CONTACT CONDUCTANCE CORRELATIONS BASED ON GREENWOOD AND WILLIAMSON SURFACE MODEL

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Abstract

Elastic and plastic thermal contact conductance correlations are proposed for conforming rough surfaces under load. The correlations compare quite well with the well established Cooper, Mikic and Yovanovich and Mikic models. An explicit expression to predict dimensionless plastic contact pressure is proposed. The correlations are shown to predict accurately thermal contact conductances for untreated and heat-treated ground-lapped interfaces of tool steel for both directions of heat flow. The correlations in conjunction with the empirical model proposed by the same authors predict contact conductance and the ratio of real area to apparent contact area for a wide range of materials (SS304, Ni200, Zirconium alloys, Titanium alloy and tool steel) without the need of microhardness measurements.

Nomenclature

A_a	=	apparent contact area, m^2
A_r	=	real contact area, m^2
a	=	mean radius of circular contact, m
c_1, c_2	=	Vickers correlation coefficients, c_1, MPa
D_{sum}	=	total number of summits/unit apparent area (m^{-2})
d_V	=	Vickers indentation diagonal, μm
E	=	elastic modulus, MPa
E'	=	equivalent elastic modulus, MPa $\equiv \left[(1-\nu_A^2)/E_A + (1-\nu_B^2)/E_B\right]^{-1}$
H_e	=	elastic contact hardness, MPa
H_p	=	plastic contact hardness, MPa
H_V	=	Vickers microhardness, MPa
$I_v(\lambda)$	=	Integral used in the Greenwood and Williamson model
I_v^{-1}	=	Inverse functions used in the text
k_s	=	harmonic mean thermal conductivity, $\equiv 2k_A k_B/(k_A + k_B)$ $(W/m \cdot K)$
$L_n^{\mathcal{A}}(x)$	=	Laguerre function; x is the argument, n is the order and $\mathcal{A}$ is the associated order
l	=	sampling interval, m
m	=	effective mean absolute surface slope, $\equiv \sqrt{m_A^2 + m_B^2}$, rad
m_{eq}	=	equivalent slope for ground surface $\sqrt{m_{mx} \cdot m_{mn}}$, rad
m_0	=	variance of surface heights, $\equiv \sigma^2 = \sigma_A^2 + \sigma_B^2$ (μm^2)
m_2	=	variance of surface slopes $\equiv m_{2A} + m_{2B}$ (radians-square)
m_4	=	variance of the second derivative of surface heights, $\equiv m_{4A} + m_{4B}$ (μm^{-2})
n	=	contact spot density, m^{-2}
P	=	nominal contact pressure, MPa
Q	=	heat transfer rate, W
Y	=	surface mean plane separation (m)

Greek Symbols

α	=	bandwidth parameter, $\equiv m_0 m_4/m_2^2$
β	=	radius of curvature of contacting asperity
ΔT_c	=	effective temperature drop across the interface (°C)
λ	=	dimensionless surface mean plane separation $\equiv Y/\sigma$
ν	=	Poisson's ratio
σ	=	RMS surface roughness heights for given surface or surface pair, $\equiv \sqrt{\sigma_A^2 + \sigma_B^2}$, μm

Subscripts

A, B	=	surfaces A and B
a	=	apparent
e	=	elastic
p	=	plastic
mx	=	maximum
mn	=	minimum
r	=	real
V	=	Vickers

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Abbreviations

CMY	=	Cooper, Mikic and Yovanovich
GW	=	Greenwood and Williamson
G	=	Ground Surface
L	=	Lapped Surface
Ni	=	Nickel
SS	=	Stainless Steel

Introduction

The Greenwood and Williamson (GW) surface model was presented in 1966. The model has been used extensively in tribology and thermal contact conductance work since that time. The GW surface geometry model has been reviewed in detail by Johnson (1985) and McCool (1986). Sridhar and Yovanovich (1994) have presented the GW elastic and plastic contact conductance models in dimensionless form. McWaid and Marschall (1992) and Aikawa and Winer (1994) have implemented GW elastic thermal contact conductance model which seems to be partially successful in predicting experimental data. In 1969, the Cooper, Mikic and Yovanovich (CMY) model was presented which has been successfully implemented by Yovanovich (1982) and Yovanovich et al. (1982) to predict experimental data (see Antonetti, 1983 and Hegazy, 1985).

The contact conductance model is a combination of three separate models: 1) the thermal model, 2) the surface geometry model, and 3) the deformation model. A complete GW contact conductance model utilizes the CMY (1969) thermal model, the GW surface geometry model and the Hertz elastic or geometric plastic models to predict contact conductance. Results of the GW elastic and plastic contact conductance models are summarized in Table 1. The integral used in the GW surface model is given by:

$$I_v(\lambda) = \frac{1}{\sqrt{2\pi}} \int_{\lambda}^{\infty} (s-\lambda)^v exp(-s^2/2) ds \qquad (1)$$

The GW surface model is applicable to isotropic rough surfaces, i.e., the models assume no variations in surface profile slopes with direction. Also the contact stresses depend only upon the relative profile of their two surfaces. Hence the system of two rough surfaces in contact can be replaced by a single flat rigid surface in contact with a body having an effective modulus E', equivalent roughness σ and mean absolute slope m (see Fig. 1):

$$E' = \left(\frac{1-\nu_A^2}{E_A} + \frac{1-\nu_B^2}{E_B} \right)^{-1} \qquad (2)$$

$$\sigma = \sqrt{\sigma_A^2 + \sigma_B^2} \qquad (3)$$

$$m = \sqrt{m_A^2 + m_B^2} \qquad (4)$$

Table 1 GW elastic and plastic models

Deformation	Results
Elastic or Plastic	$\frac{A_r}{A_a} = \kappa^* \pi D_{sum} \sigma \beta I_1(\lambda)$ $n = \frac{1}{2} D_{sum} erfc(\lambda/\sqrt{2})$ $a = \sqrt{\frac{2\kappa^* \sigma \beta I_1(\lambda)}{erfc(\lambda/\sqrt{2})}}$ $h_c = \frac{\sqrt{2\kappa^*} k_s D_{sum} \sqrt{erfc(\lambda/\sqrt{2}) \sigma \beta I_1(\lambda)}}{\left[1 - \sqrt{\kappa^* \pi D_{sum} \sigma \beta I_1(\lambda)}\right]^{1.5}}$
Elastic	$\lambda = I_{3/2}^{-1} \left(\frac{3P}{4 D_{sum} E' \sigma \sqrt{\sigma\beta}} \right)$
Plastic	$\lambda = I_1^{-1} \left(\frac{P}{2\pi D_{sum} H_p \sigma \beta} \right)$

* $\kappa = 1$ for elastic, $\kappa = 2$ for plastic

The surface model does not differentiate between profile and surface statistics, therefore:

Density of summits:

$$D_{sum} = \frac{1}{39.48} \left(\frac{m_4}{m_2} \right) \qquad (5)$$

Variance of surface heights:

$$\sigma^2 = m_0 \qquad (6)$$

Radius of curvature of summits:

$$\beta = \frac{0.798}{\sqrt{m_4}} \qquad (7)$$

where m_2 and m_4 are the variance of the surface slopes and the variance of the second derivative of the surface heights respectively. For a surface pair m_2 and m_4 are given by (McWaid, 1990):

$$m_2 = m_{2A} + m_{2B} \qquad (8)$$

$$m_4 = m_{4A} + m_{4B} \qquad (9)$$

McWaid (1990) and Maddren (1994) have presented steps for the implementation of the elastic part of the

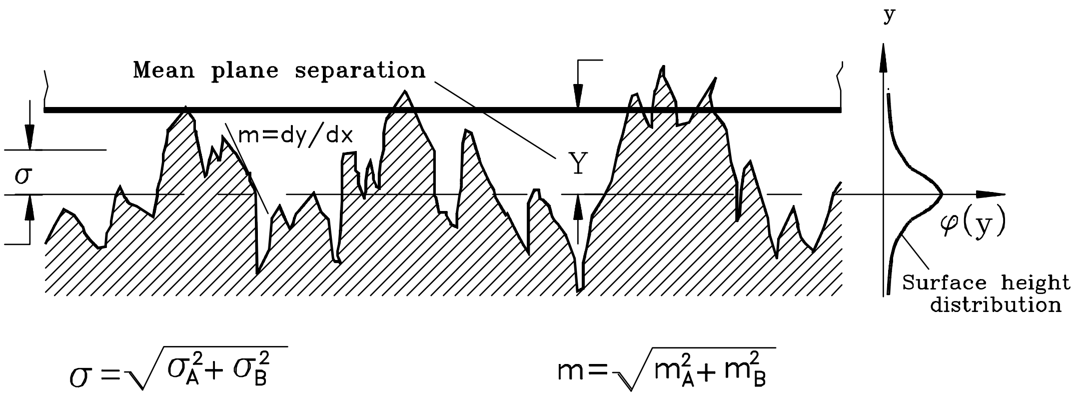

Fig. 1 A typical conforming rough surface in contact with a flat

model presented in Table 1. They first compute the dimensionless mean plane separation λ using a fifth order polynomial approximation for the integral $I_{3/2}$ (see Table 1) and the Newton-Raphson iterative method. In the next step they determine the ratio of the real area of contact to apparent contact area A_r/A_a. Again they utilize a polynomial approximation for the integral I_1. The contact spot density n and the mean contact spot radius a are then evaluated and finally they estimate the contact conductance using the thermal model presented in CMY (1969):

$$h_c = \frac{2k_s n a}{\left(1 - \sqrt{\frac{A_r}{A_a}}\right)^{1.5}} \tag{10}$$

The procedure used by McWaid (1990) and Maddren (1994) is tedious and involves numerous calculations in order to predict h_c at a particular applied pressure and is limited to surfaces undergoing only fully elastic deformation.

Blahey, Tevaarwerk and Yovanovich (1980) proposed the following correlation for contact conductance based on the GW elastic model:

$$h_c = 3.81 \ \frac{k_s}{\sigma} \left(\frac{P}{E'}\right)^{0.93} \ \left(\frac{\sigma}{\beta}\right)^{0.035} \tag{11}$$

where k_s is the harmonic mean conductivity, P is the applied pressure, β is the mean asperity peak radius. The mean asperity summit radius β which appears in the correlation does not have a fixed range and the above correlation as presented cannot be compared with the elastic correlation proposed by Mikic (1974) in dimensionless form. It is also not clear from this work the conditions (i.e. surface roughness and microhardness) under which the above correlation could be used. It is known from the work of Sridhar (1994) that the elastic correlation can underpredict contact conductance for an interface undergoing plastic deformation by as much as 800 %.

The aim of the present work is to minimise the difficulties encountered in using the procedure and correlations proposed by previous authors. From the previous work (Sridhar and Yovanovich, 1996a, 1996b) it is clear that the elastic and the plastic models form the bounds and the experimental data (NI200, SS304, Zirconium alloys and Tool Steel) lie between them. It was also seen that the elastic and plastic contact conductance models run parallel to each other and are quite close (difference $\simeq$ 30%). Based on these conclusions and that rough surfaces undergo both elastic and plastic deformation, there is a need to implement the elastic and plastic contact conductance models only. In many engineering applications it is important to predict accurately contact conductance with a minimum number of surface parameters. The work presented here is a complete implementation of the GW elastic and plastic contact conductance models with three surface parameters, namely the RMS roughness σ, the mean absolute slope m and the bandwith parameter α. The dimensionless plastic contact pressure based on surface microhardness which is an important part of the GW plastic model is also developed here.

Correlations for Dimensionless Contact Conductance

In order to generate correlations for contact conductance which are independent of surface geometry and material properties the expression for contact conductance h_c in Table 1 has to be cast in a dimensionless form. It is known since CMY (1969) that the most suitable dimensionless group for contact conductance is

$$C_c = \frac{\sigma}{m} \cdot \frac{h_c}{k_s} \tag{12}$$

The dimensionless contact conductance C_c is normally plotted against (see CMY (1969), Mikic (1974) and Sridhar and Yovanovich (1994)) the ratio of the real area to the apparent area of contact A_r/A_a which is equal to P/H_p and $\sqrt{2}P/(E'm)$ for plastic and elastic deformation respectively.

The modified and unmodified versions of the GW model give almost the same results and only the unmodified version of the model in dimensionless form will be presented here. The derivations for C_c (only elastic) is presented here. The expressions for the density of summits, variance of surface heights and radius of curvature of summits from Eq. (5) to Eq. (7) are used here. From Table 1 for elastic deformation, we have:

$$\frac{A_r}{A_a} = \pi D_{sum} \sigma \beta I_1(\lambda) \tag{13}$$

$$n = \frac{1}{2} D_{sum} erfc\left(\lambda/\sqrt{2}\right) \tag{14}$$

$$a = \sqrt{\frac{2\sigma\beta I_1(\lambda)}{erfc(\lambda/\sqrt{2})}} \tag{15}$$

Substituting values for D_{sum}, σ and β from Eqs. (5), (6) and (7) respectively into Eq. (13) and simplifying we have:

$$\frac{A_r}{A_a} = \frac{1}{15.75} \sqrt{\alpha}\, I_1(\lambda) \tag{16}$$

where the bandwidth parameter α is given by:

$$\alpha = \left(\frac{m_0 m_4}{m_2^2}\right) \tag{17}$$

The product na is given by:

$$na = \frac{1}{\sqrt{2}} D_{sum} \sqrt{\sigma\beta} \sqrt{I_1(\lambda) erfc(\lambda/\sqrt{2})} \tag{18}$$

Non-dimensionalizing the above equation as follows we have:

$$\left(\frac{\sigma}{m}\right) na = \frac{1}{\sqrt{2}} \left(\frac{\sigma}{m}\right) D_{sum} \sqrt{\sigma\beta} \sqrt{I_1(\lambda) erfc(\lambda/\sqrt{2})} \tag{19}$$

For Gaussian surfaces we know that (Nho, 1990):

$$m = \frac{\sqrt{m_2}}{\sqrt{\frac{\pi}{2}}} \tag{20}$$

The term $\left(\frac{\sigma}{m}\right) D_{sum} \sqrt{\sigma\beta}$ reduces to:

$$\left(\frac{\sigma}{m}\right) D_{sum} \sqrt{\sigma\beta} = \frac{1}{35.3} \left(\frac{m_0 m_4}{m_2^2}\right)^{3/4} = \left(\frac{\alpha^{3/4}}{35.3}\right) \tag{21}$$

Therefore

$$\left(\frac{\sigma}{m}\right) na = \frac{\alpha^{3/4}}{49.9} \sqrt{I_1(\lambda) erfc(\lambda/\sqrt{2})} \tag{22}$$

and

$$C_c = \frac{\sigma}{m} \cdot \frac{h_c}{k_s} = \frac{2\left(\frac{\sigma}{m}\right) na}{(1 - \sqrt{A_r/A_a})^{1.5}} \tag{23}$$

and

$$C_{ce} = \frac{\frac{\alpha^{3/4}}{24.95} \sqrt{I_1(\lambda) erfc(\lambda/\sqrt{2})}}{\left(1 - \sqrt{\frac{1}{15.75} \sqrt{\alpha}\, I_1(\lambda)}\right)^{1.5}} \tag{24}$$

where

$$\lambda = I_{3/2}^{-1} \left(\frac{3P}{4 D_{sum} E' \sigma \sqrt{\sigma\beta}}\right) \tag{25}$$

Dimensionless elastic contact pressure according to Mikic and Roca (1993) is given by:

$$P/H_e = \frac{P}{\left(\frac{E'}{\sqrt{2}}\, m\right)} \tag{26}$$

The term in Eq. (25) can be rewritten as follows:

$$\left(\frac{3P}{4 D_{sum} E' \sigma \sqrt{\sigma\beta}}\right) = \frac{3}{4} \cdot \frac{P}{E'} \cdot \frac{1}{m/\sqrt{2}} \cdot \frac{m/\sqrt{2}}{1} \cdot \frac{1}{D_{sum} \sigma \sqrt{\sigma\beta}} \tag{27}$$

$$= \frac{3}{4\sqrt{2}} \cdot P/H_e \cdot \frac{1}{\frac{\sigma}{m} D_{sum} \sqrt{\sigma\beta}} \tag{28}$$

$$= \frac{18.72}{\alpha^{3/4}} \cdot P/H_e \tag{29}$$

Therefore

$$\lambda = I_{3/2}^{-1} \left(\frac{18.72}{\alpha^{3/4}} \cdot P/H_e\right) \tag{30}$$

The expression for dimensionless contact conductance C_c for interfaces undergoing plastic deformation can be derived in a similar way:

$$C_{cp} = \frac{\frac{\alpha^{3/4}}{17.65} \sqrt{I_1(\lambda) erfc(\lambda/\sqrt{2})}}{\left(1 - \sqrt{\frac{1}{7.87} \sqrt{\alpha}\, I_1(\lambda)}\right)^{1.5}}, \qquad \lambda = I_1^{-1} \left(\frac{7.88}{\sqrt{\alpha}} \cdot \frac{P}{H_p}\right) \tag{31}$$

There are two different ways of developing correlation equations of Eqs. (24) and (31). One way would be to generate a table of values of C_c for different values of α and correlating them with power-law relations. The second method which is based on approximating the three different special function combinations which appear in the expressions for dimensionless contact conductance, allows derivation of the correlations without actually generating the correlations for different values of α. The second method, even though it is elegant, leads to larger errors as two approximations are multiplied in the numerator of the expression for dimensionless contact conductance.

The integrals I_1 and $I_{3/2}$ were first evaluated symbolically in *Mathematica* (1988-94) which gave the following results:

$$I_1(\lambda) = \frac{1}{\sqrt{2\pi}} \exp(-\lambda^2/2) - \frac{\lambda}{2} \quad erfc(\frac{\lambda}{\sqrt{2}}) \qquad (32)$$

and

$$I_{3/2}(\lambda) = 0.772836 \ L_{3/4}^{-1/2}(-\lambda^2/2) - 0.53895 \ L_{1/4}^{1/2}(-\lambda^2/2) \qquad (33)$$

where $L_{3/4}^{-1/2}(-\lambda^2/2)$ and $L_{1/4}^{1/2}(-\lambda^2/2)$ are fractional order Laguerre functions of the form $L_n^{\mathcal{A}}(x)$, where x is the argument, n is the order and $\mathcal{A}$ is the associated order.

After having expressed the integrals in Eqs. (24) and (31) in the form of special function combinations (Eq. (32) and Eq. (33)), the dimensionless mean plane separation λ ($=I_{3/2}^{-1}\left[18.72/\alpha^{3/4} \cdot P/H_e\right]$ or $I_1^{-1}\left[7.88/\sqrt{\alpha} \cdot P/H_p\right]$) was computed for known values of $\left[18.72/\alpha^{3/4} \cdot P/H_e\right]$ or $\left[7.88/\sqrt{\alpha} \cdot P/H_p\right]$ using the FindRoot command in *Mathematica* (1988-94). Simple correlations for λ are proposed ($2 < \lambda < 4$):

$$\lambda_e = -0.2664 \ln\left[18.72/\alpha^{3/4} \cdot P/H_e\right] + 0.789$$

$$\lambda_p = -0.2876 \ln\left[7.88/\sqrt{\alpha} \cdot P/H_p\right] + 0.676$$

The maximum and RMS differences between the correlations and computed values for λ_e were found to be 2.8 % and 1.9 % respectively. The maximum and RMS differences for λ_p were 2.2 % and 1.5 % respectively.

Once the appropriate values of λ are known, C_{ce} and C_{cp} can be computed. Figures 2 and 3 show the plots of dimensionless contact conductance against dimensionless contact pressure. The symbols in Figs. 2 and 3 represent the values computed using *Mathematica* (1988-94) for known values of α and P/H_e or P/H_p. The lines through the symbols are power-law fits. The Mikic elastic model and the CMY plastic models are also included in the plots. It can be seen that the Mikic and CMY models compare well with the GW models when $\alpha = 5$.

The constants which were obtained at different values of α (see Fig. 2 and 3) were recorrelated to derive a single correlation equation for C_{ce} and C_{cp} respectively. The GW elastic and plastic contact conductance correlations which are valid within the range $10^{-5} \leq P/H_e$ or $P/H_p \leq 10^{-2}$ and $5 \leq \alpha \leq 100$ are:

$$C_{ce} = (1.18 + 0.161 \ln \alpha) \left(\frac{P}{H_e}\right)^{0.922 \ \alpha^{1/205.54}} \qquad (34)$$

and

$$C_{cp} = 0.91 \ \alpha^{0.31} \ \left(\frac{P}{H_p}\right)^{0.971 \ \alpha^{1/251.93}} \qquad (35)$$

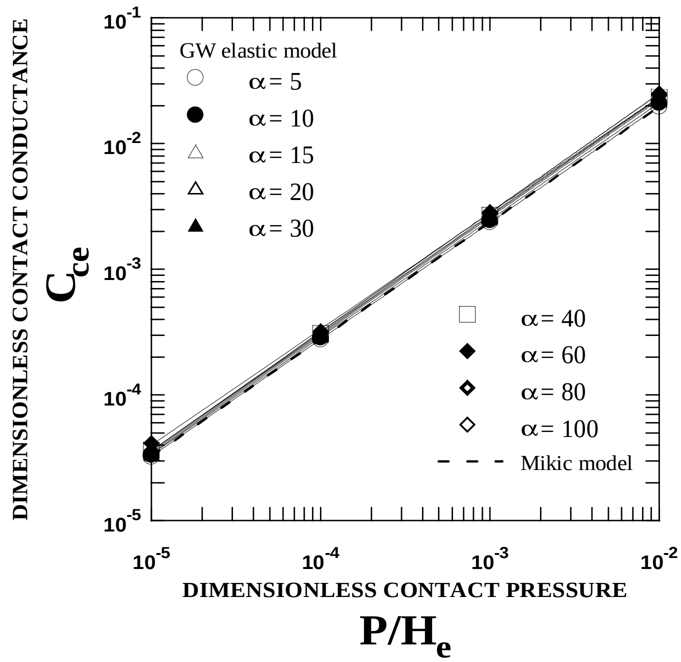

Fig. 2 Comparison of GW and Mikic elastic models

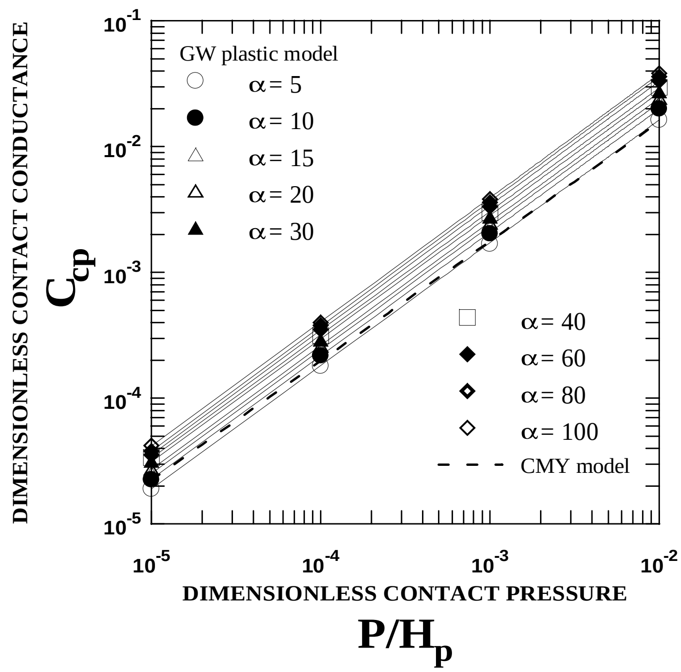

Fig. 3 Comparison of GW and CMY plastic models

The above correlations predict the computed results very accurately for α between 5 and 40. The elastic correlation predicts values for this range of α with RMS and maximum differences of 3 % and 5 % respectively. The plastic correlation for this range predicts values with RMS and maximum differences of 1 % and 2.3 % respectively. The elastic and plastic correlations have maximum RMS differences of 5.4 % and 6.8% respectively for α between 40 and 100.

Explicit Expression for Dimensionless Plastic Contact Pressure

It is known since Yovanovich et al. (1982) presented their model that the dimensionless plastic contact pressure (P/H_p) is dependent on surface roughness (σ/m), microhardness and applied pressure P. The iterative scheme presented by Yovanovich et al. (1982) was later simplified and an explicit expression for dimensionless plastic contact pressure (P/H_p) was presented by Song and Yovanovich (1988). This expression was derived using the CMY plastic model. In the present work the explicit expression will be derived based on the GW plastic model.

The dimensionless plastic contact pressure can be written as:

$$\frac{P}{H_p} = \frac{0.9272\ P}{H_V} = \frac{0.9272\ P}{c_1 {d_V}^{c_2}} \tag{36}$$

In the above equation H_p and H_V are the appropriate values of the contact microhardness and the Vickers microhardness respectively. The constant 0.9272 converts the Vickers microhardness which is based on total surface area of indentation to the contact microhardness based on the projected area of indentation. This conversion is necessary because H_p is defined based on the projected area of indentation. It is known (Hegazy, 1985) that a surface does not have a single value of Vickers microhardness, but it is indentation size dependent. Hence the Vickers microhardness in the above equation is expressed as a function of the indentation diagonal d_V of the Vickers indenter. The constants c_1 and c_2 are empirical correlation coefficients which are obtained by conducting experiments on surfaces of interest at different loads and later correlating them with power-law expressions using the method of least squares. Figure 4 shows a typical plot of Vickers microhardness H_V against indentation diagonal d_V for untreated tool steel from Sridhar (1994) measured at five different temperature levels.

The indentation diagonal d_V for a particular applied pressure P is unknown at the moment. This is evaluated using the plastic model. Since the pyramidal Vickers indenter produces a square indentation and the model assumes circular spots, they are related to each other by equating their projected areas. Therefore d_V is given by:

$$d_V = \sqrt{2\pi}\ a \tag{37}$$

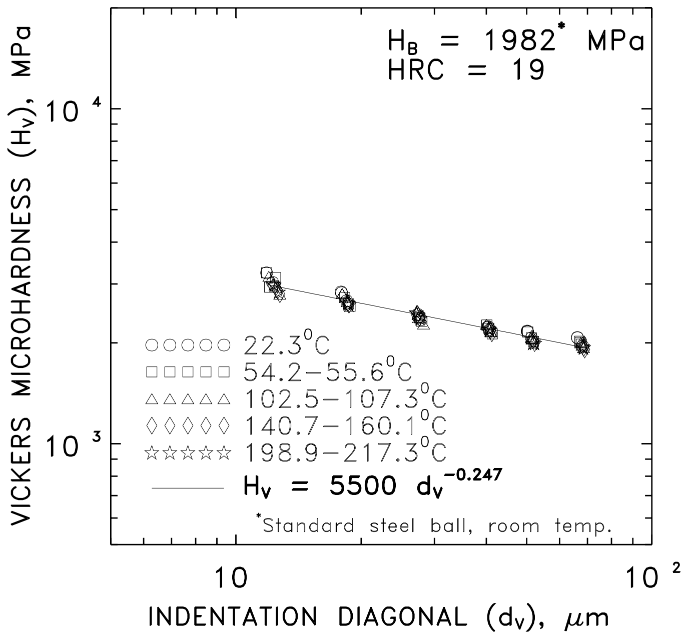

Fig. 4 Plots of H_V versus d_V for untreated tool steel at five temperature levels

Examining Table 1 the mean contact spot radius a for surfaces undergoing plastic deformation is given by the following expression:

$$a = \sqrt{\frac{4\sigma\beta I_1(\lambda)}{erfc(\lambda/\sqrt{2})}} \tag{38}$$

Substituting for λ from Eq. (31) gives

$$a = \sqrt{\frac{4\sigma\beta X_p}{erfc(I_1^{-1}(X_p)/\sqrt{2})}} \tag{39}$$

where $X_p = 7.88/\sqrt{\alpha}\cdot P/H_p$.

The product $\sigma\cdot\beta$, using Eq. (7), can be written as:

$$\sigma\ \beta = \frac{\sigma}{m}\ m\ \beta = \frac{\sigma}{m}\ m\ \frac{0.794}{\sqrt{m_4}} = \frac{0.798}{\sqrt{\pi/2}}\ \frac{\sigma}{m}\ \sqrt{\frac{m_2}{m_4}} \tag{40}$$

In the above equation the mean absolute surface slope is rewritten using the relationship $m_2 = \sqrt{\pi/2}\ m^2$ which is valid for surfaces having Gaussian distribution of surface heights and slopes. We also know from Eq. (17) that $m_2/m_4 = m_0/m_2 \cdot 1/\alpha$, where $m_0 = \sigma^2$ from Eq. (6). Substituting for m_2/m_4 in the above equation and simplifying yields:

$$\sigma\ \beta = \frac{0.798}{\sqrt{\alpha}}\ \left(\frac{\sigma}{m}\right)^2 \tag{41}$$

The special function combination $erfc(I_1^{-1}[X_p]/\sqrt{2})$ can be approximated within the range $1.3\times10^{-5} \le X_p \le 1.6\times10^{-2}$ as follows:

$$erfc\left(\frac{I_1^{-1}[X_p]}{\sqrt{2}}\right) = 3.84 \ \ X_p^{0.925} \tag{42}$$

To arrive at the above approximation *Mathematica* (1988-94) was used to compute values of the left-hand-side of Eq. (42) which were fitted using the method of least squares. The RMS error and the maximum error between the approximation and the computed values are 2.4 % and 5.5 % respectively.

The mean contact spot radius a can be expressed in terms of P/H_p and the surface parameters α and σ/m using Eq. (41) and Eq. (42):

$$a = \frac{0.986}{\alpha^{0.269}} \ \left(\frac{\sigma}{m}\right) \ \left(\frac{P}{H_p}\right)^{0.0375} \tag{43}$$

Figure 5 shows plots of dimensionless mean contact spot radius [a/(σ/m)] against dimensionless plastic contact pressure for three different values of α. The computed values and the approximations (including Antonetti, 1983 and Sridhar, 1994) for dimensionless contact radius for the CMY plastic model are also included in the plot. The RMS errors between the computed values and the approximations were less than 2 % for the range shown in the graph. The maximum error that occurs at the right end of the graph (Fig. 5) is approximately 5.5 %.

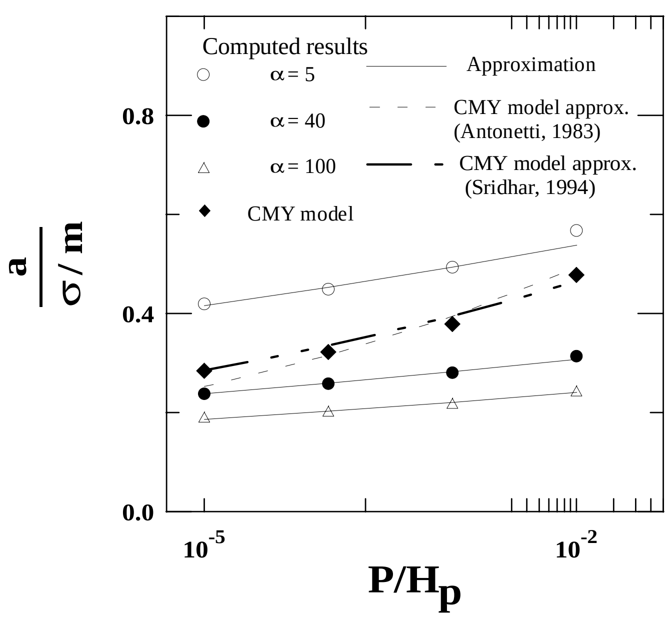

Fig. 5 Comparison of computed values and approximations of dimensionless mean contact spot radius for GW and CMY plastic models

After substituting Eq. (43) and Eq. (37) in Eq. (36) and simplifying we can derive an explicit expression for the dimensionless plastic contact pressure:

$$\frac{P}{H_p} = \left[\frac{0.9272 \ \ P}{c_1\left(\dfrac{2.47}{\alpha^{0.269}} \ \ \dfrac{\sigma}{m}\right)^{c_2}}\right]^{\dfrac{1}{1 + 0.038 \ c_2}} \tag{44}$$

The explicit expression for dimensionless plastic contact pressure (see Song and Yovanovich, 1988) based on the CMY plastic model is:

$$\frac{P}{H_p} = \left[\frac{0.9272 \ \ P}{c_1\left(1.62 \ \ \dfrac{\sigma}{m}\right)^{c_2}}\right]^{\dfrac{1}{1 + 0.071c_2}} \tag{45}$$

It should be noted that the exponents $1/(1 + 0.038c_2)$ and $1/(1 + 0.071c_2)$ in the above equations are approximately equal to 1.0 within the range $-0.28 \leq c_2 \leq 0$. The ratio $2.47/\alpha^{0.269}$ in Eq. (44) replaces the constant 1.62 in the Song and Yovanovich (1988) explicit expression (Eq. (45)).

Predicting Data With the GW Correlations

Thermal contact conductance data obtained from ground-lapped interfaces of heat-treated and untreated tool steel (Sridhar, 1994) will be predicted using the GW elastic and plastic models in this section. The experimental setup, experimental procedure and data reduction are described in detail by Sridhar (1994). The contact conductance h_c is given by:

$$h_c = \frac{Q}{A\Delta T_c} \tag{46}$$

where Q is the heat flow rate, A is the apparent area of contact and ΔT_c is the interface temperature drop. The contact conductance was measured for different values of applied pressure P ranging from 0.5 to 8.0 MPa.

To predict h_c with the GW model, the elastic and plastic correlations are rewritten as follows:

$$h_{ce} = k_s \ \ \frac{m}{\sigma} \ \ (1.18 + 0.161 \ln \alpha) \left(\frac{\sqrt{2}P}{E'm}\right)^{0.922 \ \ \alpha^{1/205.54}} \tag{47}$$

and

$$h_{cp} = 0.91 k_s \frac{m}{\sigma} \alpha^{0.31} \left[\frac{0.9272P}{c_1\left(\dfrac{2.47}{\alpha^{0.269}}\dfrac{\sigma}{m}\right)^{c_2}}\right]^{\dfrac{0.971\alpha^{1/251.93}}{1 + 0.038 \ c_2}} \tag{48}$$

The elastic contact conductance h_{ce} can be predicted knowing the thermal conductivity k_s of the interface, surface properties and the equivalent elastic modulus E' of the interface. In order to predict the plastic contact conductance h_{cp} at a particular applied pressure P one needs to know the surface Vickers microhardness parameters c_1 and c_2 of the softer material, thermal conductivity of the interface k_s and the surface properties of the ground-lapped interface: σ, m and α.

Recently Sridhar and Yovanovich (1995) developed empirical correlations to predict Vickers microhardness for a wide range of materials including SS304, Ni200, two Zirconium alloys, Titanium alloy, and untreated and heat treated tool steel. They examined Vickers microhardness variation with indentation size for tool steel for different values of the bulk hardness which was varied by heat treatment. The plots of Vickers microhardness versus indentation size were correlated with a simple power-law relation: $H_V = c_1 \, {d_V}^{c_2}$. The Vickers correlation coefficients c_1 and c_2 were found to have definite relationships with the Brinell hardness. Two methods of correlating the coefficients c_1 and c_2 with the Brinell hardness H_B were proposed. The second method was generally superior and it is given by:

$$\frac{c_1}{3178} = \left[4.0 - 5.77\,(H_B^*) + 4.0\,(H_B^*)^2 - 0.61\,(H_B^*)^3\right] \quad (49)$$

where $H_B^* = H_B/3178$, and

$$c_2 = -0.370 + 0.442\left(\frac{H_B}{c_1}\right) \quad (50)$$

The above correlations are valid over the Brinell hardness H_B range from 1300 to 7500 MPa. Even though the empirical correlations were derived from five different metals, the correlations should be applicable to any material whose Brinell hardness is between 1300 to 7500 MPa.

The surface parameters: RMS roughness σ, mean absolute slope m and variance of the second derivative of surface heights m_4 can be measured using a surface roughness measuring device. It has been found (Thomas, 1982 and Sridhar, 1994) that m and m_4 are dependent on the sampling interval. Surface measurements on ground and lapped surfaces of tool steel (Sridhar, 1994) were made using a Taylor Hobson Talysurf-5 profilometer.

A ground surface is an anisotropic surface for which the surface slope varies with direction. The maximum surface slope is observed at the line perpendicular to the grinding direction and the minimum at the line parallel to the grinding direction. The equivalent slope for a ground surface is defined as: $m_{eq} = \sqrt{m_{mx} \cdot m_{mn}}$, where m_{mx} and m_{mn} are the maximum and minimum slopes of a ground surface. The lapped surface is an isotropic surface and surface slope does not vary with direction. A mean value of two traces perpendicular to each other is a good representation of the surface slope of a ground surface. The ground and lapped surfaces were characterized at four different sampling intervals, i.e., 3.4, 4.2, 5.0 and 5.9 μm. The smaller sampling interval produces larger values of the slope at each trace orientation.

Table 2 Surface properties of ground-gapped interfaces of untreated and heat-treated tool steel from Sridhar (1994)

Sampling Interval μm	Slope m	RMS Roughness σ μm	σ/m μm	m_4 μm^{-2}	Bandwidth parameter $\alpha = \frac{4}{\pi^2}\left(\frac{\sigma}{m}\right)^2 \frac{m_4}{m^2}$
Ground-lapped interface, untreated tool steel					
3.36	0.089	0.98	11.0	0.00320	19.8
4.20	0.077	0.98	12.7	0.00159	17.5
5.04	0.067	0.98	14.6	0.00084	16.2
5.88	0.060	0.98	16.4	0.00049	14.8
Ground-lapped interface, heat-treated tool steel					
3.36	0.045	0.59	13.1	0.00076	26.1
4.20	0.041	0.59	14.5	0.00040	20.3
5.04	0.037	0.59	16.1	0.00022	16.9
5.88	0.034	0.59	17.3	0.00014	14.7

The surface properties (m, σ and m_4) for a ground-lapped interface can be calculated using Eqs. (4), (3) and (9), where the subscripts A and B refer to the equivalent surface properties of ground and lapped surfaces respectively. Table 2 lists the surface properties of ground-lapped interfaces of untreated and heat-treated tool steel. It can seen that the RMS surface roughness σ is independent of the sampling interval.

There are two correlations (Eq. (47) and Eq. (48)) available to predict contact conductance; one is based on elastic deformation and the other is based on plastic deformation. The decision to use the elastic or the plastic correlation to predict a particular data set is based on the magnitude of the elastic or plastic hardness. Therefore, the rules we prescribe for using a particular correlation is based on comparing the dimensionless elastic contact pressure P/H_e and dimensionless plastic contact pressure P/H_p for a particular interface. If P/H_e is greater than P/H_p, the interface is undergoing elastic deformation; and if P/H_e is less than P/H_p then the interface is undergoing plastic deformation. Table 3 shows the surface properties of the two interfaces (i.e. ground-lapped, untreated and heat-treated) and dimensionless contact pressures for four different sampling intervals. The dimensionless contact pressures were computed at a single value (P = 1 MPa) applied pressure since it is a weak function of P. It is clear from Table 3 that the untreated tool steel is undergoing plastic deformation and the heat-treated one is undergoing elastic deformation. It should be noted that a higher value of the equivalent elastic modulus $E_c = 1.5\ E'$ was used as recommended by Sridhar (1994) for tool steel.

The thermal contact conductance data for untreated tool steel can now be predicted knowing the Brinell hardness H_B which is 1982 MPa (Sridhar, 1994). Figure 6 shows comparisons of untreated tool steel data with the predictions, Eq. (48), at four different sampling intervals. Data for both directions of heat-flow, i.e. ground to lapped and lapped to ground have been included. The data seem to move from the 3.36 μm sampling interval bound to the 5.88 sampling interval bound. The comparison is reasonable and the data sets seem to show no directional effect and fall within the experimental uncertainty. Figure 7 shows a similar comparison between the GW elastic model, Eq. (47), and data for heat-treated tool steel. The data seem to show more scatter than the comparison between the untreated data and the GW plastic model.

Summary and Conclusions

Correlations to predict thermal contact conductance for surfaces undergoing elastic and plastic deformation based on the GW surface model have been proposed. The correlations are valid for a wide range of the dimensionless contact pressure ($10^{-5} \le P/H_e$ or $P/H_p \le 10^{-2}$) and bandwidth parameter ($5 \le \alpha \le 100$). An explicit expression to predict dimensionless plastic contact pressure based

Table 3 Comparison of dimensionless elastic and plastic contact pressure for untreated and heat-treated tool steel

Sampling Interval μm	σ/m μm	P/H_e at P = 1.0 MPa	P/H_p P = 1.0 MPa
Ground-lapped interface; untreated tool steel			
3.36	11.0	0.0000964	0.0002594
4.20	12.7	0.0001114	0.0002697
5.04	14.6	0.0001281	0.0002794
5.88	16.4	0.0001430	0.0002881
Ground-lapped interface; heat-treated tool steel			
3.36	13.1	0.0001907	0.0001107
4.20	14.5	0.0002093	0.0001130
5.04	16.1	0.0002319	0.0001151
5.88	17.3	0.0002523	0.0001166

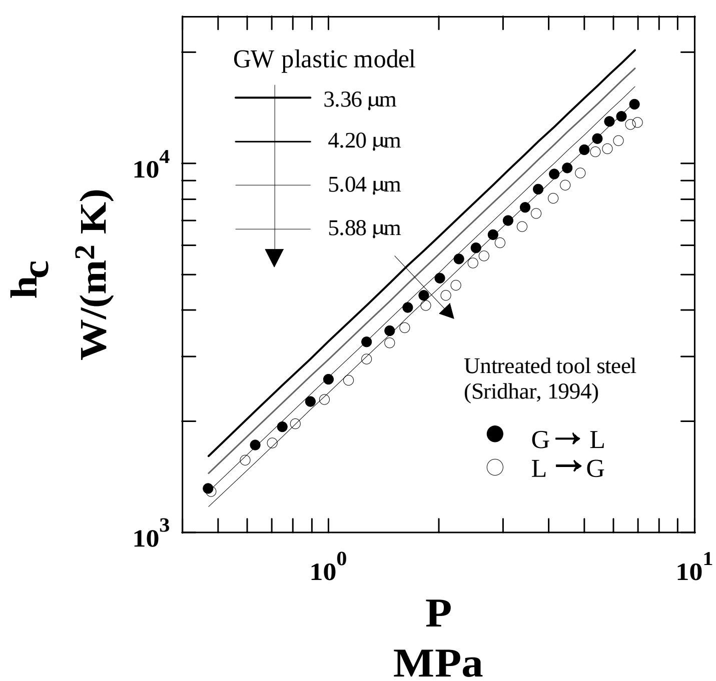

Fig. 6 Prediction of untreated tool steel data with GW plastic model at four different sampling intervals

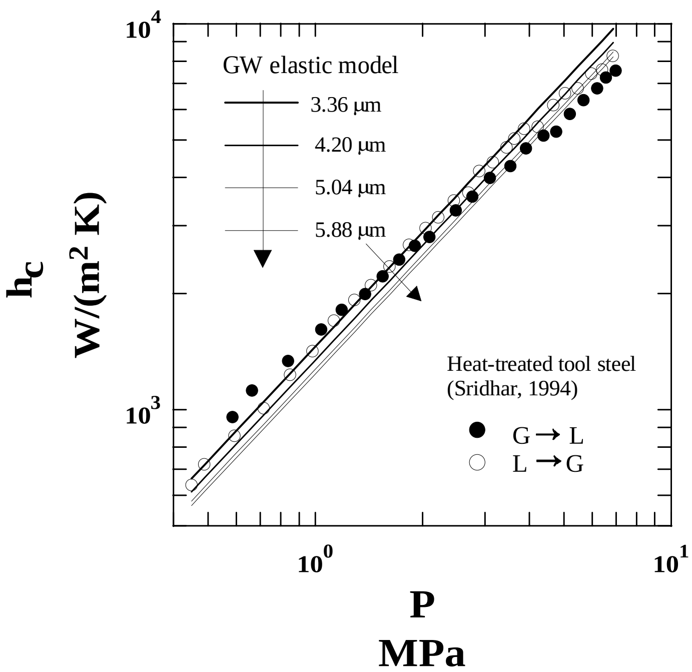

Fig. 7 Prediction of heat-treated tool steel data with GW elastic model at four different sampling intervals

on the GW plastic model has been proposed. Rules for ascertaining *a priori* if a particular interface is undergoing elastic or plastic deformation have also been proposed. The correlations were also used to predict untreated and heat-treated tool steel at four different sampling intervals. The predictions compare quite well with the experimental data. The elastic and plastic correlations in conjunction with the predictions for c_1 and c_2 in Eqs. (49) and (50) can be used by design engineers to predict thermal contact conductance for practically any material which has a Brinell hardness between 1300 and 7500 MPa. For plastic deformation the ratio of real area to apparent area, A_r/A_a, of contact between two conforming rough surfaces, which is equal to the dimensionless plastic contact pressure P/H_p, can be obtained from:

$$\frac{A_r}{A_a} = \left[\frac{0.9272 \quad P}{c_1 \left(\dfrac{2.47}{\alpha^{0.269}} \quad \dfrac{\sigma}{m}\right)^{c_2}}\right]^{\dfrac{1}{1+0.038c_2}} \tag{51}$$

The Vickers correlation coefficients c_1 and c_2 are obtained from Eqs. (49) and (50).

The GW elastic and plastic contact conductance models have been completely implemented in this work. The correlations are easy to use and can be compared with the CMY and Mikic models on dimensionless plots. The present correlations require an extra surface parameter to predict contact conductance when compared with the CMY and Mikic models. It can be concluded from predicting tool steel data at four different sampling intervals that low load data are closer to the smaller sampling interval predictions than the higher load data. It is possible that at light loads, surfaces have to be characterized at smaller sampling intervals as opposed to higher loads where larger sampling intervals give a sufficient representation of the surface geometry of the interface.

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