Correlation of Thermal Parameters Influencing the Differences between One- and Two-Dimensional Conduction within PCB'S

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ABSTRACT

Steady-state, one and two-dimensional heat conduction within microelectronic PCB's depends upon many thermophysical and geometric parameters such as the board's thermal conductivity, thickness and length; the number of surface heat sources, their lengths and locations on the board; the magnitude and distribution of the heat flux over each source; and the air inlet temperature and velocity.

The magnitude of differences between one and two-dimensional source temperatures is of importance because one-dimensional analyses or numerical studies are much simpler, faster, and more economical than fully two-dimensional studies.

One and two-dimensional models are considered and how they are used to determine the top and bottom surface temperatures of PCB's. The temperature difference of interest is the average difference over heat sources between the two-dimensional top surface temperature and the one-dimensional temperature. Temperature differences are computed for a wide range of thermal and geometric parameters for one heat source placed on the surface of the PCB and is nondimensionalized and correlated against several dimensionless groups. The correlation of the dimensionless temperature difference with the dimensionless fluid properties permits the analyst to quickly and accurately estimate one or two-dimensional analyses and it also provides a correction for the one dimensional model which may be used to approximate the two-dimensional solution.

1.0 INTRODUCTION

The thermal analysis of microelectronic circuit boards can be very complicated unless several assumptions are used to simplify the analysis procedure. Regardless of the nature of the analysis, be it finite difference, finite element, analytical or a combination of techniques, the level of uncertainty introduced by each assumption must be weighed against the speed of computation and the ease of formulating the problem. For most microelectronic applications involving forced convection above and below the circuit board a steady-state, two-dimensional analysis can be used to calculate the surface temperature at any position parallel to the fluid flow direction without significantly

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sacrificing the accuracy of the solution. Many techniques may be used to solve Laplace's two dimensional equation, as in Negus and Yovanovich (1986), to determine the temperature distribution within the circuit board, but specification of the boundary conditions at the fluid/solid interface are restricted to either a Dirichlet, Neumann or a Robin condition but not a combination of the three. Simeza(1986) has developed a Boundary Integral Equation Method(BIEM) which allows the boundary condition along the surface of the circuit board to be specified alternately as Robin and Neumann conditions. A solution to the conjugate heat transfer can then be obtained by iterating between the BIEM solution and a boundary layer solution.

A further simplification of the problem can be made for applications where the Biot number given as $(h_0 + h_1)t/k_s$ is less than 0.2. In this instance, the internal resistance to heat flow$(t/k_s A)$ is much less than the external resistance to heat flow$(1/(h_0 + h_1)A)$ and a one-dimensional solution to Laplace's equation can be used. Since the resistance within the solid is small, the temperature difference between the upper and lower surface is small and the mean cross sectional temperature is a good estimate of the surface temperature of the circuit board. Culham and Yovanovich(1987) have presented a non-iterative technique for calculating the cross sectional temperature in flat plates with boundary conditions alternating between Robin and Neumann conditions.

This paper presents a method for relaxing the limitation of the one- dimensional analysis provided that $Bi < 0.2$. A correction factor is given which can be added to the mean cross sectional temperature to give an accurate estimate of the surface temperature in applications where the Biot number is as high as 4.0.

1.1 The One-Dimensional Model

Steady-state heat conduction within a material with no internal sources is described by Laplace's equation:

$$\nabla^2 T = 0 \tag{1}$$

If a Neumann (flux specified) boundary condition is specified at one boundary it can be shown that the maximum temperature will also occur at that boundary. Similarily, it can be shown that the minimum temperature will occur at the opposite boundary. The difference between the two surface temperatures becomes small when the internal thermal resistance of the solid material is much less than the thermal resistance at the convection boundary, ie, $Bi < 0.2$. As the Biot number becomes small, the two surface temperatures can be approximated by a single mean value of the cross sectional temperature.

The temperature field within a flat plate can be defined in terms of the distance from the leading edge of the plate as

$$T(x) - T_\infty = C_1 e^{-mx} + C_2 e^{+mx} + \frac{n}{m^2} \tag{2}$$

where

$$m^2 \; = \; \frac{h_0 + h_1}{k_s t} \tag{3}$$

$$n \; =_j \; \frac{q + h_0(T_{f_0} - T_{f_1})}{k_s t} \tag{4}$$

and $q = 0$ for non-source sections. C_1 and C_2 are constants of integration which can be determined by applying boundary conditions at the leading and trailing edge of the board and the interior interfaces between each source and non-source section.

The conduction solution is readily coupled with a boundary layer solution based upon the integral form of the energy equation. A unique surface temperature distribution can be determined by iterating between the conduction and the boundary layer solution. The advantage of this technique is the speed in which an accurate solution of the surface temperature distribution can be computed.

1.2 The Two-Dimensional Model

Simeza(1986) has developed a method for solving Laplace's equation in a two dimensional domain using a BIEM technique. The method allows the cross sectional temperature to be calculated at any position within a flat plate where the boundary conditions alternate between regions where the heat flux is specified and regions where the film coefficient is specified. Using a method similar to that described in the one dimensional model, the BIEM technique can be coupled with a boundary layer analysis to give an iterative model which can be used to predict surface temperatures in the presence of conjugate heat transfer. The method is valid for any value of the Biot number, however the time required to compute surface temperatures is roughly 12 times greater than the one dimensional model. Also, the matrix algebra involved in the BIEM is memory intensive and can exceed the hardware capability especially when using a personnal computer.

2.0 PARAMETRIC STUDY

The BIEM necessitates solving two fully populated N × N matrices where N is the total number of elements chosen over the surface of the plate. Since the programing language selected for the computations contained in this paper was IBM-BASIC, the total addressable memory was restricted to one 64 kilobyte block. This limited the total number of boundary elements to 70 and, therefore, a board geometry as shown in Fig. 1 was selected.

The input parameters, as given in Table 1, where selected to cover a range which would include most applications found in the microelectronic industry.

TABLE 1. Range of Input Parameters

0.2	<	k	<	8.0	W/m K
2.5	<	U_∞	<	10.0	m/s
500	<	q	<	3000	W/m^2
0.001	<	t	<	0.005	m

The objective of this study is to find a correlation in terms of dimensionless groups which allows the difference between the average heat source temperature calculated using the one-dimensional and the two-dimensional models to be correlated using a single relationship for the full range of input data presented above. As an initial attempt the dimensionless temperature given as

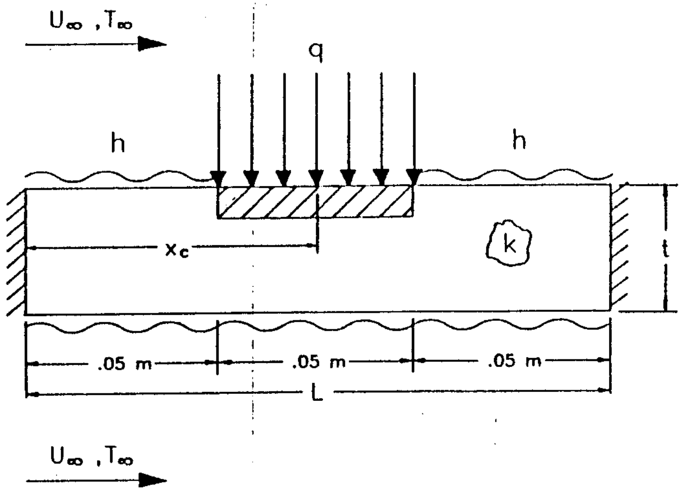

FIGURE 1. PCB Test Configuration

$$\theta = \frac{(T_{2D} - T_{1D})\, h}{q} \tag{5}$$

was correlated versus the average Biot number over the heat source region. But as shown in Fig. 2, the relationship between the dimensionless temperature and the Biot number did not take the full effect of the plate thickness into consideration. A second relationship including the aspect ratio of the flat plate(L/t) is plotted in Fig. 3 where the abscissa is given as $Bi(L/t)^{0.1}$. The dimensionless temperature collapses onto a single curve for the range of $0.2 < Bi(L/t)^{0.1} < 3.6$. The curve shown in Fig. 3 can be correlated to give

$$\theta = 0.3014 \left[\ln\left(0.27Bi(L/t)^{0.1} + 1\right)\right]^{0.94} \tag{6}$$

Equation 6 is applicable for a single heat source located symmetrically on a flat plate as shown in Fig. 1 where the design conditions are varied over the range of data given in Table 1. The maximum relative error between the calculated (using the BIEM model) and the predicted temperature difference as given by Eqn. 6 was observed to be 5.6 %. Moving the location of the heat source away from the center of the PCB introduced more error in the predicted values. Therefore, an improved correlation which includes the effects of the heat source location is given as

$$\theta = 0.3014 \left[\ln\left(0.27Bi(2)^{0.1}(x_c/t)^{0.1} + 1\right)\right]^{0.94} \tag{7}$$

where x_c is the position of the center of the heat source. For an extreme case of $Bi = 3.8$ and the heat source leading edge 0.01 m from the PCB leading edge, the improvement of Eqn. 7 reduced the relative error from 15% to 4%.

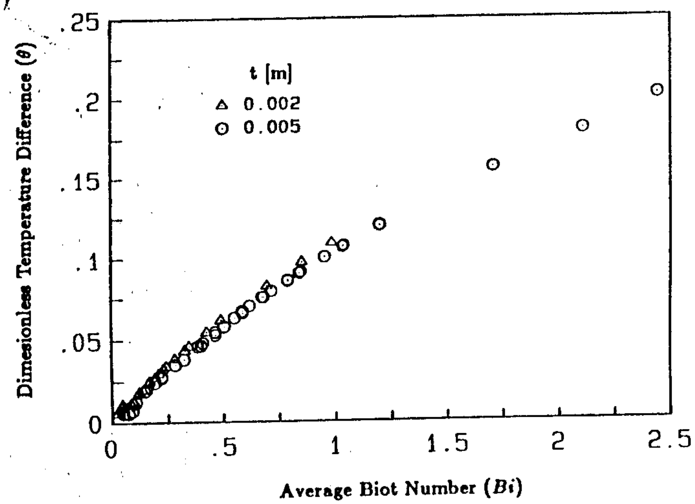

FIGURE 2. Dimensionless Temperature Difference (θ) vs. Average Biot Number (Bi)

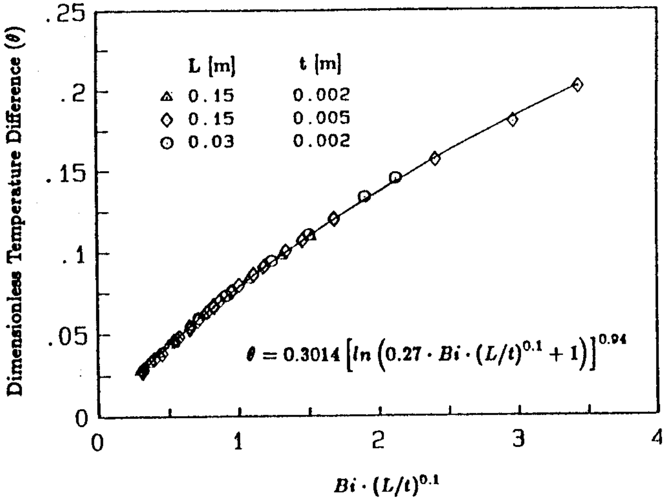

FIGURE 3. Correlation of Dimensionless Temperature Difference (θ)

3.0 DISCUSSION

The correlation presented in Eqn.7 provides a correction on temperature that can be added to the 1-D model results to obtain a better estimate of the temperature profile over a heat source. Implied in the correlation is that the 1-D model be executed first in order to obtain the average h over the heat source. With h, the average difference between the 1-D and 2-D temperature over a heat source can be determined.

To model a PCB as 1-D or 2-D depends largely on the accuracy required by the analyst. For high accuracy requirements, a 2-D solution is always required. However, in the preliminary design stages of PCB's, quick, rough estimates are more useful, necessitating the use of 1-D models.

A means for establishing the validity of the 1-D assumption can be done by defining a new dimensionless temperature of the form

$$\phi = \frac{(T_{2D} - T_{1D})}{(T_{max_{1D}} - T_\infty)} \tag{8}$$

An arbitrary limit on ϕ can be set which defines values for which the 1-D assumption is acceptable or not acceptable. If the limit on ϕ is set to 0.04, then for any $\phi \leq 0.04$ the 1-D assumption is acceptable. In terms of temperatures, this implies that if the temperature difference is less than 1°C, and the maximum temperature excess is 25°C over the heat source, the 1-D approximation is valid. The limit on ϕ can be arbitrarily varied to suit the particular design application at hand.

For the numerous cases run, ϕ was less than 0.04 for all Bi < 0.2 as seen on Figure 4. Therefore, if a designer can tolerate a temperature error of 1°C on the front face of the board for each 25°C of maximum temperature excess, then the 1-D assumption is valid and sufficiently accurate. However, the main purpose of this analysis was to provide a correction for the temperature over a heat source when 2-D effects prevail. When the Biot number over a heat source is large, then it is much faster to execute the 1-D solution and then add on the average correction temperature to obtain a better estimation of the temperature profile with the 2-D effects accounted for. The advantage of this correlation analysis is that in validating the 1-D assumption, it also provides the error in so doing and allows 2-D results to be estimated without actually executing a complete 2-D model.

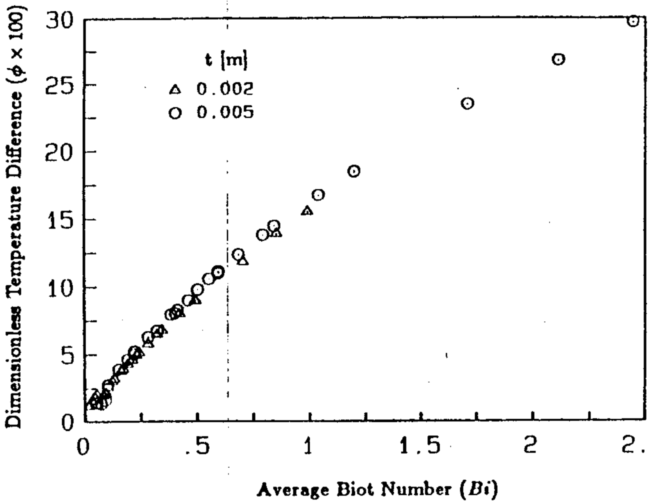

FIGURE 4. Dimensionless Temperature Difference (ϕ) vs. Average Biot Number (Bi)

From this study the parameters affecting the difference between the 2-D and the 1-D model are observed to be Bi, and x_c/t. The Biot number has the strongest effect on the temperature difference. As a rough guide, it was observed that for all $Bi < 0.2$ the 1-D assumption is valid based on a limit on ϕ of 0.04.

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NOMENCLATURE

Bi - average heat source Biot number, $(h_0 + h_1)t/k_s$
C_1, C_2 - constants of integration
h - average heat source film coefficient, W/m^2 K
k - board thermal conductivity, W/m K
L - total board length, m
m - as in Eqn.3
n - as in Eqn.4
q - total heat flux, W/m^2
t - board thickness, m
T - average heat source temperature, K
T_{max} - maximum heat source temperature, K
U - fluid velocity, m/s
x - distance from leading edge, m

Greek Symbols

ϕ - dimensionless temperature, $(T_{2D} - T_{1D})/(T_{max_{1D}} - T_\infty)$
θ - dimensionless temperature, $(T_{2D} - T_{1D})h/q$

Subscripts

f - fluid
c - center of heat source
s - solid
0 - back side of board
1 - front side of board
$1D$ - one-dimensional
$2D$ - two-dimensional
∞ - free stream

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The Effect of Metal Powder on Thermal Contact Resistance

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ABSTRACT

Thermal contact resistance between two pieces of metal in contact formed a measurable temperature difference across the interface. It could be reduced by putting interstitial material like thermal grease into the gap. The present study tries to further promote the thermal contact conductance by adding metal powder into the grease. The results show that within 30 weight percentage of the metal powder content, thermal contact resistance could decrease steadily from the value for pure thermal grease to less than half of that value. Further increase of the metal powder content makes the mixture too dry to be uniformly applied on the surface and the mixture begin to lose its advantage.

INTRODUCTION

Conduction heat transfer through composite walls and cylinders have normally been idealized where there is no resistance to the flow of heat at the interfaces between the walls and cylinders in contact. However, due to the surface roughness, a thermal contact resistance to heat flow does exist at the region of the interface. Heat transfer across an interface formed by two contacting bodies is hence accompanied by a measurable temperature difference across the interface.

In modern electronic equipment, where high reliability is of such importance, many physical contacts exist between the electronic components and the heat dissipation surfaces. Such contacts with accompanying thermal resistance can cause excessive component temperature. It is clear that an electronic equipment designer should be able to evaluate and then minimize this contact resistance. A rather complete survey of the problem has been performed during the past decades [1, 2, 3]. Unfortunately, there is no satisfactory theory which will predict thermal contact resistance for all types of engineering applications, nor have experimental studies yielded completely reliable empirical correlations. It is understandable because of the many complex surface affecting parameters are encountered in practice. Thus the contact resistance problem still plagued the engineers in the electronic industry.