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Laminar free convective heat transfer from isothermal spheres: a new analytical method

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Abstract—A simple but accurate approximate analytical method based on a linearization of the energy equation is developed for the area mean Nusselt number for free convection heat transfer from isothermal spheres for the range of Rayleigh number $0 \leqslant Ra \leqslant 10^8$ and all Prandtl numbers. In the process of linearization, the energy equation is reduced to the form of the transient heat conduction equation for which the solution exists. Comparison of the final correlation of Nu against Ra (which is an explicit form of linear superposition of the diffusive limit and boundary layer solution) with other correlation and experimental air data reveals very good agreement with a maximum difference of less than 5%.

INTRODUCTION

LAMINAR natural convection heat transfer from isothermal spheres has been the subject of numerous analytical, computational and experimental investigations. This is due to the fact that a sphere presents an important geometry for the study of free convection flow in many engineering applications (spherical storage tanks, packed beds of spherical bodies, etc.). Many of the above mentioned investigations have been reviewed by Clift *et al.* [1] and Churchill [2], and also in a recent paper by Lo Choy and Yovanovich [3].

In general, the experimental approach has the capability of supplying realistic heat transfer data; however, it is time consuming and expensive. Numerical solutions are also expensive and require a wide computational effort to be implemented. Since obtaining complete exact solutions of the boundary layer momentum and energy equations presents an impossible task for general cases, theoretical studies of free convection have mainly been focused on approximate solutions. They include integral [4, 5] and perturbation [6, 7] methods, and asymptotic solutions for small and large Grashof, Gr, and Prandtl, Pr, numbers [8, 9].

While such theoretical studies have produced valuable and interesting results, they are limited either to the diffusive regime ($GrPr = Ra < 10^{-8}$; see Yovanovich [10]) or to the range of Rayleigh numbers for which the postulate of laminar boundary layer theory is applicable ($10^4 < Ra < 10^8$; see Merk and Prins [4] and Chiang *et al.* [11]). Although a few researchers have tried to extend their schemes for the transition from the diffusive regime to the laminar regime ($10^{-4} < Ra < 10^4$; see Lo Choy and Yovanovich [3]), they have had less success in obtaining the final result in an explicit form.

The objective of this study is to present a new approximate analytical method to predict external natural convection heat transfer from isothermal spheres for a wide range of Rayleigh numbers $0 \leqslant Ra \leqslant 10^8$ and all Prandtl numbers, $0 < Pr < \infty$. The present study will adopt a simple analytical method based on linearization of the energy equation as well as the usual assumptions made for natural convection problems, i.e. the boundary layer and Boussinesq approximations. In the process of linearization, the energy equation will be reduced to the form of the transient heat conduction equation. These simplifications make otherwise intractable equations amenable to analysis and open a new door to the solution of external natural convection problems.

THEORETICAL ANALYSIS

In processes dealing with free convective heat transfer, the temperature field is linked with the flow; therefore the nonlinear momentum and energy equations are coupled through the variation of the density. To allow a more convenient procedure for obtaining approximate solutions to these nonlinear, coupled governing equations, several simplifying assumptions and various approximation schemes will be used. Boussinesq and boundary layer approximations, as mentioned earlier, are the two widely used simplifying approximations in modeling natural convection heat transfer.

Another method of simplification, which will be exploited in this work, involves modification of the inertial (convective) terms, i.e. $\mathbf{V}\cdot\nabla(\ )$. Oseen first suggested that the inertial terms $\rho\mathbf{V}\cdot\nabla\mathbf{V}$ in the momentum equation could be uniformly approximated by the term $\rho\mathbf{V}_\infty\cdot\nabla\mathbf{V}$ [12]. Following this linearization, Oseen was able to obtain an approximate solution to the Navier–Stokes equations for creeping flow past a sphere. Subsequently, several researchers have worked on Oseen's equations to obtain:

NOMENCLATURE

A	sphere surface area [m^2]
$\mathscr{A}$	constant coefficient
c_p	specific heat at constant pressure [kJ kg^{-1} K^{-1}]
D	sphere diameter [m]
$f(Pr)$	function of Pr
g	gravitational acceleration [m s^{-2}]
$Gr_{\mathscr{L}}$	Grashof number, $g\beta(T_s - T_\infty)\mathscr{L}^3/\nu^2$
h	heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
$\mathscr{L}$	characteristic length of the body [m]
n	constant power
$Nu_{\mathscr{L}}$	Nusselt number, $h\mathscr{L}/k$
Pr	Prandtl number, ν/α
q	heat flux [W m^{-2}]
$Ra_{\mathscr{L}}$	Rayleigh number, $Gr_{\mathscr{L}}Pr$
t	time [s]
T	temperature [K]
ΔT	temperature rise, $T - T_\infty$ [K]
T^*	dimensionless temperature rise, $(T-T_\infty)/(T_s - T_\infty)$
v_i	velocity component in ith direction (i denotes different coordinate directions) [m s^{-1}]
V	velocity [m s^{-1}]
V_c	characteristic velocity given by equation (13) or (21) [m s^{-1}]
V_e	effective velocity given by equation (27) [m s^{-1}]
x	distance along the surface [m].

Greek symbols

α	thermal diffusivity, $k/\rho c_p$ [m^2 s^{-1}]
β	volumetric expansion coefficient [K^{-1}]
δ_t	thermal boundary layer thickness [m]
μ	dynamic viscosity [kg m^{-1} s^{-1}]
ν	kinematic viscosity, μ/ρ [m^2 s^{-1}]
ρ	mass density [kg m^{-3}].

Subscripts

D	based on D, as the characteristic length
$\mathscr{L}$	based on $\mathscr{L}$, as the characteristic length
s	at the surface
∞	at points far from the body.

Superscripts

0	at Ra or $Pr \to 0$
-	an averaged quantity
∞	at very large Prandtl numbers.

Coordinates

x, y, z	Cartesian coordinates
r, θ, φ	spherical coordinates.

- results with higher order approximations [13, 14];
- solutions of forced convection heat transfer from isothermal spheres and cylinders [15–17]; and
- a matched solution of inner and outer expansions of their perturbation schemes in free convection [6, 18].

In this work, the extension of the Oseen modification to the convection terms of the energy equation in free convection will be studied.

Linearization of energy equation

Consider an isothermal sphere of temperature T_s, and diameter D, which is immersed in an extensive, quiescent medium at constant temperature T_∞ as shown in Fig. 1. For this time-steady, small scale problem with zero heat generation and relatively large temperature difference, for which the Eckert number can be taken as zero, one can write the simplified constant property energy equation in the spherical coordinate system (r, θ, φ) as follows:

$$\rho c_p\left[v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\varphi}{r\sin\theta}\frac{\partial T}{\partial \varphi}\right] = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\varphi^2}\right]. \quad (1)$$

Neglecting conduction in the θ-direction, and using the equality

$$\frac{\partial T}{\partial\varphi} = \frac{\partial^2 T}{\partial\varphi^2} = 0 \quad (2)$$

due to the symmetry of the problem, the above energy equation becomes

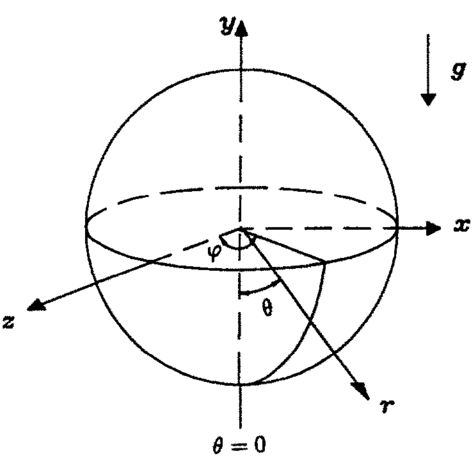

FIG. 1. Spherical polar coordinate system.

$$\rho c_p\left[v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}\right]=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right]. \quad (3)$$

The two terms on the left-hand side of equation (3) are further approximated by a single equivalent term, $(V_e/r)(\partial T/\partial\theta)$, where V_e is some constant effective velocity yet to be determined. Therefore, the *linearized energy equation* can be written as

$$\rho c_p\frac{V_e}{r}\frac{\partial T}{\partial \theta}=k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right] \quad (4)$$

in the region:

$$r \geqslant D/2, \quad 0 \leqslant \theta \leqslant \pi.$$

Transient conduction-type equation and its solution

For large Grashof numbers ($Gr > 10^4$), heat transfer occurs in a thin boundary layer, therefore r in the advection term appearing on the left-hand side of equation (4) can be approximated by $D/2$. Let $k/\rho c_p = \alpha$, thermal diffusivity; $x = D\theta/2$, distance along the surface from the stagnation point, and $t = x/V_e = D\theta/2V_e$, the effective particle residence time. The linearized energy equation, equation (4), can therefore be written as

$$\frac{1}{\alpha}\frac{\partial T^*}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T^*}{\partial r}\right) \quad (5)$$

where

$$r \geqslant D/2, \quad 0 \leqslant t \leqslant \pi D/2V_e$$

and T^* is the dimensionless temperature rise, $(T-T_\infty)/(T_s-T_\infty)$. The solution to equation (5) is [19]

$$T^*=\frac{D}{2}\frac{1}{r}\operatorname{erfc}\left(\frac{r-D/2}{2\sqrt{(\alpha t)}}\right) \quad (6)$$

which consists of the linear superposition of the steady-state solution, $D/2r$, and the transient solution. The local Nusselt number, $Nu_D(\theta)$, defined as $h(\theta)D/k$, is related to the local wall heat flux and the overall temperature rise as follows:

$$Nu_D(\theta)=\frac{q_s D}{(T_s-T_\infty)k}. \quad (7)$$

The wall heat flux, q_s, is determined from the Fourier rate equation

$$q_s=-k(T_s-T_\infty)\left.\frac{\partial T^*}{\partial r}\right|_{r=D/2} \quad (8)$$

which gives

$$q_s=\frac{k(T_s-T_\infty)}{D/2}+\frac{1}{\sqrt{\pi}}\frac{k(T_s-T_\infty)}{\sqrt{(\alpha t)}}. \quad (9)$$

The wall heat flux, equation (9), consists of the linear sum of two asymptotes: the steady-state value and the transient solution for a half-space. Substituting for q_s in equation (7), and setting $t = D\theta/2V_e$, one obtains the local Nusselt number:

$$Nu_D(\theta)=2+\sqrt{\left(\frac{2}{\pi}\right)}\sqrt{\left(\frac{DV_e}{\alpha\theta}\right)}. \quad (10)$$

Thus, the area-averaged Nusselt number, $Nu_D = (1/2)\int_0^\pi Nu_D(\theta)\sin\theta\,d\theta$, becomes:

$$Nu_D=2+0.714\sqrt{\left(\frac{DV_e}{\alpha}\right)}. \quad (11)$$

It is worthwhile noting that equation (11) at this point is applicable to both forced and free convection heat transfer. This, of course, depends on the type of transformation equation to be used for the effective velocity, V_e, which will be discussed next.

Transformation equation for effective velocity, V_e

The type of transformation equation to be used for the velocity, V_e, which is a characteristic of the flow field around the body, will determine the type of convective heat transfer correlation that results. Before proceeding to the analysis of the problem and the procedure to obtain the constant effective velocity, let us see what this velocity actually represents. In the case of forced convection, clearly, the effective velocity could be some fraction of the 'free stream' velocity which is considered constant and uniform (or averaged) over the surface of the body. However, in free convective boundary layer problems, although there is no such thing as 'free stream' velocity (or it is actually zero), this effective velocity could still represent some constant velocity which may be the result of averaging the velocity over the boundary layer thickness as well as over the surface of the body. Of interest to this study is free convective heat transfer from spheres, the development of the velocity transformation equation which will be discussed now.

Assuming $Gr_D > 10^4$, the fluid motion is confined to a thin boundary layer; therefore the basic equation for this part of the analysis is the boundary layer momentum equation (θ-component) expressed in spherical coordinates [20]:

$$v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}=\nu\frac{\partial^2 v_\theta}{\partial r^2}+g\beta\sin\theta(T-T_\infty). \quad (12)$$

Previous works on free convection suggest that the investigation be split into two different studies. One study should be concerned with very small Prandtl numbers ($Pr \ll 1$), while the other should pertain to very large Prandtl numbers ($Pr \gg 1$). In each part, first the characteristic velocity for each limit will be given and then the possibility of using this characteristic velocity as the effective velocity will be investigated through a simple analysis.

Small Prandtl numbers ($Pr \ll 1$). For very small Prandtl numbers buoyancy and inertial forces must be of the same order of magnitude; thus one can show through scale analysis that the characteristic velocity is [21, 22]

$$V_c = \sqrt{(g\beta\Delta TD)}. \tag{13}$$

Since the thermal boundary layer is much thicker than the velocity boundary layer for cases of very small Prandtl numbers, ΔT is close to $(T_s - T_\infty)$. This is more obvious if one divides the whole boundary layer into two regions—*inner and outer regions*—and uses the so-called double-boundary layer concept [23].

On the other hand, one can start with the momentum equation, equation (12), and attempt to find the velocity averaged over the surface of the sphere. Then, for cases of very small Prandtl numbers, a balance of buoyancy and inertial forces simplifies equation (12) to

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} = g\beta\Delta T \sin\theta. \tag{14}$$

If the two terms on the left-hand side of equation (14) are approximated by the single equivalent term, $(v_\theta/(D/2))(\mathrm{d}v_\theta/\mathrm{d}\theta)$, one may use the following steps to get the area-averaged velocity, $\bar{v}$:

$$\frac{v_\theta}{D/2}\frac{\mathrm{d}v_\theta}{\mathrm{d}\theta} = g\beta\Delta T \sin\theta \tag{15}$$

$$\frac{\mathrm{d}v_\theta^2}{\mathrm{d}\theta} = g\beta\Delta TD \sin\theta. \tag{16}$$

Integration of equation (16) with the condition that $v_\theta = 0$ at $\theta = 0$ gives:

$$v_\theta^2 = g\beta\Delta TD \int_0^\theta \sin\theta \,\mathrm{d}\theta \tag{17}$$

or

$$v_\theta = \sqrt{(g\beta\Delta TD)}(1-\cos\theta)^{1/2}. \tag{18}$$

Taking the average over the total surface area,

$$\bar{v} = \frac{1}{A}\iint_A v_\theta \,\mathrm{d}A, \tag{19}$$

gives

$$\bar{v} = 0.943\sqrt{(g\beta\Delta TD)} \tag{20}$$

which is almost the same as V_c, equation (13), found above. Thus, using the characteristic velocity instead of the effective velocity required in equation (11) could be a reasonable estimate of the effective velocity for very small Prandtl numbers.

Large Prandtl numbers ($Pr \gg 1$). For very large Prandtl numbers buoyancy forces must be of the same order of magnitude as viscous forces; then one can obtain the following expression for the characteristic velocity through scale analysis [21, 22]:

$$V_c = \sqrt{\left(\frac{g\beta\Delta TD}{Pr}\right)}. \tag{21}$$

In this case, the velocity boundary layer is much thicker than the temperature boundary layer; in the temperature boundary layer the velocity grows rapidly and reaches its maximum value near the outer edge of the temperature boundary layer. Thus, ΔT would be between 0 and $(T_s - T_\infty)$ in this inner region (the region between zero and maximum velocity), or taking its mean value $\frac{1}{2}(T_s - T_\infty)$. This inner region is known to be the driving region, which is why it is considered here.

To see how well this characteristic velocity represents some kind of averaged velocity over the surface, we will follow Merk and Prins' [4] argument and equate the buoyancy force to the viscous drag for large Prandtl numbers for a slab of unit height of the boundary layer. The buoyancy force in the boundary layer is proportional to the temperature difference. Taking its mean value as $\frac{1}{2}(T_s - T_\infty)$, the upward force on the slab, supposed to have approximately the thickness δ_t, is $\frac{1}{2}\rho g\beta(T_s - T_\infty)\delta_t$ when $\sin\theta$ is not very different from 1. However, as was pointed out earlier, the velocity v decreases almost linearly in the thermal boundary layer to its zero value at the wall, where it causes a drag $\mu\bar{v}/\delta_t$ (which is an approximation for $\mu(\partial v/\partial r)$ at the wall). Equating the two forces for steady flow one obtains

$$\bar{v} = \tfrac{1}{2}g\beta(T_s - T_\infty)\delta_t^2/\nu. \tag{22}$$

Merk and Prins [4] suggested the following equation for δ_t by considering the diffusion of heat perpendicular to the wall into the fluid moving with an average upward velocity $\bar{v}$:

$$\delta_t^2 \approx \alpha t = \alpha\frac{D}{\bar{v}} = \frac{\nu}{\bar{v}}\frac{D}{Pr}. \tag{23}$$

Substituting for δ_t^2 in equation (22) gives

$$\bar{v} = \sqrt{\left(\frac{g\beta(T_s - T_\infty)D}{2Pr}\right)} \tag{24}$$

which is the same as equation (21) found for the characteristic velocity, V_c, when $\frac{1}{2}(T_s - T_\infty)$ is substituted for ΔT.

Effective velocity for all Prandtl numbers. Recall equations (13) and (21) obtained for the two limits:

$$Pr \to 0, \quad V_c^0 = \sqrt{(g\beta\Delta TD)} = \frac{\nu}{D}Gr_D^{1/2}$$

and

$$Pr \to \infty, \quad V_c^\infty = \sqrt{\left(\frac{g\beta\Delta TD}{Pr}\right)} = \frac{\nu}{D}Gr_D^{1/2}(2Pr)^{-1/2}.$$

Based on the above equations, valid for small and large Prandtl numbers, and using the Churchill and Usagi [24] blending technique, one may obtain the following equation for the effective velocity applicable for all Prandtl numbers:

$$\frac{1}{(V_e)^n} = \frac{1}{(V_e^0)^n} + \frac{1}{(V_e^\infty)^n}. \tag{25}$$

Therefore, the effective velocity can be determined from the following blended equation:

$$V_e = \frac{V_e^\infty}{\left[1+\left(\frac{V_e^\infty}{V_e^0}\right)^n\right]^{1/n}}. \tag{26}$$

Substituting for V_e^0 and V_e^∞ into equation (26) gives the effective velocity valid for all Prandtl numbers

$$V_e = \frac{\frac{\nu}{D}(Gr_D/2Pr)^{1/2}}{\left[1+\left(\frac{0.5}{Pr}\right)^{n/2}\right]^{1/n}} \tag{27}$$

where a value of 9/8 can be used for n as recommended by Churchill and Chu [25] for their $f(Pr)$.

RESULTS AND DISCUSSION

Substituting the effective velocity of equation (27) into equation (11), the area-averaged Nusselt number, Nu_D, becomes

$$Nu_D = 2 + \frac{0.600 Ra_D^{1/4}}{\left[1+\left(\frac{0.5}{Pr}\right)^{9/16}\right]^{4/9}} \tag{28}$$

where a value of 9/8 is used for n as mentioned earlier.

A few remarks should be made regarding the form and the constants of equation (28). First of all, the form of the equation, namely

$$Nu_{\mathscr{L}} = Nu_{\mathscr{L}}^0 + \mathscr{A} f(Pr) Ra_{\mathscr{L}}^{1/4} \tag{29}$$

was not forced to this format to fit the experimental data [26] or to have $Nu_{\mathscr{L}}^0$ as a correction to account for curvature effects [27]. Rather, this is simply the explicit form of the end result of the present analysis.

Moreover, it has the correct diffusive limit, $Nu_D^0 = 2.0$, representing the contribution of the molecular diffusion into an infinite, stagnant fluid which corresponds to Rayleigh numbers approaching zero.

Now let us examine the asymptotic values of the area-averaged Nusselt number for small and large Prandtl numbers in the laminar boundary layer regime. From equation (28), one can obtain

$$Nu_D = 0.714 Ra_D^{1/4} Pr^{1/4} \quad Pr \to 0 \tag{30}$$

and

$$Nu_D = 0.600 Ra_D^{1/4} \quad Pr \to \infty. \tag{31}$$

These limiting values for Nu_D are independent of the value of n.

Clearly, the two equations for Nu_D found above are consistent with the results of dimensional analyses for small and large Prandtl numbers [23]. In addition, the constant of 0.600 (for $Pr \to \infty$) and 0.714 (for $Pr \to 0$) are in good agreement with previously found asymptotic values. Churchill [2] reported 0.603 (Churchill's calculation was based on the analysis of Chiang *et al.* [11]) and Stewart [28] 0.589 for large Prandtl numbers, while for small Prandtl numbers, Churchill [2] has 0.727 in his correlation and the Raithby and Hollands [29] correlation gives 0.703 as the asymptotic value.

Finally, in Fig. 2, the predictions of free convective heat transfer based on equation (28) are compared with the results of a prior correlation [2]

$$Nu_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[1+\left(\frac{0.43}{Pr}\right)^{9/16}\right]^{4/9}}$$

as well as Chamberlain's [30] quasi-steady state experimental air data. The comparison reveals that the results of the present study are in very good agreement with the experimental data with a maximum difference of 4.3% (which is within the maximum error bounds of the experimental data, reported as 6% [30]) and r.m.s. of the differences of 0.25.

SUMMARY AND CONCLUSIONS

A new simple but accurate approximate analytical method is developed for the mean Nusselt number for free convection from isothermal spheres for the range of Rayleigh numbers $0 \leqslant Ra \leqslant 10^8$ and all Prandtl numbers. The proposed method is novel not only because of the linearization of the convective terms in the energy equation, but also for the method of solving the problem, i.e. transforming the governing equation into a transient conduction-type equation. The latter

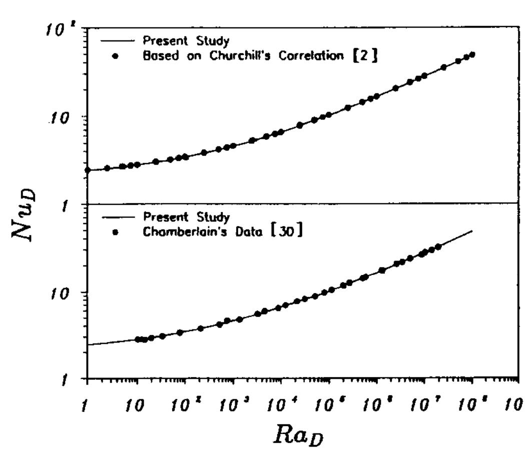

FIG. 2. Comparison of the present study's results with experimental data and other correlations.

technique provides a method of reducing a convective problem to a simpler conduction one, for which the solution exists. In addition, the final result, a correlating equation of Nu against Ra, equation (28), which is a linear superposition of the diffusive limit and laminar boundary layer limit, was derived through analysis and it was not forced to this format. It also has the correct diffusive limit ($Nu_D^0 = 2$), as well as the proper asymptotic values for small and large Prandtl numbers, equations (30) and (31). The present study also addresses the correlating equation of the velocity in the free convective boundary layer as a function of Prandtl number, which by itself is a new subject. Although it uses a simple correlating equation for the velocity, the comparison of the resultant Nu–Ra correlation with other correlation and experimental data shows very good agreement, with a maximum difference of less than 5%.

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CONVECTION NATURELLE THERMIQUE LAMINAIRE AUTOUR DE SPHERES ISOTHERMES: UNE NOUVELLE METHODE ANALYTIQUE

Résumé—Une méthode analytique approchée, simple mais précise, basée sur la linéarisation de l'équation de l'énergie, est développée pour le nombre de Nusselt moyen relatif à la convection naturelle thermique laminaire autour de sphères isothermes, pour le domaine de nombre de Rayleigh $0 \leqslant Ra \leqslant 10^8$ et pour tout nombre de Prandtl. Dans le procédé de linéarisation, l'équation d'énergie est réduite à la forme d'une équation de conduction thermique pour laquelle la solution existe. Une comparaison de la formule finale de Nu en fonction de Ra (qui est une forme explicite de superposition linéaire des solutions limites de diffusion et de couche limite) avec d'autres formules et données expérimentales sur l'air révèle un très bon accord, avec une différence maximale inférieure à 5%.

EIN NEUES ANALYTISCHES VERFAHREN ZUR BEHANDLUNG DER LAMINAREN FREIEN KONVEKTION AN ISOTHERMEN KUGELN

Zusammenfassung—Eine einfache und zugleich präzise analytische Näherungsmethode, die auf einer Linearisierung der Energiegleichung beruht, wurde für die Bestimmung der flächengemittelten Nusselt-Zahl bei freier Konvektion an isothermen Kugeln entwickelt, und zwar für Rayleigh–Zahlen im Bereich $0 \leqslant Ra \leqslant 10^8$ und alle Prandtl-Zahlen. Durch die Linearisierung wird die Energiegleichung auf die Form der instationären Wärmeleitungsgleichung reduziert, für die eine Lösung existiert. Es wird schließlich eine Korrelation von Nu abhängig von Ra gebildet, welche eine explizite Form der linearen Überlagerung des Grenzfalls reiner Leitung und der Grenzschichtlösung ist. Ein Vergleich mit anderen Korrelationen und mit experimentell gewonnenen Daten für Luft zeigt sehr gute Übereinstimmung innerhalb weniger als 5%.

ЛАМИНАРНЫЙ СВОБОДНОКОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС ОТ ИЗОТЕРМИЧЕСКИХ СФЕР, НОВЫЙ ПРИБЛИЖЕННЫЙ АНАЛИТИЧЕСКИЙ МЕТОД

Аннотация—Разработан простой приближенный аналитический метод для определения среднего по площади числа Нуссельта при свободноконвективном теплопереносе от изотермических сфер для чисел Рейнольдса, изменяющехся в диапазоне $0 \leqslant Ra \leqslant 10^8$ и всех чисел Прандтля. Метод основан на линеаризации уравнения энергии, приводимого к нестационарному уравнению теплопроводности, для которого существует решение. Сравнение полученной зависимости Nu от Ra (представляющий собой линейную суперпозицию диффузионного предельного случая и решения для пограничного слоя) с другими зависимостями и экспериментальными данными для воздуха показывает очень хорошее согласие с максимальным расхождением менее 5%.