9880	0.9880	0.9878	0.9866
10^{1}	0.1437	0.1437	0.1437	0.1437
10^{2}	0.0152	0.0152	0.0152	0.0152
10^{3}	0.0015	0.0015	0.0015	0.0015

Table 1b

M	k_{ge}^{*} ($L=1000$), $\frac{Y}{a}=0$	0.001	0.01	0.1
10^{-6}	19.4782	18.7372	16.7114	13.6880
10^{-5}	16.8323	16.7114	15.9343	13.5650
10^{-4}	13.7024	13.6880	13.5650	12.6930
10^{-3}	10.2816	10.2801	10.2665	10.1369
10^{-2}	6.7721	6.7719	6.7706	6.7576
10^{-1}	3.4392	3.4392	3.4391	3.4379
10^{0}	1.0162	1.0162	1.0162	1.0161
10^{1}	0.1474	0.1474	0.1474	0.1473
10^{2}	0.0156	0.0156	0.0156	0.0156
10^{3}	0.0016	0.0016	0.0016	0.0016

where I is given by Eqs. (11), (16) and (22). The expressions for radiation resistance R_r^* are given below.

Kitscha Macrosphere Test Results

Kitscha (10) performed experiments on the heat flow through 25.4 mm (1 inch) sphere-flat contacts in an air environment at various pressures, and different mechanical loads. For air, the gas parameter varies as follows:

$$M = \frac{1.3725\times10^{-4}}{D}\cdot\frac{T_c}{P_g} \quad (28)$$

where D is in cm, T_c in K, and P_g in mm Hg. The numerical portion of Eq. (28) is based on properties at $T_o=288$ and $P_o=760$ mm Hg. The values of T_c and P_g may be found in the Appendices of (10). The temperature $T_c = \frac{T_1+T_2}{2}$ is the extrapolated contact temperature. For the roughness modified model, the values of σ and $\frac{Y}{\sigma}$ were

$$\sigma = 0.3\mu m \ (10^{-5}\text{inches}), \quad (29a)$$

$$\frac{Y}{\sigma} = 4, \quad (29b)$$

and were held constant for all the calculations. The value of 4 for $\frac{Y}{\sigma}$ is slightly high for the loads used in the tests. Even though the values of σ and $\frac{Y}{\sigma}$ chosen may not be entirely accurate, they serve to illustrate the behaviour of the roughness modified model. The radiation resistance was calculated according to

$$R_r^* = \frac{k_s\left(\frac{1}{2}\frac{1-\epsilon_{sphere}}{\epsilon_{sphere}} + \frac{1-\epsilon_{flat}}{\epsilon_{flat}} + 1.1036\right)}{\pi\sigma D_{flat}T_c^3} \quad (30)$$

Table 2
Gap Resistance for the Kitscha Experiments

$L=115.1$	25.4 mm Sphere-Flat		Air		
T_c (K)	P_g (mm Hg)	M	R_g^* (YO)	R_g^* (YOM)	R_g^* (RM)
309.2	740.0	2.26×10^{-5}	73.3	86.2	83.4
309.0	400.0	4.17×10^{-5}	76.9	88.8	84.8
310.0	100.0	1.68×10^{-4}	87.4	96.2	91.3
311.0	40.0	4.20×10^{-4}	97.1	103.2	99.3
314.0	10.0	1.70×10^{-3}	118.3	120.8	119.2
316.0	4.4	3.88×10^{-3}	137.2	138.1	137.7
318.0	1.8	9.55×10^{-3}	167.2	166.9	167.6
321.0	0.6	2.89×10^{-2}	227.9	226.7	228.1
322.0	0.5	3.66×10^{-2}	246.6	245.1	246.8
325.0	0.2	8.78×10^{-2}	345.4	343.3	345.5

where ϵ is the emissivity and σ is the Stefan-Boltzmann constant in this context. The subscripts *flat* and *sphere* refer to the flat and sphere respectively.

Figure 8 plots the total dimensionless resistance versus M for each of the loads tested in air using the YO model and including the experimental results. Note that the total resistance is underpredicted at high pressure (low M). This is due to the underprediction of R_g^* by the YO model. Table 2 shows R_g^* for the three models. Figure 9 and Table 3 display R_t^* for $L=115.1$ calculated by the YO and modified models. They show clearly that the modified models decrease the discrepancy between the experimental data and the theory at high pressures. The Ogniewicz modified model slightly overpredicts the resistance, and the roughness model gives better agreement with the data. This is due to the roughness parameters chosen, which are not necessarily correct. Although the roughness model is somewhat unsophisticated, it may be improved by incorporating the work of Burde and Yovanovich (8).

Ogniewicz Macrosphere Test Results

Ogniewicz (3) conducted experiments to determine the resistance of a basic cell of a face centred cubic (FCC) packed bed in air for a range of loads. The FCC cell consists of one sphere resting in the space formed by three adjacent spheres. Ogniewicz assumed that Eq. (11) may be altered for a non-simple cubic cell by changing the upper limit of integration of I. For the FCC bed, the upper limit is $\frac{\sqrt{5}}{3}L = 0.7454L$.

For air, the gas parameter is

$$M = \frac{1.3567\times10^{-4}}{D}\cdot\frac{T_c}{P_g}. \quad (31)$$

In Reference (3), $D=1.125$ inches $=$ 2.8565 cm. The numerical value in Eq. (31) is slightly different from that in Eq. (28) because of differences in the values of α_1 and α_2.

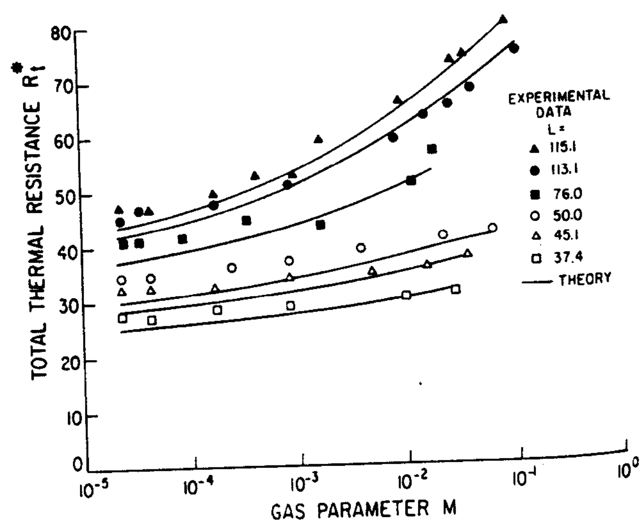

Figure 8
Total Resistance for Kitscha Air One Inch Tests (YO Model)

Table 3
Comparison of Models with Kitscha Experimental Results

M	R_r^*	R_t^* (YO)	R_t^* (YOM)	R_t^* (RM)	R_t^* (Expt.)
2.26×10^{-5}	1290.4	43.3	47.5	46.6	47.6
4.17×10^{-5}	1292.9	44.5	48.3	47.1	46.8
1.68×10^{-4}	1280.4	47.8	50.3	49.0	49.6
4.20×10^{-4}	1268.1	50.6	52.2	51.2	52.3
1.70×10^{-3}	1232.1	55.7	56.2	55.9	52.3
3.88×10^{-3}	1208.8	59.5	59.7	59.6	59.0
9.55×10^{-3}	1186.2	64.5	64.4	64.5	65.7
2.89×10^{-2}	1153.2	71.7	71.6	71.7	73.1
3.66×10^{-2}	1142.5	73.4	73.3	73.4	74.3
8.78×10^{-2}	1111.2	80.1	80.0	80.1	80.3

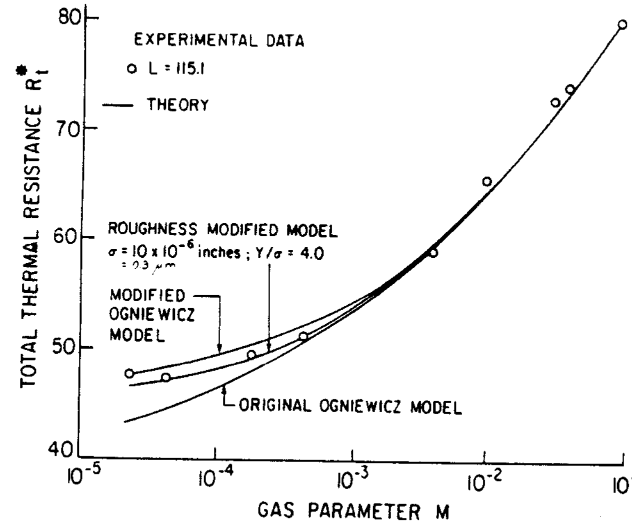

Figure 9
Total Resistance for Kitscha Tests (All Models) for $L=115.1$

The values of P_g are found in [3], but the temperatures T_c are not explicitly listed. However, the radiation resistance is given as

$$R_r = \frac{5.75\times10^{10}}{T_c^3}, \tag{32}$$

which is the resistance for the entire cell. The radiation resistance is listed in (3), hence T_c may be obtained from that.

Figure 10 illustrates the theoretical resistances versus pressure for all three loads tested using the YO model. The experimental results are also plotted. Again, the agreement between the experimental results and the theory is good, except at high pressures. Figure 11 and Table 5 display R_t^* for $L=200$, for which all three models for R_g were used (Table 4 lists R_g^*). These show that the modified models improve the agreement between theory and experiment at high pressures. There is better agreement between the Ogniewicz modified and the data, but it is also evident that the roughness modified model corrects the prediction in the proper direction. At heavier loads, the agreement of the YO theory with the data is better, and the modified models improve the prediction most at $L=200$.

Table 4
Gap Resistance for the Ogniewicz Experiments

$L=200$		FCC Basic Cell		Air	
T_c (K)	P_g (mm Hg)	M	R_g^* (YO)	R_g^* (YOM)	R_g^* (RM)
365.5	0.0	1.74×10^{2}	299674.9	298397.8	299675.0
362.5	0.009	1.91×10^{0}	3833.4	3817.2	3833.5
354.1	0.028	6.01×10^{-1}	1545.6	1539.0	1545.8
343.1	0.10	1.63×10^{-1}	711.1	708.2	711.2
333.0	0.29	5.45×10^{-2}	450.4	448.9	450.6
321.4	1.0	1.53×10^{-2}	310.0	310.1	310.4
315.8	3.0	5.00×10^{-3}	242.1	244.3	242.8
312.5	12.0	1.24×10^{-3}	190.4	197.4	192.0
310.3	32.0	4.60×10^{-4}	166.7	178.5	169.9
309.7	100.0	1.47×10^{-4}	146.6	163.7	152.9
308.7	300.0	4.89×10^{-5}	132.6	153.9	144.0
307.8	740.0	1.98×10^{-5}	123.9	147.7	140.5

CONCLUSIONS

It was shown that the YO model for the total resistance of basic cells in packed beds agrees with the experimental data of (3) and (10). However, the model underpredicts the resistance at high gas pressure due to the modelling of the gap resistance. Modifications to the R_g model improve the accuracy without affecting the pressure regimes in which the YO model is already accurate. The Ogniewicz modified model improves the accuracy to a greater extent than the roughness model. The Ogniewicz modified model blends

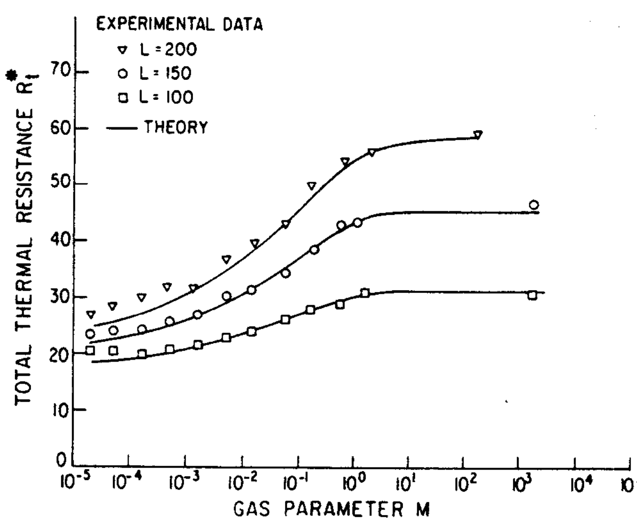

Figure 10
Total Resistance for Ogniewicz Air Tests (YO Model)

Table 5
Comparison of Models with Ogniewicz Experimental Results

M	R_r^*	R_t^* (YO)	R_t^* (YOM)	R_t^* (RM)	R_t^* (Expt.)
1.74×10^{2}	501.8	58.8	58.8	58.8	59.6
1.91×10^{0}	514.3	56.4	56.4	56.4	56.4
6.01×10^{-1}	551.5	53.3	53.3	53.3	54.7
1.63×10^{-1}	606.2	47.9	47.9	47.9	50.2
5.45×10^{-2}	663.3	43.2	43.1	43.2	43.2
1.53×10^{-2}	737.9	38.4	38.4	38.4	40.0
5.00×10^{-3}	777.6	34.9	35.0	34.9	37.0
1.24×10^{-3}	802.5	31.2	31.8	31.4	32.0
4.60×10^{-4}	819.8	29.2	30.3	29.5	31.8
1.47×10^{-4}	824.8	27.3	28.9	27.9	30.1
4.89×10^{-5}	832.3	25.8	28.0	27.0	28.6
1.98×10^{-5}	839.7	24.7	27.4	26.6	27.1

the YO model with the classic unidirectional heat flow model and is thus not based on physical interpretations of the system. The roughness modified model does take the physics into account—the surface roughness of the spheres—and is thus considered to be a better approach to improving R_g.

The roughness modification presented here is a primitive model, and yet has a visible effect in improving the accuracy. In addition, this modification is only a possible explanation for the discrepancy between the YO model and the experimental observations. Other effects, such as two dimensional heat flow within the gas gap, may also be important, and these should be investigated.

It is recommended that further work on the roughness modified model should be undertaken, and should include the work of Burde and Yovanovich (8) to upgrade the model. In particular, the definition of the edge of the contact spot should be clarified in order to better assess the lower limit of integration of the gap resistance. This also includes the determination of Y at the edge of the contact.

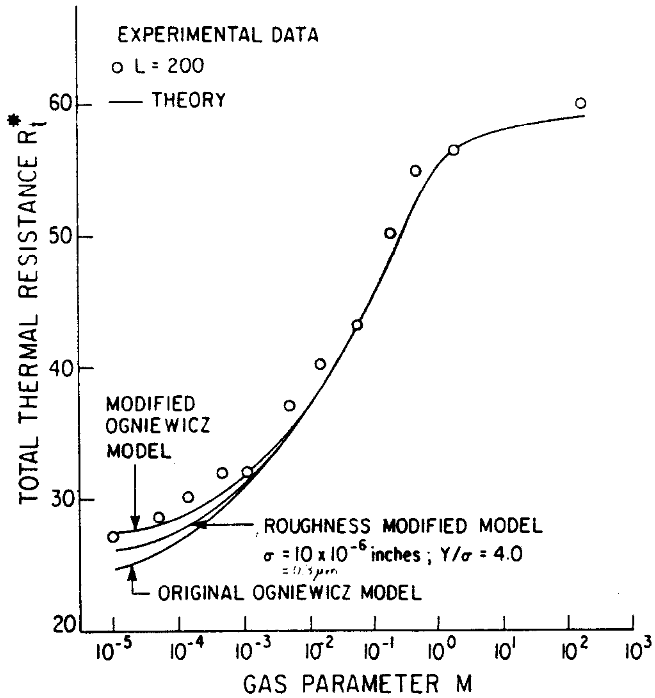

Figure 11
Total Resistance for Ogniewicz Tests (All Models) for $L=200$

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