Explicit Relative Contact Pressure Expression: Dependence Upon Surface Roughness Parameters And Vickers Microhardness Coefficients

S. Song[1] and M. M. Yovanovich[2]
Thermal Engineering Group
Department of Mechanical Engineering, University of Waterloo
Waterloo, Ontario

Abstract

An explicit expression for the contact hardness and the relative contact pressure is developed. This expression is simple and clearly shows the dependence of the contact hardness and the relative contact pressure upon the Vickers microhardness correlation coefficients and the surface roughness parameters. Parametric studies of the contact conductance are performed and it is shown that theoretical predictions and experimental data are in good agreement.

Nomenclature

a_c	mean contact radius (m)
A_a	apparent contact area (m^2)
c_1, c_2	constants for Vickers microhardness correlation
C	dimensionless conductance, $C = \dfrac{\sigma h}{m k_s}$
d_v	Vickers indentation diagonal (μm)
h	conductance $(W/m^2 \cdot K)$
H_c	contact microhardness (MPa)
H_v	Vickers microhardness (MPa)
k	thermal conductivity $(W/m \cdot K)$
k_s	harmonic mean thermal conductivity $(W/m \cdot K)$
m	mean absolute asperity slope
n_1, n_2	constants in Eq. (20) and Eq. (21)
M	gas parameter, $M = \left(\dfrac{2-\alpha_1}{\alpha_1} + \dfrac{2-\alpha_2}{\alpha_2}\right)\dfrac{2\gamma}{\gamma+1}\dfrac{1}{Pr}\Lambda$
P	apparent contact pressure (MPa)
Pr	Prandtl number
Q	heat flow rate (W)
s	constant in Eq. (18)
t	local gap thickness (m)
T	temperature (K)
Y	mean plane separation (m)

Greek Symbols

α	thermal accommodation coefficient
γ	ratio of specific heats
Λ	molecular mean free path (m)
σ	rms surface roughness (m)

Subscripts

$1,2$	two solids in contact
c	contact
g	gap
j	joint

Introduction

Heat transfer through the interfaces formed by the mechanical contact of two solids occurs in three forms: conduction through the contacting spots, conduction through the gas filled voids, and by radiation. Under normal conditions radiation effects are small compared to the other two, and therefore can be ignored. An important geometric parameter, which controls the rate of heat transfer through the contacting spots, is the ratio of actual to apparent areas of contact. This area ratio is determined by the relative contact pressure defined as the ratio of the applied pressure to the contact microhardness. The relative contact pressure also influences the effective thickness of the layers of gas entrapped in the interface voids, and thus directly affects the rate of gas conduction. A firm understanding of the nature of the relative contact pressure, therefore, is a prerequisite for studying the phenomenon of contact heat transfer.

For contacts of nominally flat but microscopically rough surfaces, Yovanovich et al. [1] developed an implicit geometric/mechanical model, which relates relative contact pressure with surface roughness characteristics and Vickers microhardness test results. This model allows one to estimate, through iterations, the contact microhardness and the relative contact pressure and thus to predict the rate of heat transfer across the interface. An explicit expression for relative contact pressure is, now, available, and it is the purpose of this paper to present its development. The explicit expression not only simplifies the calculation for contact heat transfer prediction, but readily reveals the quantitative relationships between the dependent and independent parameters. The latter part of this paper contains the analysis and verification, through experimental data, of the effects of various parameters on contact heat transfer.

[1] Graduate Research Assistant
[2] Professor of Mechanical Engineering, Associate Fellow AIAA

Review of Contact Conductances and Relative Contact Pressure

Contact, Gap, and Joint Conductance

Heat transfer through the interface of two nominally-flat surfaces, when radiation effects are neglected, takes the following form:

$$Q_j = Q_c + Q_g \qquad (1)$$

where Q_c, Q_g, Q_j are the rate of heat transfer through the total real contact area, through the interstitial gas layer, and the total rate of heat transfer, respectively. The conductance coefficients are introduced in the same manner as the film coefficient in convection heat transfer:

$$h_c = \frac{Q_c/A_a}{\Delta T}$$
$$h_g = \frac{Q_g/A_a}{\Delta T} \qquad (2)$$
$$h_j = \frac{Q_j/A_a}{\Delta T}$$

where h_c, h_g, h_j are the coefficients of contact, gap, and joint conductance, respectively, ΔT is the effective temperature difference across the interface, and A_a is the apparent contact area. One can use A_a for A_g in the definition of the gap conductance because the gap area is approximately equal to the apparent contact area. Dimensionless conductances are, then, defined as:

$$C_c = \frac{\sigma h_c}{m k_s}$$
$$C_g = \frac{\sigma h_g}{m k_s} \qquad (3)$$
$$C_j = \frac{\sigma h_j}{m k_s}$$

where C_c, C_g, C_j are the dimensionless contact, gap, and joint conductances, respectively. The parameters σ, m, k_s are, respectively, the effective rms surface roughness, the mean absolute asperity slope and the harmonic mean thermal conductivity of the two contacting solids. In terms of dimensionless conductances, Eq. (1) reduces to:

$$C_j = C_c + C_g \qquad (4)$$

Yovanovich [2] developed a simple, accurate correlation for the contact conductance model:

$$C_c = \frac{\sigma h_c}{m k_s} = 1.25 \left(\frac{P}{H_c}\right)^{0.95} \qquad (5)$$

where P is the apparent contact pressure, and H_c is the contact microhardness of the softer surface. The above expression for contact conductance is valid for $10^{-6} \leq P/H_c \leq 2.3 \cdot 10^{-2}$, and its agreement with experimental data was verified by Hegazy [3].

Yovanovich, DeVaal and Hegazy [4] also developed a gap conductance model, which takes into consideration the statistical nature of the contacting surfaces and the behavior of gases in very small gaps:

$$h_g = \frac{k_g}{\sigma\sqrt{2\pi}} \int_0^\infty \frac{exp[-(\frac{Y}{\sigma} - \frac{t}{\sigma})^2/2]}{\frac{t}{\sigma} + \frac{M}{\sigma}} d\left(\frac{t}{\sigma}\right) \qquad (6)$$

where k_g, Y and t are the gas conductivity, the mean plane separation, and the local gap thickness, respectively. The gas parameter, M, is defined as:

$$M = \left(\frac{2-\alpha_1}{\alpha_1} + \frac{2-\alpha_2}{\alpha_2}\right) \frac{2\gamma}{\gamma+1} \frac{1}{Pr} \Lambda \qquad (7)$$

where α_1 and α_2 are the thermal accommodation coefficients of the two surfaces and γ, Pr and Λ are the ratio of specific heats, the Prandtl number and the molecular mean free path, respectively. The mean plane separation, Y, is related to the relative contact pressure P/H_c by [2]:

$$\frac{Y}{\sigma} = \sqrt{2} erfc^{-1}\left(\frac{2P}{H_c}\right) \qquad (8)$$

Relative Contact Pressure and Contact Microhardness

Relative contact pressure is defined as P/H_c, the ratio of apparent applied pressure to contact microhardness. Its influence on contact heat transfer is clearly exhibited by Eqs. (5), (6) and (8). Physically, the ratio P/H_c controls three geometric elements important in contact heat transfer: contact spot density, mean contact spot radius, and the separation distance of the mean planes of the two contacting surfaces.

Contact microhardness H_c, by itself, depends on several parameters: mean surface roughness, the mean absolute slope of the asperities, the type of materials, the method of surface preparation and the applied pressure. Sufficient information regarding the type of material and the surface hardness characteristics can be introduced into the calculation of relative contact pressure in the form of Vickers microhardness correlations corresponding to a range of contact pressures. Vickers microhardness tests are performed for a range of indentation diagonals. Typical results of such tests for several different materials are shown in Fig. 1. The results of Vickers hardness tests can be correlated in a power form[3]:

$$H_v = c_1 d_v^{c_2} \qquad (9)$$

where H_v is Vickers microhardness in MPa, d_v, mean indentation diagonal in μm, and c_1, c_2 are the correlation coefficients.

In the computation of contact microhardness, an assumption is made that the contact microhardness of the surface being penetrated by the asperities of the harder surface is the same as the Vickers microhardness corresponding to the equivalent Vickers indentation diagonal[1,3]:

$$H_c = c_1 d_v^{c_2} \qquad (10)$$

and,

$$d_v = \sqrt{2\pi} \cdot 10^6 a_c \quad (11)$$

where a_c is the mean contact spot radius in m. The Vickers indentation diagonal d_v in Eq. (9) is measured in μm, and thus the conversion factor 10^6 is introduced in Eq. (11). The mean contact spot radius, in turn, is related to the relative mean plane separation, Y/σ[5]:

$$a_c = \sqrt{\frac{8}{\pi}\frac{\sigma}{m}} exp\left(\frac{Y}{\sqrt{2}\sigma}\right)^2 erfc\left(\frac{Y}{\sqrt{2}\sigma}\right) \quad (12)$$

As mentioned previously, the relative mean plane separation depends upon the relative contact pressure, as shown in Eq. (8). Finally, Eqs. (8) and (10) through (12) form a set of 4 nonlinear equations, which defines a unique value of H_c. Once the values for P, c_1, c_2, and σ/m are given, this set of equations can be solved by iteration for H_c or P/H_c.

It is evident from the above brief review that the implicit formulation does not permit one to ascertain directly the effect of the contact pressure, the surface roughness parameters and the Vickers microhardness correlations upon the contact and gap conductances. Any parametric study of the joint conductance requires the computation of the contact and gap conductances which requires iteration upon the relative contact pressure. It is therefore necessary to develop an explicit relationship which will permit direct computation of the relative contact pressure and the joint conductance. The explicit expression will give additional physical insights into the effect of the pressure, surface roughness parameters and the Vickers microhardness distribution upon the relative contact pressure. Also, it will allow one to compute efficiently the relative contact pressure whenever it is required.

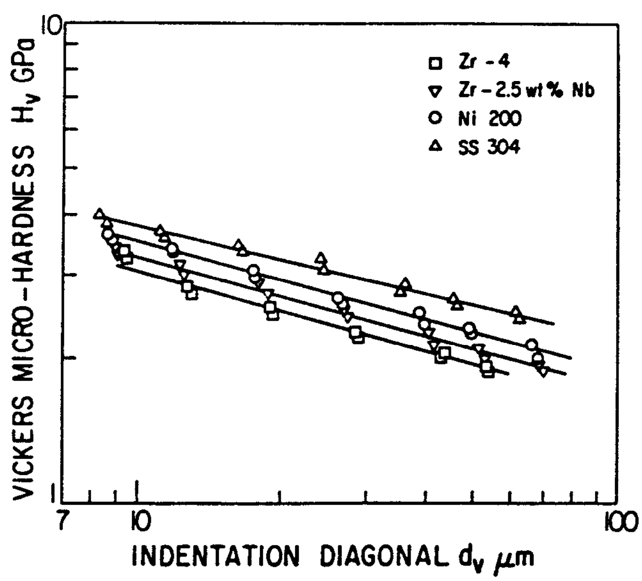

Fig. 1 Vickers microhardness variation of different materials with indentation diagonal, Hegazy[3].

Explicit Expression for Relative Contact Pressure

Explicit Expression Development

Equations (8) and (10) through (12) can be combined to result in the following implicit equation for the relative contact pressure, P/H_c:

$$\frac{P}{H_c} = \frac{P}{c_1}\left[\frac{4 \cdot 10^6 \sigma}{m}\right]^{-c_2}\left[exp\left\{\left(erfc^{-1}\left(\frac{2P}{H_c}\right)\right)^2\right\}\left\{\frac{2P}{H_c}\right\}\right]^{-c_2} \quad (13)$$

Collecting terms, and taking the natural logarithm of both sides, the equation becomes:

$$ln\left(\frac{P}{H_c}\right) + \frac{c_2}{c_2+1}\left\{erfc^{-1}\left(\frac{2P}{H_c}\right)\right\}^2 = \frac{1}{c_2+1} ln\left\{\frac{P}{c_1}\left(\frac{8 \cdot 10^6 \sigma}{m}\right)^{-c_2}\right\} \quad (14)$$

The $erfc^{-1}(2P/H_c)$ term, appearing in the above equation, is approximated by the following accurate expression:

$$erfc^{-1}\left(\frac{2P}{H_c}\right) \simeq 0.9638\left[-ln\left(5.589\frac{P}{H_c}\right)\right]^{\frac{1}{2}} \quad (15)$$

This approximation is accurate to the maximum relative error of 2 percent for the range $10^{-6} \le P/H_c \le 2 \cdot 10^{-2}$ (see Table 1). Finally, substituting Eq. (15) for $erfc^{-1}(2P/H_c)$ in Eq. (14) and manipulating terms leads to the following explicit expression:

$$\frac{P}{H_c} = \left[\frac{P}{c_1\left(\frac{1.62 \cdot 10^6 \sigma}{m}\right)^{c_2}}\right]^{\frac{1}{1+0.071c_2}} \quad (16)$$

which is valid for $10^{-6} \le P/H_c \le 2 \cdot 10^{-2}$. Equation (16) clearly reveals the quantitative dependence of the relative contact pressure upon the geometric and mechanical parameters σ, m, c_1, c_2, and P. The relative contact pressure depends on the apparent pressure as $P/H_c \propto P^{\frac{1}{1+0.071c_2}}$. Thus the power coefficient c_2 of the Vickers microhardness correlations solely influences the pressure dependence of P/H_c.

Table 1 Accuracy of $erfc^{-1}(2P/H_c)$ approximation

P/H_c	$erfc^{-1}(2P/H_c)$	Eq. (15)	% diff.
$1 \cdot 10^{-6}$	3.362	3.352	0.3
$5 \cdot 10^{-6}$	3.123	3.121	0.1
$1 \cdot 10^{-5}$	3.016	3.017	0.0
$5 \cdot 10^{-5}$	2.751	2.757	−0.2
$1 \cdot 10^{-4}$	2.630	2.638	−0.3
$5 \cdot 10^{-4}$	2.327	2.337	−0.5
$1 \cdot 10^{-3}$	2.185	2.195	−0.5
$5 \cdot 10^{-3}$	1.822	1.823	−0.1
$1 \cdot 10^{-2}$	1.645	1.637	0.5
$2 \cdot 10^{-2}$	1.452	1.427	1.8

It is interesting to note that the remaining parameters σ, m and c_1 appear with c_2 as a group, $c_1\left(\frac{1.62\cdot 10^6\sigma}{m}\right)^{c_2}$. This term, in comparison with Eq. (10), may be thought of as some characteristic microhardness representing the specific surface condition of a work hardened material. And the relative contact pressure is related to this characteristic microhardness, in conjuction with the apparent pressure, by the power coefficient $\frac{1}{1+0.07c_2}$. Equation (16) also suggests that for materials with $c_2 = 0$ (such as some annealed aluminum alloys) the contact microhardness has a constant value c_1.

Comparison of Explicit Approximation with Implicit Iterative Expression

Differences in the calculation of relative contact pressure by the two expressions arise solely from the approximation made for the inverse complementary error function, $erfc^{-1}(2P/H_c)$. In this section the two expressions will be compared for the calculation of relative contact pressure P/H_c, contact conductance C_c, and relative mean plane separation Y/σ. To simplify the task of the comparison, some reduction in the number of parameters can be made. Hegazy [3] showed that the power coefficient c_2 of the Vickers microhardness correlation for a number of materials which he studied can be fixed to a value of -0.26 without introducing a significant error. Table 2 shows typical values of c_1 and c_2 obtained for several different materials[3]. Also, Eq. (16) suggests that the remaining four parameters may be grouped into two: P/c_1 and σ/m.

Table 3 shows the percent difference in the values of relative contact pressure computed by the two expressions. The maximum difference is -3.4 percent and it occurs when $P/c_1 = 0.01$ and $\sigma/m = 100\times 10^{-6}m$. In most practical cases, however, the difference is less than 2 percent. The range of σ/m chosen represents the practical range of σ, $0.1 - 50\times 10^{-6}m$, and m, $0.05 - 0.2$, for conforming rough surfaces.

Table 2 Vickers Microhardness Correlations, $H_v = c_1 d_v^{c_2}$ (Hegazy[3])

material	c_1 (MPa)	c_2	% diff. max	% diff. rms
Zr–4	5367	−0.26	3.9	1.8
Zr–2.5wt%Nb	5750	−0.26	9.7	2.7
Ni200	6217	−0.26	5.2	1.8
SS304	6906	−0.26	5.9	2.4

Table 3 Percent difference of P/H_c ($c_2 = -0.26$)

P/c_1	$\sigma/m \times 10^6$: 0.1	1	10	100
$5\cdot 10^{-7}$	−2.9	−2.3	−1.8	−1.3
$1\cdot 10^{-6}$	−2.2	−1.7	−1.2	−0.7
$1\cdot 10^{-5}$	−0.4	0.0	0.4	0.7
$1\cdot 10^{-4}$	0.9	1.1	1.3	1.4
$1\cdot 10^{-3}$	1.3	1.2	1.0	0.6
$1\cdot 10^{-2}$	0.1	−0.7	−1.8	−3.4

Table 4 shows the the difference in predicted values of contact conductance, C_c, using Eq. (5). Again the maximum difference occurs at $P/c_1 = 0.01$ and $\sigma/m = 100\times 10^{-6}m$, and in general the difference is less than 2 percent.

The difference in the predicted values of relative mean plane separation, Y/σ is even smaller, as shown in Table 5. In all cases examined, the difference is less than 1 percent.

Dependence of Contact Conductance upon Surface and Microhardness Parameters

Theoretical Prediction

The explicit expression for relative contact pressure, Eq. (16), makes it possible to readily examine the effect on contact conductance of its parameters. Substituting Eq. (16) for P/H_c in Eq. (5) yields a contact conductance expression in terms of measurable surface and mechanical parameters:

$$C_c = \frac{h_c\sigma}{k_s m} = 1.25\left[\frac{P}{c_1\left(\frac{1.62\cdot 10^6\sigma}{m}\right)^{c_2}}\right]^{\frac{0.95}{1+0.071c_2}} \tag{17}$$

Defining, for convenience, $s = \frac{0.95}{1+0.071c_2}$, we obtain

$$\frac{h_c}{k_s} = 1.25(1.62\cdot 10^6)^{-c_2 s}\left(\frac{P}{c_1}\right)^s\left(\frac{\sigma}{m}\right)^{-(c_2 s+1)} \tag{18}$$

As mentioned previously, for a number of materials c_2, one of the two Vickers microhardness correlation coefficients, can be set to a value of -0.26[3], and thus $s = 0.97$ and $c_2 s = -0.25$. Then Eq. (18) reduces to the semi-general expression:

$$\frac{h_c}{k_s} = 44.6\left(\frac{P}{c_1}\right)^{0.97}\left(\frac{\sigma}{m}\right)^{-0.75} \tag{19}$$

Table 4 Percent difference of C_c ($c_2 = -0.26$)

P/c_1	$\sigma/m \times 10^6$: 0.1	1	10	100
$5\cdot 10^{-7}$	−2.7	−2.2	−1.7	−1.2
$1\cdot 10^{-6}$	−2.1	−1.6	−1.1	−0.7
$1\cdot 10^{-5}$	−0.4	0.0	0.4	0.7
$1\cdot 10^{-4}$	0.9	1.1	1.2	1.3
$1\cdot 10^{-3}$	1.3	1.2	0.9	0.5
$1\cdot 10^{-2}$	0.1	−0.7	−1.7	−3.2

Table 5 Percent difference of Y/σ ($c_2 = -0.26$)

P/c_1	$\sigma/m \times 10^6$: 0.1	1	10	100
$5\cdot 10^{-7}$	0.1	0.1	0.1	0.1
$1\cdot 10^{-6}$	0.1	0.1	0.1	0.0
$1\cdot 10^{-5}$	0.1	0.0	0.0	0.0
$1\cdot 10^{-4}$	−0.1	−0.1	−0.1	−0.1
$1\cdot 10^{-3}$	−0.1	−0.1	−0.1	−0.1
$1\cdot 10^{-2}$	0.0	0.1	0.4	0.9

The contact conductance, as one would expect, increases with the applied contact pressure. Equation (19) predicts that for the materials with $c_2 = -0.26$ the power index for the proportionality of h_c with respect to P is 0.97. The contact conductance increases also with the mean asperity slope but decreases with the rms roughness at the power index of 0.75.

Verification of Pressure Dependence by Experimental Data

For a given material and surface characteristics (c_1, σ, m), the contact conductance would be directly proportional to the applied contact pressure, and the degree of dependence may be approximately expressed as:

$$\frac{h_c}{k_s} \propto P^{n_1} \qquad (20)$$

Equation (19) predicts the power constant, n_1 to be equal to $s = 0.97$ for materials with $c_2 = -0.26$. Table 6 shows the values of n_1 obtained from the experimental data by Hegazy[3]. It is seen from the table that the values of n_1 obtained for the four different materials (Nickel 200, Stainless Steel 304, Zircaloy–4, Zr–2.5wt%Nb) for the range of σ/m, 6 to 60 $\times 10^{-6} m$ and P/c_1, 10^{-4} to 2×10^{-3} agree quite well with the predicted value of 0.97. Estimates of n_1 from the contact conductance data reported by other authors are also in agreement with the predicted value; these are shown in Table 7.

Verification of Surface Roughness Dependence by Experimental Data

For a given material and under a specific contact pressure, the dependence of contact conductance on surface roughness parameters can be approximately expressed as:

$$\frac{h_c}{k_s} \propto \left(\frac{\sigma}{m}\right)^{n_2} \qquad (21)$$

Theory predicts, according to Eq. (18), that $n_2 = -(c_2 s + 1)$, and thus for materials with $c_2 = -0.26$ the power coefficient, n_2, is -0.75. The values of n_2 obtained from the experimental data of Hegazy[3] are shown in Table 8. Again the agreement between the predicted and experimental values is found to be very good.

Table 6 Experimental values of power constant, n_1 (Hegazy[3])

contact materials	power constant, n_1
Ni200 pair	0.95
SS304 pair	0.97
Zr–4 pair	0.94
Zr–2.5wt%Nb pair	0.95

Table 7 Power constant, n_1, obtained from various authors

contact materials	power constant, n_1	authors
UO_2 — Zr-2	0.93	Ross & Stoute[6]
aluminum pair	0.95	Boeschoten &
Al — Fe	0.99	Van Der Held[7]
steel pair	0.97	Shlykov &
duralumin pair	0.94	Ganin[8]
copper pair	1.05	

Table 8 Experimental values of power constant, n_2 (Hegazy[3])

contact materials	power constant, n_2
Nickel 200 pair	−0.76
Stainless Steel 304 pair	−0.77
Zircaloy–4 pair	−0.76
Zr–2.5wt%Nb pair	−0.75

Conclusion

An explicit expression for relative contact pressure has been developed which considerably simplifies the prediction of contact heat transfer. The difference in the computed values of relative contact pressure between the implicit and the explicit expression is negligible. Furthermore, the explicit expression allows parametric studies of contact heat transfer. The parametric study performed on contact conductance shows that theoretical predictions and experimental data are in good agreement.

Acknowledgements

The authors acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada under operating grant A7455 for Dr. Yovanovich.

References

[1] Yovanovich, M.M., Hegazy, A.H., and DeVaal, J., *Surface Hardness Distribution Effects Upon Contact, Gap and Joint Conductances,* Paper presented at AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, June 7–11, 1982, St. Louis, Missouri.

[2] Yovanovich, M.M., *Thermal Contact Correlations,* Spacecraft Radiative Transfer and Temperature Control, ed. by T.E. Horton, Vol. 83 of Progress in Astronautics and Aeronautics, New York, 1982.

[3] Hegazy, A.A., *Thermal Joint Conductances of Conforming Rough Surfaces: Effects of Surface Micro-Hardness Variation,* Ph.D Thesis, Department of Mechanical Engineering, University of Waterloo, 1985.

[4] Yovanovich, M.M., DeVaal, J., Hegazy, A.H., *A Statistical Model to Predict Thermal Gap Conductance Between Conforming Rough Surfaces,* Paper presented at AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference, June 7–11, 1982 St. Louis, Missouri.

[5] Mikic, B.B., *Analytical Studies of Contact of Nominally Flat Surfaces; Effect of Previous Loading,* Journal of Lubrication Technology, Vol. 93, No. 4, 1971, pp. 451–459.

[6] Ross, A.M., and Stoute, R.L., *Heat Transfer Coefficient between UO_2 and Zircaloy-2,* Report CRFD-1075, Atomic Energy of Canada Ltd., Chalk River, Ontario, June 1962.

[7] Boeschoten, F., and Van Der Held, E.F.M., *The Thermal Conductance of Contacts between Aluminum and Other Metals,* Phsyca, Vol. 23, pp. 37–44, 1957.

[8] Shlykov, Y.U.P, and Ganin, Y.E.A., *Thermal Resistance of Metallic Contacts,* Int. J. Heat Mass Transfer, Vol. 1, pp. 921–929, 1964.