THERMAL CONSTRICTION RESISTANCE IN MULTILAYERED CONTACTS: APPLICATIONS IN THERMAL CONTACT RESISTANCE

Y.S. Muzychka*, M.R. Sridhar† and M.M. Yovanovich‡
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada

V. W. Antonetti§
Department of Mechanical Engineering
Manhattan College
Riverdale, New York

Abstract

Application of highly conductive coatings to contacting surfaces is a commonly employed method to enhance the thermal contact conductance. In many applications it is often necessary to apply an intermediate coating such that the conductive coating may be applied to a non-adhering substrate. In these instances, it is desirable to predict the effect that the intermediate and final coatings have on the constriction resistance. A solution for computing the thermal constriction resistance of a planar circular contact on a doubly coated substrate is presented. Also a model is developed to compute the contact conductance between a bare substrate and a coated substrate. Comparisons are made with data obtained in the literature for which no analytical model was available. Solution of the governing equations and numerical computation of the constriction resistance were obtained using Computer Algebra Systems (CAS).

Nomenclature

a, b	=	two radii with $a < b$, m
A_c, A_t, A_a	=	area, m^2
A_{i_n}, B_{i_n}	=	Fourier-Bessel coefficients
CAS	=	Computer Algebra System
C_L	=	constriction correction factor
DLC	=	Diamond-Like Coating
e	=	natural log base
$J_0(x)$	=	Bessel Function of the first kind of order zero
$J_1(x)$	=	Bessel Function of the first kind of order one
h_c	=	contact conductance, W/m^2K
H_c	=	contact micro-hardness, MPa
k_o, k_1, k_2, k_3	=	thermal conductivity, W/mK
K_{21}	=	k_2/k_1 thermal conductivity ratio
K_{32}	=	k_3/k_2 thermal conductivity ratio
$\mathcal{L}$	=	characteristic length
N	=	number of contacts
P	=	contact pressure, MPa
Q	=	total heat flow rate, W
r, z	=	cylindrical coordinates, m
R_c	=	constriction resistance, K/W
$t_1, t_2, \ldots, t_n$	=	coating thickness, m
T	=	temperature, K
$\overline{T}$	=	area mean temperature, K
TEF	=	Thermal Enhancement Factor
$z_1, z_2, \ldots, z_n$	=	interface location, m

Greek Symbols

β	=	equation parameter
δ_n	=	n^{th} eigenvalue
ϵ	=	contact spot aspect ratio, a/b or $\sqrt{A_c/A_t}$
γ_n	=	equation parameter
λ	=	separation constant
ψ	=	constriction parameter
ϕ	=	equation parameter
ρ_n	=	boundary condition modification factor
σ/m	=	rms roughness/mean asperity slope
τ_i	=	relative coating thickness, Δ_i/a

Subscripts

a	=	denotes apparent contact area
c	=	denotes contact spot
i	=	denotes the i^{th} layer
n	=	denotes a term in a series
o	=	bare surface
t	=	denotes flux tube

*Graduate Research Assistant

†Post-Doctoral Fellow

‡Professor and Director, Fellow AIAA

§Professor

Introduction

This paper presents the general theory of multilayered flux tubes and discusses a particular case of a flux tube having two applied coatings. An application of the results in thermal contact resistance models is also presented through the development of a new model. Comparisons are then made with experimental data presented in Marotta et al[1].

Thermal constriction resistance has applications in predicting the contact conductance across semi-conductor junctions and in thermal contact resistance models. Solution for the thermal constriction resistance of a planar heat source in perfect contact with a semi-infinite region has been examined by numerous researchers. Yovanovich and Antonetti[2] and Yovanovich[3] present a comprehensive review on the theory and application of constriction resistance for bare and singly coated surfaces.

Of particular interest, is the solution for the constriction resistance of an array of contacts. As the spacing between contacts approaches the characteristic dimension of the contact, it becomes necessary to model the contact as a heat source in perfect contact with an insulated semi-infinite cylinder or flux tube. The theory of flux tubes is presented in Yovanovich[3] for bare surfaces and in Negus, Yovanovich, and Beck[4] for bare surfaces having arbitrarily shaped contacts. Antonetti[5] presents the complete solution for a circular flux tube with a single and double coating.

Problem Statement and Solution

The contact between two conforming rough surfaces in a vacuum may be modelled as an array of circular contact spots. The total heat transfer is then determined by combining all of the elemental flux tubes in parallel. The governing equation for each elemental flux tube is Laplace's equation in circular cylinder coordinates. If the flux tube is composed of N layers in the axial direction as shown in Fig. 1, then Laplace's equation must be written for each layer, resulting in a system of N equations and $2(N+1)$ boundary conditions.

A system consisting of a cylindrical substrate and two base coatings is presented in Fig. 2. The governing equation in each coating and substrate is

$$\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r} + \frac{\partial^2 T_i}{\partial z^2} = 0 \tag{1}$$

for $i = 1, 2, 3$. The appropriate boundary conditions are summarized in Table 1.

Solution for Temperature Distribution in Each Layer

The resulting system of three equations and eight boundary conditions is easily solved by analytical methods. Solutions to heat conduction problems in composite systems using integral transforms and separation of variables are discussed in Ozisik[6]. The problem as stated above, may be solved by separation of variables. Applying the method of separation of variables results in

$$T_i(r,z) = J_0(\lambda r)[A_i e^{-\lambda z} + B_i e^{\lambda z}] \tag{2}$$

where the Bessel function $Y_0(\lambda r)$ has been eliminated due to the singularity at $r = 0$. Application of the boundary condition along $r = b$ yields the characteristic equation

$$J_1(\delta_n) = 0 \tag{3}$$

where $\delta_n = \lambda_n b$. The eigenvalues for this equation are well tabulated in Abramowitz and Stegun[7], or they may be approximated to single precision using Stokes' approximation (Gray and Mathews[8]). A modified Stokes approximation developed by Yovanovich[9] which provides greater accuracy is given below:

$$\delta_n = \frac{\beta_n}{4}\left[1 - \frac{6}{\beta_n^2} + \frac{6}{\beta_n^4} - \frac{4716}{5\beta_n^6} + \frac{3902918}{70\beta_n^8}\right] \tag{4}$$

where $\beta_n = \pi(4n+1)$ and $n \geq 3$.

Table 1 Boundary conditions

	Radial	$z \geq 0, i = 1,2,3$
i	$r = 0$	$\left.\frac{\partial T_i}{\partial r}\right\vert_{r=0} = 0$
ii	$r = b$	$\left.\frac{\partial T_i}{\partial r}\right\vert_{r=b} = 0$
	Axial	$0 \leq r \leq b$
iii	$z \to \infty$	$T_3(r, z \to \infty) = 0$
iv	$z = z_2 = t_1 + t_2$	$\begin{cases} T_2(r,z_2) = T_3(r,z_2) \\ k_2 \left.\frac{\partial T_2}{\partial z}\right\vert_{z=z_2} = k_3 \left.\frac{\partial T_3}{\partial z}\right\vert_{z=z_2} \end{cases}$
v	$z = z_1 = t_1$	$\begin{cases} T_1(r,z_1) = T_2(r,z_1) \\ k_1 \left.\frac{\partial T_1}{\partial z}\right\vert_{z=z_1} = k_2 \left.\frac{\partial T_2}{\partial z}\right\vert_{z=z_1} \end{cases}$
vi	$z = 0$	$\begin{cases} \left.\frac{\partial T_1}{\partial z}\right\vert_{z=0} = -\frac{Q}{\pi a^2 k_1}, & 0 \leq r < a \\ \left.\frac{\partial T_1}{\partial z}\right\vert_{z=0} = 0, & a < r \leq b \end{cases}$

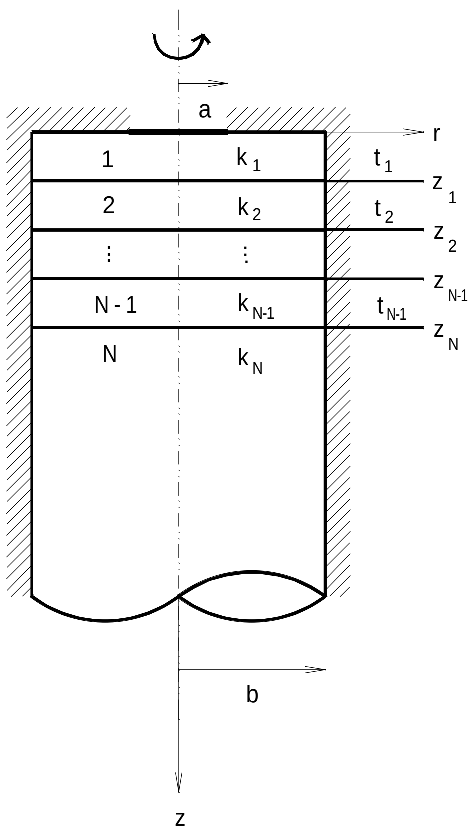

Fig. 1 Multilayered flux tube

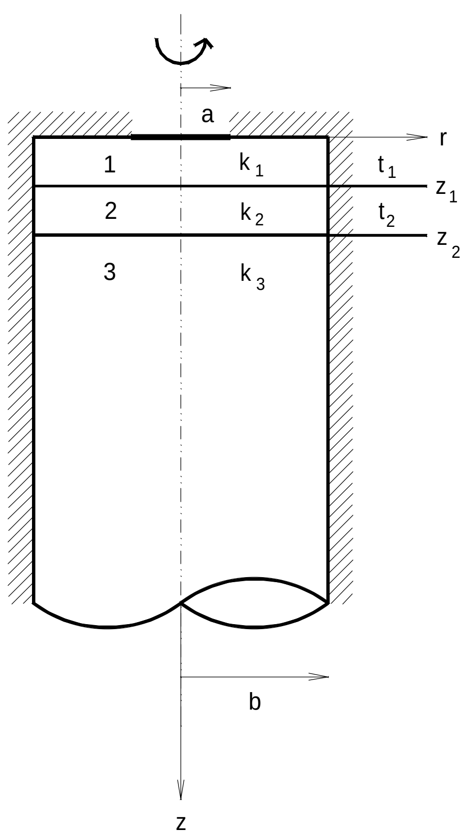

Fig. 2 Flux tube with two applied coatings

In this paper, exact values are used for the first five eigenvalues and the remainder are computed using Eq. (4). The solution for each coating becomes

$$T_1(r,z) = \sum_{n=1}^{\infty} J_0(\lambda_n r)[A_{1_n} e^{-\lambda_n z} + B_{1_n} e^{\lambda_n z}] \tag{5}$$

and

$$T_2(r,z) = \sum_{n=1}^{\infty} J_0(\lambda_n r)[A_{2_n} e^{-\lambda_n z} + B_{2_n} e^{\lambda_n z}] \tag{6}$$

Now eliminating the singularity at $z \to \infty$ requires that $B_{3_n} = 0$, and the solution for the temperature in the substrate becomes

$$T_3(r,z) = \sum_{n=1}^{\infty} A_{3_n} J_0(\lambda_n r) e^{-\lambda_n z} \tag{7}$$

The five constants may be determined by applying the remaining boundary conditions at each interface and at the contact plane. Application of Eqs. (iii-v) in Table 1 results in a system of four equations which may be solved for the constants: $A_{1_n}, B_{1_n}, A_{2_n}$, and B_{2_n}. The solution to this system of equations was easily obtained using the Computer Algebra Systems *Maple*[10] and *Mathematica*[11]. The four constants in terms of the unknown constant A_{3_n} are

$$A_{1_n} = \frac{A_{3_n}}{4} \Big((1+K_{21})(1+K_{32}) + (1-K_{21})(1-K_{32}) e^{-2\delta_n \epsilon \tau_2}\Big) \tag{8}$$

$$B_{1_n} = \frac{A_{3_n}}{4} \Big((1-K_{21})(1+K_{32}) e^{-2\delta_n \epsilon \tau_1} + (1+K_{21})(1-K_{32}) e^{-2\delta_n \epsilon(\tau_1+\tau_2)}\Big) \tag{9}$$

$$A_{2_n} = \frac{A_{3_n}}{2}(1+K_{32}) \tag{10}$$

$$B_{2_n} = \frac{A_{3_n}}{2}(1-K_{32}) e^{-2\delta_n \epsilon(\tau_1+\tau_2)} \tag{11}$$

where $K_{21} = k_2/k_1$,$K_{32} = k_3/k_2$ are the relative conductivity of adjacent layers, $\tau_1 = t_1/a$, $\tau_2 = t_2/a$ are the relative thicknesses of each coating, $\delta_n = \lambda_n b$ are the eigenvalues of Eq. (3), and $\epsilon = a/b$ is the contact spot aspect ratio.

The final constant A_{3_n} is obtained by taking a Fourier-Bessel series expansion of Eq. (vi) in Table 1. This results in

$$A_{3_n} = \frac{8}{\pi} \frac{Q}{k_1 a} \frac{J_1(\delta_n \epsilon)}{\delta_n^2 J_0^2(\delta_n) \gamma_n} \tag{12}$$

where the dimensionless parameter γ_n which accounts for the effects of each layer's conductivity and thickness, is defined as

$$\begin{aligned} \gamma_n = & (1+K_{21})(1+K_{32}) \\ & -(1-K_{21})(1+K_{32})e^{-2\delta_n\epsilon\tau_1} \\ & +(1-K_{21})(1-K_{32})e^{-2\delta_n\epsilon\tau_2} \\ & -(1+K_{21})(1-K_{32})e^{-2\delta_n\epsilon(\tau_1+\tau_2)} \end{aligned} \quad (13)$$

Contact Area Boundary Conditions

The solution given above may also be obtained for an arbitrarily prescribed heat flux. Two cases which are often considered are the isoflux and isothermal contacts. In the case of an isoflux contact, the temperature profile which results is parabolic, with the maximum temperature occurring at the centroid of the contact (see Fig. 3). Alternatively, if one prescribes a parabolic heat flux distribution with the minimum at the centroid of the contact spot, a uniform temperature distribution will result. The equations presented so far are for the isoflux case.

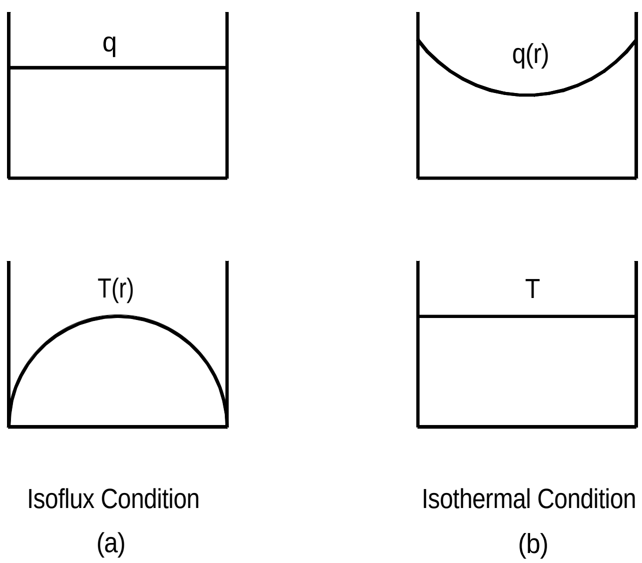

Fig. 3 Isoflux and isothermal boundary conditions

Yovanovich[12] and Negus, Yovanovich and Thompson[13] discuss the solution for constriction resistance in semi-infinite domains for uncoated and coated surfaces for arbitrary heat flux distributions. Using the results of Yovanovich[12], it can be shown that the isothermal contact may be modelled using the isoflux case by simply multiplying each term in the series by the factor

$$\rho_n = \begin{cases} \dfrac{sin(\delta_n\epsilon)}{2J_1(\delta_n\epsilon)} & isothermal \\ \\ 1 & isoflux \end{cases} \quad (14)$$

The results presented in this paper are based upon the isothermal contact condition, rather than the isoflux contact condition. The difference between these two cases is approximately eight percent as $\epsilon \to 0$ and $\tau_1 \to \infty$. In this limit, the solution approaches that of an isolated contact on a semi-infinite region.

Thermal Constriction Parameter

In order to use the results of the previous section in contact resistance models, we must define the constriction resistance and the dimensionless constriction parameter. The constriction resistance is defined as

$$R_c = \frac{\overline{T}_{contact} - \overline{T}_{cp}}{Q} \quad (15)$$

where

$$\overline{T}_{contact} = \frac{1}{\pi a^2}\int_0^a T_1(r,0)2\pi r dr \quad (16)$$

is the mean temperature of the contact spot, and

$$\overline{T}_{cp} = \frac{1}{\pi b^2}\int_0^b T_1(r,0)2\pi r dr \quad (17)$$

is the mean temperature of the contact plane.

The constriction parameter ψ is defined with respect to the substrate thermal conductivity k_3:

$$\psi = 4k_3\mathcal{L}R_c \quad (18)$$

where $\mathcal{L}$ is some characteristic length of the contact spot geometry. For the case of a circular contact, $\mathcal{L} = a$, the radius of the contact. Extension of this solution for non-circular contacts is discussed later, and an alternative length $\mathcal{L}$ is proposed.

Special Cases

In order to examine the effect that each coating has on the constriction resistance the solution for the constriction parameter in the singly and doubly coated contacts will be presented in terms of the bare surface constriction parameter. The constriction parameters for the singly coated and bare surfaces may be obtained as special cases from the constriction parameter for the doubly coated surface.

Bare Surface

By setting $K_{21} = 1$ and $K_{32} = 1$, the bare contact constriction parameter becomes

$$\psi_{bare} = \frac{16}{\pi\epsilon}\sum_{n=1}^{\infty}\rho_n\frac{J_1^2(\delta_n\epsilon)}{\delta_n^3 J_0^2(\delta_n)} \quad (19)$$

Simple correlations which may be used in place of Eq. (19) have been developed by Yovanovich[3] which cover the range $0 \leq \epsilon \leq 0.9$ with a maximum error of 0.02 percent. These are

$$\psi^T_{bare}(\epsilon) = 1 - 1.40978\epsilon + 0.34406\epsilon^3 + 0.04305\epsilon^5 + 0.02271\epsilon^7 \quad (20)$$

and

$$\psi^q_{bare}(\epsilon) = 1.08076 - 1.41042\epsilon + 0.26604\epsilon^3 - 0.00016\epsilon^5 + 0.058266\epsilon^7 \quad (21)$$

where the superscripts T and q indicate the isothermal and isoflux boundary conditions respectively.

Single Coated Surface

By setting $K_{32} = 1$ the single layer contact constriction parameter becomes

$$\psi_{single} = \frac{16}{\pi\epsilon} \sum_{n=1}^{\infty} \rho_n \frac{J_1^2(\delta_n \epsilon)}{\delta_n^3 J_0^2(\delta_n)} * K_{21} \frac{(1+K_{21}) + (1-K_{21})e^{-2\delta_n \epsilon \tau_1}}{(1+K_{21}) - (1-K_{21})e^{-2\delta_n \epsilon \tau_1}} \quad (22)$$

Double Coated Surface

The two layer contact constriction parameter is

$$\psi_{double} = \frac{16}{\pi\epsilon} \sum_{n=1}^{\infty} \rho_n \frac{J_1^2(\delta_n \epsilon)}{\delta_n^3 J_0^2(\delta_n)} \frac{\phi^+}{\phi^-} K_{21} K_{32} \quad (23)$$

where

$$\begin{aligned} \phi^{\pm} &= (1+K_{21})(1+K_{32}) \\ &\pm (1-K_{21})(1+K_{32})e^{-2\delta_n \epsilon \tau_1} \\ &+ (1-K_{21})(1-K_{32})e^{-2\delta_n \epsilon \tau_2} \\ &\pm (1+K_{21})(1-K_{32})e^{-2\delta_n \epsilon (\tau_1+\tau_2)} \end{aligned} \quad (24)$$

The effect that each coating has on the constriction parameter is easily seen in Eqs. (19, 22 and 23). Antonetti[5] computed results for the bare surface constriction parameter and also tabulated values of the constriction resistance correction parameter for a wide range of parameters. The correction parameter for the constriction resistance in a layered system is defined as

$$C_L = \frac{\psi_{single,double}}{\psi_{bare}} \quad (25)$$

Results for the single layer constriction correction parameter have been reported in graphical form in Antonetti[5] and Antonetti and Yovanovich[14]. Tabulation of the double layer constriction parameter would be too involved due to the large number of parameters involved. In a later section, a parametric analysis is conducted for comparison of experimental data with a new contact conductance model.

Isolated Contact $\epsilon \rightarrow 0$

As $\epsilon \rightarrow 0$, the contact becomes isolated and the solution for a single contact on a half-space is obtained. Computing this special case requires several thousand terms, thus the half-space solution should be used instead, if computing resources are limited. However, with most computer algebra systems such as *Maple*[10] and *Mathematica*[11], the computation time is quite reasonable and there is no need to resort to the half-space solutions. The interested reader should refer to Negus, Yovanovich, and Thompson[13] for the procedure to obtain the half-space contact solution.

Effect of Contact Spot Geometry

In many applications of constriction resistance the contact spot may not be circular. Other common shapes include square and triangular contacts. The model presented above is easily modified to account for a contact spot of arbitrary shape. The effect of contact spot geometry on constriction resistance was studied by Yovanovich, Burde and Thompson[15]. It was shown that the bare surface constriction parameter for an isolated contact on a semi-infinite region is a weak function of geometry when the constriction resistance is non-dimensionalized using $\mathcal{L} = \sqrt{A_c}$ as a characteristic length, where A_c is the area of the contact spot. Negus, Yovanovich, and Beck[4] also showed that the constriction parameters for semi-infinite flux tubes having various shapes are also weak functions of geometry if non-dimensionalized using the square root of the contact area.

It can be shown that the constriction parameter for the singly and doubly coated contacts are also weak functions of the contact spot geometry. Thus the solution given above may also be used for contact spots of arbitrary shape if the constriction parameter is defined as

$$\psi = 4k_3\sqrt{A_c}R_c \quad (26)$$

and the relative contact spot size is defined as

$$\epsilon = \sqrt{A_c/A_t} \quad (27)$$

where A_t is the cross-sectional area of the flux tube. The equivalent contact spot radius a is chosen such that $a = \sqrt{A_c/\pi}$.

Application in Thermal Contact Conductance Models

An important application of thermal constriction resistance arises in the prediction of the thermal contact resistance between two contacting, nominally flat, rough surfaces. In many applications the contact conductance is enhanced if one of the surfaces is coated with a high conductivity material such as a metallic coating. In certain instances it is necessary to apply an intermediate coating to promote the adherence of the metallic coating. It is therefore desirable to assess the overall effect that each coating has on the enhancement (or reduction) of the thermal contact conductance. The authors have derived a general expression for determining the contact conductance of a doubly coated substrate in contact with a bare surface. Comparisons are then made to experimental data for Diamond-Like Coatings (DLC) which are presented in Marotta et al[1].

Contact Conductance of Coated Interfaces

Contact resistance in a vacuum environment for the configuration shown in Fig. 4 is given by Antonnetti[5]:

$$R_c = \frac{1}{h_c A_a} = \frac{1}{N}\left[\frac{\psi_{bare}}{4k_0 a} + \frac{\psi_{coated}}{4k_3 a}\right] \qquad (28)$$

where a is the mean contact spot radius, k_0 and k_3 are the conductivities of the upper bare surface and the substrate of the lower surface (see Fig. 4), h_c is the contact conductance, A_a is the apparent contact area, N is the total number of contact spots and ψ_{bare} and ψ_{coated} are the thermal constriction parameters for the bare and coated surfaces respectively.

Therefore we have

$$h_c = \frac{2}{A_a}\left[\frac{2k_0k_3}{(k_3 + k_0\psi_{coated}/\psi_{bare})}\right]\frac{Na}{\psi_{bare}} \qquad (29)$$

From Yovanovich[16] for a bare interface, the contact conductance is given by

$$h_{c,bare} = \frac{2}{A_a}\frac{2k_0k_3}{k_0+k_3}\frac{Na}{\psi_{bare}} \qquad (30)$$

where k_0 and k_3 represent the conductivities of the two bare surfaces in contact. Comparing the expression for contact conductance for the coated configuration in Fig. 4 and the bare interface given by Yovanovich[16] we can rewrite the expression for contact conductance for the coated interface as follows:

$$h_c = h_{c,bare}\frac{k_0+k_3}{k_3+C_Lk_0} = h_{c,bare}TEF \qquad (31)$$

where TEF $= (k_0+k_3)/(k_3+C_Lk_0)$ is called the thermal enhancement factor and $C_L = \psi_{coated}/\psi_{bare}$ is the constriction correction factor defined earlier.

From Yovanovich[16] $h_{c,bare}$ is correlated as:

$$h_{c,bare} = 1.25\left(\frac{m}{\sigma}\right)k_s\left(\frac{P}{H_c}\right)^{0.95} \qquad (32)$$

Therefore,

$$h_c = 1.25\left(\frac{m}{\sigma}\right)k_s\left(\frac{P}{H_c}\right)^{0.95}\text{TEF} \qquad (33)$$

where $k_s = 2k_0k_3/(k_0+k_3)$ is the harmonic mean thermal conductivity of the bare interface. If the TEF > 1 there is an enhancement in the contact conductance over the bare interface due to coatings.

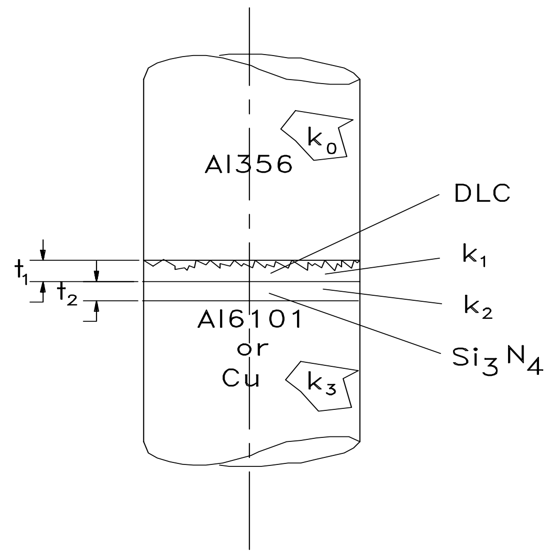

Fig. 4 Configurations considered for parametric analysis

Parametric Analysis and Comparison with Data

In this section we examine the effect of using Diamond-Like Coatings (DLC) to enhance contact conductance. In practice DLC's cannot be directly applied on a substrate. Once the substrate surfaces are prepared, each test surface must be coated with a 3 μm layer of silicon nitride (see Marotta et al[1]). This coating was necessary to ensure the stability of the surface for the deposition of a DLC.

The experimental study of Marotta et al[1] included two types of interfaces, each with three to four different coating thicknesses. The first type consisted of two aluminum substrates with upper specimen bare and lower specimen coated with silicon nitride and DLC on top of it. The silicon nitride coating thickness was fixed at 3 μm whereas the thickness of the DLC was varied from 0 μm (i.e. no coating) to 5 μm. The second type consisted of the same bare aluminum alloy on top but the substrate of the coated specimen was changed to copper.

In the present work a comparison is made of the experimental results from Marotta et al[1] with the present model using the bulk values of thermal conductivities. Then an estimate of the actual thermal conductivies of the coatings will be made with the aid of the present model.

In order to generate parametric plots of contact conductance h_c versus dimensionless contact pressure P/H_c one must calculate the correction factor C_L at each value of applied pressure for the given surface, material and thermal properties. In order to calculate the correction factor C_L at particular values of the dimensionless contact pressure (P/H_c) one requires the thermal conductivities: k_0, k_1, k_2 and k_3, the thicknesses of the two coatings: t_1 and t_2 and the mean contact spot radius a. The mean contact spot radius is determined using the approximation developed by Sridhar[17]:

$$a = 0.645\frac{\sigma}{m}\left(\frac{P}{H_c}\right)^{0.071} \tag{34}$$

The upper aluminum specimen (Al356) is the softer one and it is assumed to undergo full plastic deformation. It is known from past experience (Nho[18]) that aluminum alloys do not generally possess a hard surface layer and the microhardness of the alloy is almost equal to the bulk hardness. Based on this assumption, the experimental data from Marotta et al[1] were reduced using a single hardness value $H_c = 1256\ MPa$.

The correction factor C_L, TEF, and thus contact conductance h_c, were computed by means of the Computer Algebra System *Mathematica*[11], using about two thousand terms for each computation for the configurations tested by Marotta et al[1]. Each computation of contact conductance took about 80 seconds on a PC with 16 Megabytes of memory and a 486 DX4 100 MHz CPU.

Figure 5 (a) shows a plot of contact conductance versus dimensionless plastic contact pressure for the configuration without the DLC. The upper and lower specimens are aluminum (see Fig. 4) with a 3 μm silicon nitride coating on the lower specimen. The surface roughness parameter σ/m was varied from 5 μm to 60 μm. The data had a roughness $\sigma/m = 5.6\ \mu m$. The sampling interval at which the roughness data was obtained was not available. The computed values lie well above the data most probably because the conductivities of DLC and silicon nitride used were that of the bulk material which are 2100 $W/(mK)$ and 15 $W/(mK)$ respectively.

A similar trend is seen in Figs. 5 (b) – 5 (d) where the DLC is introduced and its thickness is varied. The experimental data lie well below the predictions. The roughness parameter σ/m for data in Figs. 5 (b) – 5 (d) are between 5.8 μm and 6.0 μm.

Predictions for the second configuration where the lower substrate is changed to copper are shown in Figs. 6 (a) – 6 (d). Marotta et al[1] did not report contact conductance data for the configuration without the DLC. Comparison of data with the predictions in Figs. 6 (b) – 6 (d) are similar to the previous configuration with data falling well below the model.

The numerical values of the thermal enhancement factor (TEF) for each set of computations are included in Figs. 5 (a)–(d) and Figs. 6 (a)–(d). If the TEF is greater than 1 there is enhancement over a bare interface for the same configuration, and if TEF is less than 1 there is a decrease in heat transfer through the joint over the bare interface. The TEF was found to be almost independent of applied load for the computations in this work and hence a single value for the three computations is reported.

It is clearly seen from the comparisons that the model overpredicts the data with bulk values of conductivities for thin films. It has been shown by Lambropoulos et al[19,20] through measurements of the thermal conductivity of thin films that the value may be as much as two orders of magnitude lower than that of the corresponding bulk solid. A comparison of bulk and film conductivity is presented in Table 2. The measurements were made in air for a wide variety of thin films of oxides, flourides, nitrides, amorphous metals and superconductors.

Table 2 Thermal conductivity of bulk materials and thin films[20]

Material	k_{film} (W/mK)	k_{bulk} (W/mK)
SiO_2	$0.4 - 1.1$	$1.2 - 10.7$
TiO_2	$0.5 - 0.6$	$7.4 - 10.4$
ZrO_2	0.04	–
Al_2O_3	0.72	$20 - 46$
MgF_2	0.58	$15 - 30$
Air	–	0.025
Oxides/Fluorides	–	$1.0 - 10$
Diamond I, II	–	$1200 - 2300$
Silicon	–	150

With the aid of the experimental data of Marotta et al[1], an attempt is made to estimate the conductivity of coatings by decreasing the conductivity of silicon nitride and DLC in the model until the model and the data coincide. The first step was to estimate the conductivity of silicon nitride coating. Figure 7 shows a comparison between the model (i.e. computed values) and data for different values of conductivity. As the conductivity of the silicon nitride coating is decreased from its bulk value of 15 $W/(mK)$ to 2.4 $W/(mK)$ the computed values coincide with the experimental data.

Having estimated the conductivity of the silicon nitride coating, the conductivity of the DLC coating was then determined by again decreasing the conductivity and comparing with data (see Fig. 8). For the configuration shown in Fig. 8 the conductivity of 1 μm DLC coating was 0.05 $W/(mK)$.

A similar estimate of the conductivities for different configurations tested by Marotta et al[1] was made. It was found that the conductivity of 3 μm and 5 μm thick coatings of DLC on Al6106 was between 0.02 $W/(mK)$ – 0.03 $W/(mK)$. The conductivities of the coatings on copper were slightly lower with 1 μm coating having a conductivity between 0.035 $W/(mK)$ – 0.04 $W/(mK)$. The thicker coatings (i.e. 3 μm and 5 μm) had conductivities ranging from 0.015 $W/(mK)$ – 0.02 $W/(mK)$.

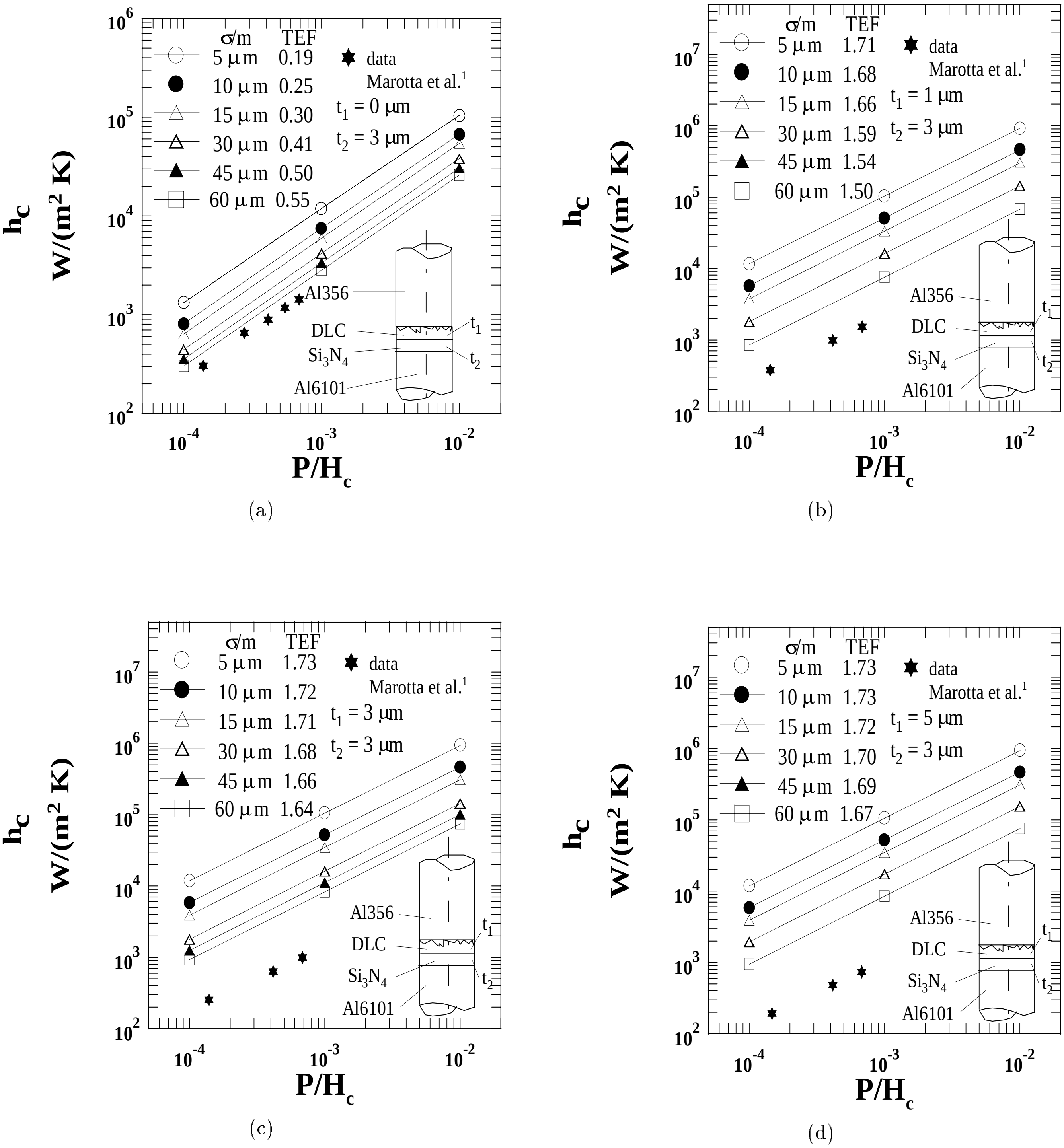

Fig. 5 Contact conductance versus dimensionless contact pressure for different interface roughnesses for specimens with aluminum substrate

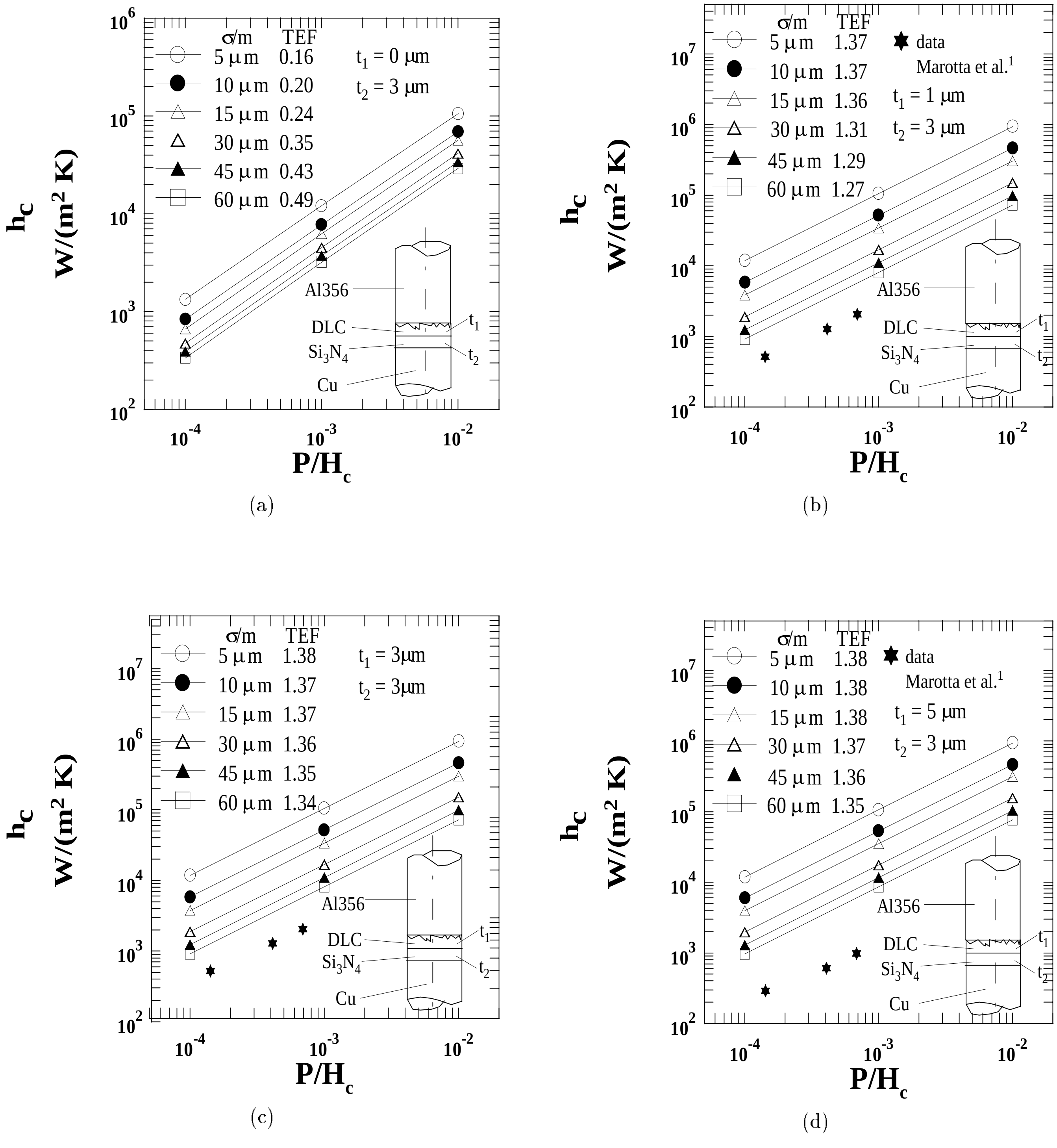

Fig. 6 Contact conductance versus dimensionless contact pressure for different interface roughnesses for specimens with copper substrate

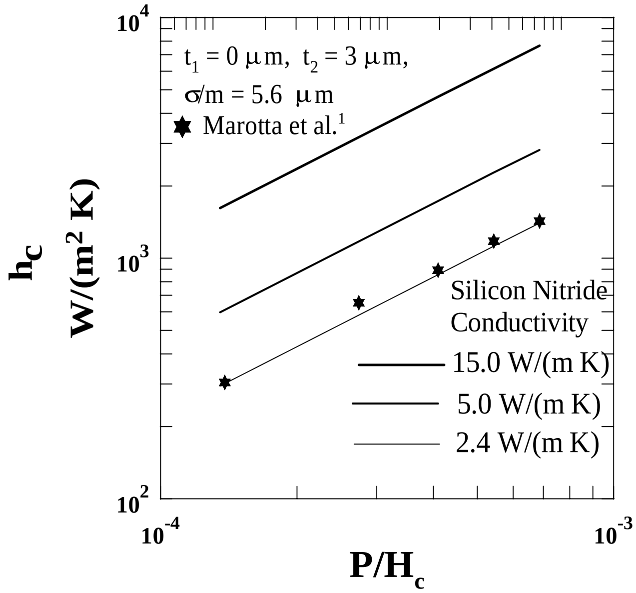

Fig. 7 Estimate of the conductivity of the Silicon Nitride layer for a typical interface

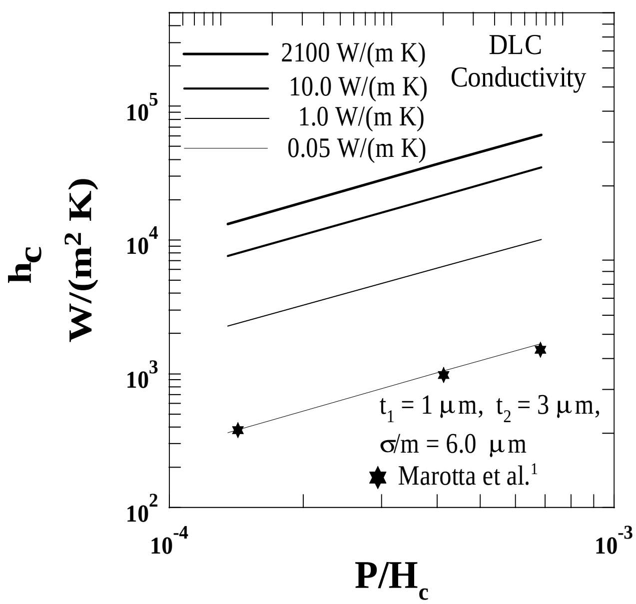

Fig. 8 Estimate of the conductivity of the DLC layer for a typical interface

Comparison of the predicted values of thermal conductivity compare quite well with data reported in Lambropoulos et al[19,20], where the authors reported measurements for the thermal conductivity of thin films of various materials. It should also be noted that the predictions of Lambropoulos et al[19,20] were computed using the analysis of Dryden[21] for a point contact on a single coated half-space. The method outlined earlier for fitting the experimental data of Marotta et al[1] to the analytic model is similar to the procedure used by Lambropoulos et al[19,20] to determine the conductivity of thin films. Thus, an alternate method for determining the conductivity of thin films has been developed.

Summary and Conclusions

This paper presents the general theory for determining the constriction resistance for an isoflux or isothermal planar heat source in contact with a multilayered semi-infinite flux tube. The solution was presented for several special cases which result for particular combinations of the physical parameters. In addition, extension of the solution for contacts of arbitary shape was also discussed.

The solution to the governing equations and computation of numerical results were performed using the Computer Algebra Systems *Maple*[10] and *Mathematica*[11]. Both of these packages are capable of performing symbolic and numerical computations, and provide an efficient means for computing the special functions which appear in the solutions.

Finally, a simple application of the theory of constriction resistance in multilayered contacts was discussed for the particular case of predicting the thermal contact resistance between two contacting planes. In this particular case, one of the substrates has been coated in order to enhance the thermal contact conductance between planes. It was found that the experimental data fell below the values computed using the model when the bulk values of the thermal properties were used. However, the correct trend in the data was predicted by the model. By using the model to match the experimental data, the thermal conductivity of each layer was predicted. The resulting values were much smaller than the reported bulk properties, but compared quite well with experimental results reported for thin films.

Acknowledgements

The authors acknowledege the support of Natural Sciences and Engineering Research Council operating grant A6490. The authors also thank E. Marotta for his advice and information on the thermal conductivities of dielectric films.

References

[1]Marotta, E.E., Blanchard, D.G., and Fletcher, L.S., 1993, "The Thermal Contact Conductance of Diamond-Like Films", AIAA paper 93-0845.

[2]Yovanovich, M.M. and Antonetti, V.W., 1984, "Application of Thermal Contact Resistance Theory to Electronic Packages", in *Advances in Thermal Modeling of Electronic Components and Systems Vol. 1*, edited by A. Bar-Cohen and A.D. Kraus, pp. 79-128.

[3]Yovanovich, M.M., 1987, "Theory and Applications of Constriction and Spreading Resistance Concepts for Microelectronics Thermal Management", in *Advances in Cooling Techniques for Computers*, edited by W. Aung, pp. 277-332.

[4]Negus, K.J., Yovanovich, M.M., and Beck, J.V., 1989, "On the Non-dimensionalization of Constriction Resistance for Semi-infinite Heat Flux Tubes", *Journal of Heat Transfer*, Vol. 111, pp. 804-807.

[5]Antonetti, V.W., 1983, *On the Use of Metallic Coatings to Enhance Thermal Contact Conductance*, Ph.D. Thesis, University of Waterloo.

[6]Ozisik, N., 1968, *Boundary Value Problems of Heat Conduction*, International Textbook Company.

[7]Abramowitz, M. and Stegun, I.A., 1965, *Handbook of Mathematical Functions*, Dover Publications.

[8]Gray, A. and Mathews, G.B., 1952, *A Treatise on Bessel Functions and Their Applications to Physics*, Macmillan and Co. Limited.

[9]Yovanovich, M.M., 1993, *ME 651 Advanced Heat Conduction*, Course Notes, University of Waterloo.

[10]*Maple*, 1984-1996, Waterloo Maple Software, Waterloo, Ontario, Canada.

[11]*Mathematica*, 1987-1996, Wolfram Research Incorporated, Champaign, Illinois, USA.

[12]Yovanovich, M.M., 1976 "General Expression for Circular Constriction Resistances for Arbitrary Heat Flux Distribution", Progress in Astronautics and Aeronautics: Radiative Transfer and Thermal Control, edited by A. M. Smith, AIAA, Vol. 49, pp. 301-308.

[13]Negus, K.J., Yovanovich, M.M., and Thompson, J.C., 1988, "Constriction Resistance of Circular Contacts on Coated Surfaces: Effect of Boundary Conditions", *Journal of Thermophysics and Heat Transfer*, Vol. 2 , pp. 158-164.

[14]Antonetti, V.W. and Yovanovich, M.M., 1984, "Enhancement of Thermal Contact Conductance by Metallic Coatings: Theory and Experiment", *Journal of Heat Transfer*, Vol. 107, pp. 513-519.

[15]Yovanovich, M.M., Burde, S.S., and Thompson, J.C., 1977, "Thermal Constriction Resistance of Arbitrary Planar Contacts with Constant Flux", AIAA Progress in Astronautics and Aeronautics, Thermophysics of Spacecraft and Outer Planet Entry Probes, Vol. 56, edited by A.M. Smith, pp. 127-139.

[16]Yovanovich, M.M., 1982, "Thermal Contact Correlations", Spacecraft Radiative Transfer and Temperature Control, Edited by T.E. Horton, Vol. 83 of Progress in Astronautics and Aeronautics, New York, pp. 82-95.

[17]Sridhar, M.R., 1994, *Elastoplastic Contact Models for Sphere-Flat and Conforming Rough Surface Applications*, Ph.D. Thesis, University of Waterloo, Canada.

[18]Nho, K.M., 1990, *Experimental Investigation of Heat Flow Rate and Directional Effect on Contact Conductance of Anisotropic Ground/Lapped Interfaces*, Ph.D. Thesis, University of Waterloo, Canada.

[19]Lambropoulous J.C., Jolly M.R., Amsden C.A., Gilman S.E., Sinicropi, M.J., Diakomihalis, D., and Jacobs, S.D., 1989, "Thermal Conductivity of Dielectric Films", *Journal of Applied Physics*, Vol. 66, pp. 4230-4242.

[20]Lambropoulous J.C., Jacobs S.D., Burns S.J., Shaw-Klien L. and Hwang S.S., 1991, "Thermal Conductivity of Thin Films: Measurement and Microstructural Effects", HTD- Vol. 184, Film Heat Transfer - Properties and Processing, Book No. H00671.

[21]Dryden, J. R., 1983, "The Effect of a Surface Coating on the Constriction Resistance of a Spot on an Infinite Half Plane", *Journal of Heat Transfer*, Vol. 105, pp. 408-410.