Pressure Loss Modeling for Surface Mounted Cuboid-Shaped Packages in Channel Flow

Pete Teertstra, M. Michael Yovanovich, and J. Richard Culham

Abstract—**An analytical model is presented that predicts pressure loss for fully developed flow for air in a parallel plate channel with an array of uniformly-sized and spaced cuboid blocks on one wall. The model is intended for use in optimizing enclosure designs for air cooled electronics equipment containing arrays of printed circuit boards. Using a composite solution, based on the laminar and turbulent smooth wall channel limiting cases, the friction factor for periodic fully developed flow can be calculated as a function of the array geometry and fluid velocity. The resulting model is applicable for a full range of Reynolds numbers, 1 $\leq \mathrm{Re}_{D_h} \leq$ 100000 and accurately predicts the available measured values to within a 15% average difference.**

Index Terms—**Friction factor, parallel plate channel, pressure loss.**

NOMENCLATURE

A	Duct cross-sectional area (m^2).
$\mathcal{A}, \mathcal{B}$	Composite solution coefficients.
B	Block height (m).
D_h	Hydraulic diameter, $\equiv 4A/P$ (m).
f_{D_h}	Friction factor, (2).
H	Channel height (m).
L	Block length (m).
N	Number of blocks.
p	Pressure (N/m^2).
P	Duct perimeter (m).
Re_{D_h}	Reynolds number, (1).
s	Effective flow distance (m).
S	Block spacing (m).
V	Average velocity (m/s).
W	Channel width (m).
x	Coordinate in flow direction.
X	Distance in flow direction (m).
Greek Symbols	
ν	Kinematic viscosity (m^2/s).
ρ	Mass density (kg/m^3).
χ, γ, ξ, ζ	Coefficient parameters.
Subscripts	
a	Array parameter.

I. INTRODUCTION

THE pressure loss for fully developed laminar or turbulent flow through a parallel plate channel with an array of cuboid-shaped blocks attached to one or both wall surfaces is of interest to engineers designing air cooled electronic equipment. Vented equipment enclosure designs common in the microelectronics and telecommunications industries often use an array of printed circuit cards, cooled by buoyancy-induced or forced airflow through the resulting parallel plate channels. The pressure loss for airflow through these channels, calculated as a function of board pitch, package size and spacing, and velocity, is required by system designers to optimize fan size and board pitch for single and multi-channel applications.

In order to characterize the wide range of package sizes and spacings of typical printed circuit boards, an equivalent array of uniformly-sized and spaced cuboid-shaped blocks is often used. This simplified array reduces the complexity of the problem, allowing the geometry to be fully described by the following dimensionless parameters:

$$\frac{B}{H},\ \frac{H}{L} \text{ and } \frac{L}{L+S}$$

where B and L are the height and length (width) of the blocks, S is the spacing between adjacent blocks, and H is the channel height, as shown in Fig. 1. Values for these dimensionless parameters are chosen based on averages for all packages on a "real" board.

Many researchers have published experimental pressure loss measurements for fully developed flow through a cuboid array, often as part of a combined heat transfer and pressure loss analysis. Sparrow, Niethammer, and Chaboki [1] presented fully developed, per-row pressure loss results for both uniform arrays and arrays with implanted barriers or missing blocks. Moffat, Arvizu, and Ortega [2] reported empirical results for six different block geometries while Souza-Mendes and Santos [3] examined experimentally the effects of nonuniform elements, such as double height blocks. A comparison of the pressure loss for inline and staggered arrays is reported by Wirtz and Colban [4] based on their empirical results.

Numerical simulations of the pressure loss in an array are described in two papers by Asako and Faghri [5], [6]. In the first [5], these authors report pressure loss predictions for nine array geometries for low Reynolds number, laminar flow. The latter publication [6] extends their numerical modeling to include turbulent flow through twenty four different array geometries.

Only two analytical models or correlations for pressure drop are available in the current literature. Ashiwake *et al.* [7] presented an analytical model for the per-row pressure drop that predicted their empirical results for turbulent flow through three array geometries to within 15%. Based on their extensive measurements of local pressure loss for nine

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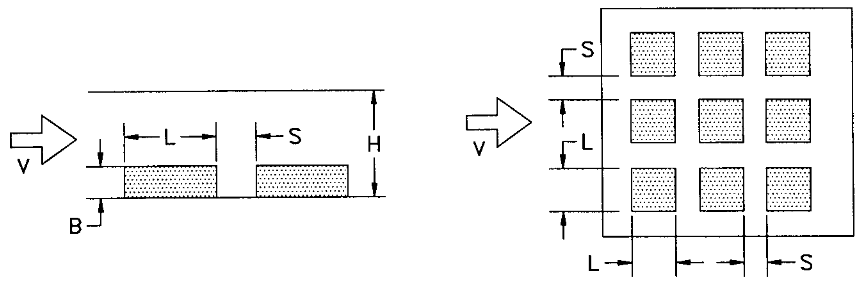


Fig. 1. Schematic of array geometry.

different array geometries, Molki, Faghri, and Ozbay [8] developed a correlation of the local pressure loss which predicted their empirical results to within 54%.

Each of the empirical or numerical studies conclude with either a graphical or tabular presentation of the pressure loss or a correlation applicable for only a limited range of values. No models are currently available in the literature to predict fully developed flow pressure loss as a function of array geometry for the full range of Reynolds numbers from laminar to turbulent flow.

The goal of this research is to develop an analytical model for pressure loss for fully developed flow of air in a parallel plate channel with a uniformly-sized and spaced array of cuboid block packages attached at one wall. This model will be a function of the three dimensionless array parameters and the average velocity and will be valid for the full range of Reynolds number, $1 \leq \mathrm{Re}_{D_h} \leq 100\,000$. The resulting model will predict pressure drop for the fully developed portion of the channel flow only; the pressure drop at the channel entrance or in the developing portion of the array will not be considered.

II. Model Development

A. Parameter Definitions

Both the independent and dependent variables are nondimensionalized using the hydraulic diameter of the duct as the characteristic length. The Reynolds number is defined as

$$\mathrm{Re}_{D_h} = \frac{VD_h}{\nu} \tag{1}$$

where, in the limit of a smooth wall channel, the hydraulic diameter can be shown to equal twice the channel height, $D_h = 2H$. For the dependent variable, the friction factor is used for the nondimensionalization in order to allow the resulting model to characterize the pressure loss by a single value. The friction factor, defined by

$$f_{D_h} = \frac{-\left(\frac{dp}{dx}\right)D_h}{\frac{1}{2}\rho V^2} \tag{2}$$

becomes independent of position when periodic, fully developed flow is achieved; that is, when the pressure gradient in the flow direction becomes constant. Once again, the hydraulic diameter of the duct is chosen as the characteristic length.

B. Available Solutions

Solutions for the fully developed friction factor, as a function of Reynolds number, are available for the limiting case of a smooth wall channel, achieved when either $B/H \to 0$ or $L/(L+S) \to 0$. For laminar flow, an analytical solution can be developed based on a solution of the x-momentum equation for fully developed flow. The resulting expression for the wall shear is substituted into the definition of the Darcy friction factor [9] to give

$$f_{2H} = \frac{96}{\mathrm{Re}_{2H}} \tag{3}$$

where the Reynolds number definition from (1) with $D_h = 2H$ has been used.

A number of correlations of empirical results for turbulent, fully developed flow are available in the literature. Patel and Head [10] present the following correlation:

$$f_{2H} = \frac{0.167}{\mathrm{Re}_{2H}^{1/6}} \tag{4}$$

valid for the range $5000 \leq \mathrm{Re}_{2H} \leq 2\times\ 10^5$, while the correlation of Beavers, Sparrow and Lloyd [11], valid over the same range of Re_{2H}, differs in both the coefficient and the exponent

$$f_{2H} = \frac{0.507}{\mathrm{Re}_{2H}^{0.3}}. \tag{5}$$

Dean [12] collected data from many sources, including Patel and Head [10] and Beavers *et al.* [11], and developed a correlation valid for a wider range of Re_{2H}

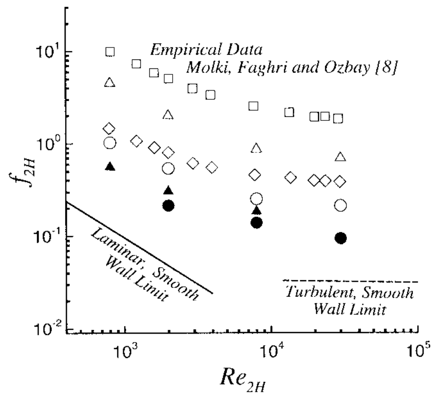
$$f_{2H} = \frac{0.347}{\mathrm{Re}_{2H}^{1/4}} \tag{6}$$

where $1.2 \times 10^4 \leq \mathrm{Re}_{2H} \leq 1.2 \times 10^6$.

The analytical model for laminar flow and the turbulent flow correlation of Dean [12] are plotted in Fig. 2, along with empirical data from Molki, Faghri, and Ozbay [8]. Fig. 2 illustrates the behavior of these limiting cases in comparison with selected array geometries.

C. Proposed Model

A close examination of the results plotted in Fig. 2 illustrate some important characteristics of the empirical data and the models for the limiting cases. First, all of the data displays a smooth transition from laminar to turbulent flow, with none



	B/H	H/L	L/(L+S)
□	0.8	0.625	0.889
△	0.67	0.75	0.889
◇	0.67	0.75	0.752
○	0.5	1.0	0.752
▲	0.5	1.0	0.67
●	0.25	2.0	0.67

Fig. 2. Friction factor for smooth channels and measured values for various arrays.

of the discontinuous transition behavior of smooth wall pipe or duct flow. Second, the plot shows that the laminar and turbulent smooth wall models act as lower limits for the friction factor data. Finally, Fig. 2 clearly demonstrates that the functional dependence of the data on the Reynolds number at both small and large Re_{2H} is virtually identical to that of the laminar and turbulent smooth wall channels, respectively.

Based on these observations, a model for the friction factor is proposed using the Churchill and Usagi [13] composite solution technique and the available analytical results

$$f_{D_h} = \left[\left(\frac{96\mathcal{A}}{\mathrm{Re}_{D_h}}\right)^3 + \left(\frac{0.347\mathcal{B}}{\mathrm{Re}_{D_h}^{1/4}}\right)^3\right]^{1/3} \tag{7}$$

where the laminar and turbulent smooth wall models are clearly used as asymptotes for this model. Recognizing that transition occurs at approximately the same range of Re_{2H} for all the data shown in Fig. 2, the coefficients $\mathcal{A}$ and $\mathcal{B}$ in (7) can be expressed as functions of geometry alone, or

$$\mathcal{A} = g_1\left(\frac{B}{H}, \frac{H}{L}, \frac{L}{L+S}\right) \tag{8}$$

$$\mathcal{B} = g_2\left(\frac{B}{H}, \frac{H}{L}, \frac{L}{L+S}\right). \tag{9}$$

These functional relationships for the coefficients will be formulated separately for laminar and turbulent flow in the following sections.

D. Laminar Flow

The coefficient for the laminar flow asymptote, $\mathcal{A}$, can be determined by introducing new definitions for hydraulic diameter, D_h, average velocity in the array, V_a and pressure gradient that depend on the geometry of the array.

The classical definition for the hydraulic diameter, based on the cross-sectional area and perimeter of the duct, is:

$$D_h = \frac{4A_a}{P_a}. \tag{10}$$

From the schematic shown in Fig. 3, the cross-sectional area and perimeter of the duct in the array can be calculated based on its physical dimensions

$$D_h = \frac{4[HW - WB + NSB]}{[W + 2H + N(L + S + 2B)]} \tag{11}$$

where N is the number of packages across the channel. Substituting the relationship

$$N = \frac{W}{L + S} \tag{12}$$

into the hydraulic diameter expression and simplifying gives

$$D_h = \frac{4\left[H - B + \frac{SB}{L+S}\right]}{\left[1 + \frac{2H}{W} + \frac{L+S+2B}{L+S}\right]}. \tag{13}$$

Introducing the parallel plate assumption $W \to \infty$ and re-arranging (13) in terms of the dimensionless geometric parameters results in a new definition for hydraulic diameter

$$D_h = 2H\frac{\left[1 - \frac{B}{H}\frac{L}{L+S}\right]}{\left[1 + \frac{B}{H}\frac{H}{L}\frac{L}{L+S}\right]}. \tag{14}$$

The average fluid velocity through the array, V_a, can be related to the average inlet velocity V by continuity

$$A_a V_a = AV \tag{15}$$

where from Fig. 3 it can be shown that

$$A = HW \tag{16}$$

$$A_a = (HW - WB + NSB). \tag{17}$$

Substituting the relationship for the number of packages N, (12), applying the parallel plate assumption $W \to \infty$, and simplifying yields

$$V_a = V\frac{1}{\left[1 - \frac{B}{H}\frac{L}{L+S}\right]}. \tag{18}$$

Using these new definitions for average velocity and hydraulic diameter, the Reynolds number for the array can be related to its smooth wall channel equivalent by

$$\mathrm{Re}_{Dh} = \frac{\mathrm{Re}_{2H}}{\left[1 - \frac{B}{H}\frac{L}{L+S}\right]} \cdot \frac{\left[1 - \frac{B}{H}\frac{L}{L+S}\right]}{\left[1 + \frac{B}{H}\frac{H}{L}\frac{L}{L+S}\right]}. \tag{19}$$

This expression can be further simplified to

$$\mathrm{Re}_{D_h} = \frac{\mathrm{Re}_{2H}}{\gamma} \tag{20}$$

where

$$\gamma = \left[1 + \frac{B}{H}\frac{H}{L}\frac{L}{L+S}\right]. \tag{21}$$

In preliminary comparisons of the laminar model, including the new definitions of D_h and V_a, with the numerical data of Asako and Faghri [5] it was found that the model

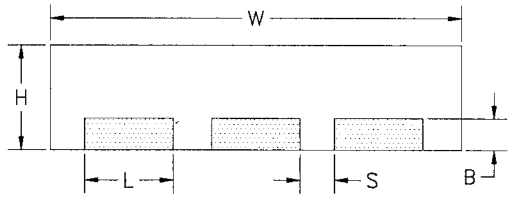


Fig. 3. Schematic of array cross section.

consistently overpredicted the friction factor. In addition, as B/H increased, the level of overprediction also increased. This behavior suggested that the predicted pressure gradient was larger than the measured value due to an underprediction of the path length over which the pressure gradient occurs.

Fig. 4(a) presents a schematic of the channel and array geometry in the flow direction, including projected laminar flow streamlines along the top and bottom walls. From these streamlines it can be deduced that for a given distance in the flow direction X, the path length for laminar (creeping) flow through the array, X_a, will be larger than X. The effective flow distance through the channel will be bounded by these two limiting values

$$X < s < X_a.$$

Therefore, f_{D_h} will overestimate the flow restriction in the channel if X is used to calculate the pressure gradient.

The path length for laminar flow over the surface of the array can be modeled by

$$X_a = N(S + L + 2B) \tag{22}$$

where the number of packages in the flow direction, N, can be related to the flow distance along the smooth channel wall by

$$N(L + S) = X. \tag{23}$$

As the ratio of package height to channel height B/H increases, it is reasonable to assume that not all of the surface area of the array will contribute to the wall shear. To include this behavior in the model, B/H is used as a weighting factor in the following expression for the effective pressure gradient for the channel

$$\frac{dp}{ds} = \frac{\Delta p}{\left[X\left(\frac{B}{H}\right) + X\left(1 - \frac{B}{H}\right)\left(\frac{S+L+2B}{L+S}\right)\right]}. \tag{24}$$

By assuming that a linear approximation can be used for the pressure gradient for fully developed flow

$$\frac{dp}{dx} \approx \frac{\Delta p}{X} \tag{25}$$

the effective pressure gradient is defined in terms of the smooth wall channel gradient

$$\frac{dp}{ds} = \frac{dp}{dx}\frac{1}{\left[\frac{B}{H} + \left(1 - \frac{B}{H}\right)\left(1 + \frac{2B}{H}\frac{H}{L}\frac{L}{L+S}\right)\right]}. \tag{26}$$

Using these definitions for hydraulic diameter, average array velocity and effective pressure gradient, the friction factor for the array can be related to the smooth wall channel result by the following simplified expression:

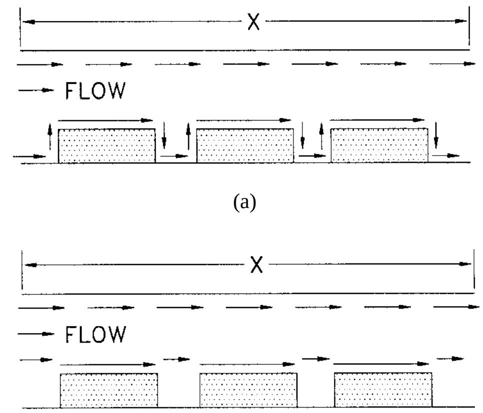


Fig. 4. Schematic of projected streamlines: (a) laminar flow and (b) turbulent flow.

$$f_{D_h} = f_{2H}\frac{\zeta^3\chi}{\gamma} \tag{27}$$

where γ is defined in (21) and

$$\zeta = \left[1 - \frac{B}{H}\frac{L}{L+S}\right] \tag{28}$$

$$\chi = \left[\frac{B}{H} + \left(1 - \frac{B}{H}\right)\left(1 + \frac{2B}{H}\frac{H}{L}\frac{L}{L+S}\right)\right]. \tag{29}$$

Therefore, the laminar asymptote for the composite solution, (7), can be determined by

$$\begin{aligned} f_{2H} &= \frac{\gamma}{\zeta^3\chi} f_{D_h} \\ &= \frac{\gamma}{\zeta^3\chi}\left(\frac{96}{\mathrm{Re}_{D_h}}\right) \\ &= \frac{\gamma^2}{\zeta^3\chi}\left(\frac{96}{\mathrm{Re}_{2H}}\right) \end{aligned} \tag{30}$$

which results in the following expression for the coefficient $\mathcal{A}$

$$\mathcal{A} = \frac{\gamma^2}{\zeta^3\chi} \tag{31}$$

where γ, ζ, and χ are defined by (21), (28), and (29), respectively.

E. Turbulent Flow

The coefficient for the turbulent flow asymptote, $\mathcal{B}$, in the composite solution is determined in a manner similar to that used in the previous section for $\mathcal{A}$. The new definitions for hydraulic diameter D_h and average velocity V_a of the array developed for laminar flow can be applied directly to this asymptote, resulting in the following relationship between the array and smooth wall channel Reynolds numbers

$$\mathrm{Re}_{D_h} = \frac{\mathrm{Re}_{2H}}{\gamma} \tag{32}$$

where γ is defined in (21).

Preliminary comparison of the turbulent asymptote formulation using the array definitions of D_h and V_a with the experimental measurements of Molki, Faghri and Ozbay [8] for $\mathrm{Re}_{2H} \approx 30\,000$ shows that the model consistently underestimates the actual pressure drop. From the projected streamlines shown in the schematic in Fig. 4(b) it can be demonstrated that for turbulent, separated flow, the flow through the array tends to "skip" from the top surface of one package to the next. Therefore, for a given distance in the flow direction X, the path length for the flow through the array, X_a, will be less than X. The effective flow distance s is bounded by these two limiting values

$$X_a < s < X$$

and the friction factor will be underestimated if X is used in place of s in the pressure gradient.

Based on the flow path suggested in Fig. 4(b), the path length for separated flow through the array can be modeled by

$$X_a = NL \tag{33}$$

where the number of packages in the flow direction can be related to X by (23).

As in the laminar case, the dimensionless block height B/H is used as a weighting parameter in the following expression for the array pressure gradient

$$\frac{dp}{ds} = \frac{\Delta p}{\left[X\left(\frac{B}{H}\right) + X\left(1 - \frac{B}{H}\right)\frac{L}{L+S}\right]}. \tag{34}$$

By assuming a linear pressure gradient in the flow direction, as in (25), the array pressure gradient for turbulent flow becomes

$$\frac{dp}{ds} = \frac{dp}{dx}\frac{1}{\left[\frac{B}{H} + \left(1 - \frac{B}{H}\right)\frac{L}{L+S}\right]}. \tag{35}$$

Substituting this expression, along with D_h and V_a into the friction factor definition gives the relationship between f_{D_h} and its smooth wall channel equivalent. Using the parameters γ and ζ defined in (21) and (28), this expression can be simplified as

$$f_{D_h} = f_{2H}\frac{\zeta^3\xi}{\gamma} \tag{36}$$

where

$$\xi = \left[\frac{B}{H} + \left(1 - \frac{B}{H}\right)\frac{L}{L+S}\right]. \tag{37}$$

The turbulent asymptote for the composite pressure loss solution is determined as

$$\begin{aligned} f_{2H} &= \frac{\gamma}{\zeta^3\xi} f_{D_h} \\ &= \frac{\gamma}{\zeta^3\xi}\left(\frac{0.347}{\mathrm{Re}_{D_h}^{1/4}}\right) \\ &= \frac{\gamma^{5/4}}{\zeta^3\xi}\left(\frac{0.347}{\mathrm{Re}_{2H}^{1/4}}\right). \end{aligned} \tag{38}$$

Therefore, the coefficient $\mathcal{B}$ for the composite solution, (7), is

$$\mathcal{B} = \frac{\gamma^{5/4}}{\zeta^3\xi} \tag{39}$$

where γ, ζ and ξ are given by (21), (28), and (37), respectively.

F. Model Summary

The composite solution for friction factor for fully developed flow through an array of uniformly-sized and spaced cuboid blocks in a parallel plate channel can be summarized as

$$f_{2H} = \left[\left(\frac{96\mathcal{A}}{\mathrm{Re}_{2H}}\right)^3 + \left(\frac{0.347\mathcal{B}}{\mathrm{Re}_{2H}^{1/4}}\right)^3\right]^{1/3} \tag{40}$$

where

$$\mathcal{A} = \frac{\gamma^2}{\zeta^3\chi} \quad \mathcal{B} = \frac{\gamma^{5/4}}{\zeta^3\xi}$$

and γ, ζ, χ and ξ are defined by (21), (28), (29), and (37), respectively.

III. Model Validation

The proposed friction factor model is validated using available data from the open literature in two steps. First, the laminar flow asymptote of the composite solution is compared to the numerical results of Asako and Faghri [5] for nine different array geometries, as presented in Fig. 5. These plots demonstrate the good agreement between the model and the numerical predictions for all cases, with an average percent difference of 5% and a maximum difference of less than 10%.

The composite model for the full range of Re_{2H} is validated using available empirical data from three different references. Fig. 6 presents a comparison of the model with measured values for six different array geometries from Molki, Faghri, and Ozbay [8] and with results from Sparrow *et al.* [1] and Souza-Mendes and Santos [3]. Once again, good agreement is shown between the model predictions and the data, although maximum and average differences are somewhat larger than those in the previous, laminar case. Much of this deviation can be attributed to experimental error, as suggested by the large uncertainty value of ±15% on f_{2H} presented by Molki *et al.* [8].

IV. Conclusion

An analytical model for the pressure loss for fully developed flow through a parallel plate channel with an array of uniformly-sized and spaced cuboid blocks on one wall is developed. This model is validated for a wide range of array geometries and Reynolds numbers. The average percent difference between the model and existing numerical results from the literature for laminar flow is approximately 5%, with a maximum difference of less than 10%. Agreement between the model and available empirical data is also good, with an average deviation of approximately 15%, equal to experimental uncertainty values.

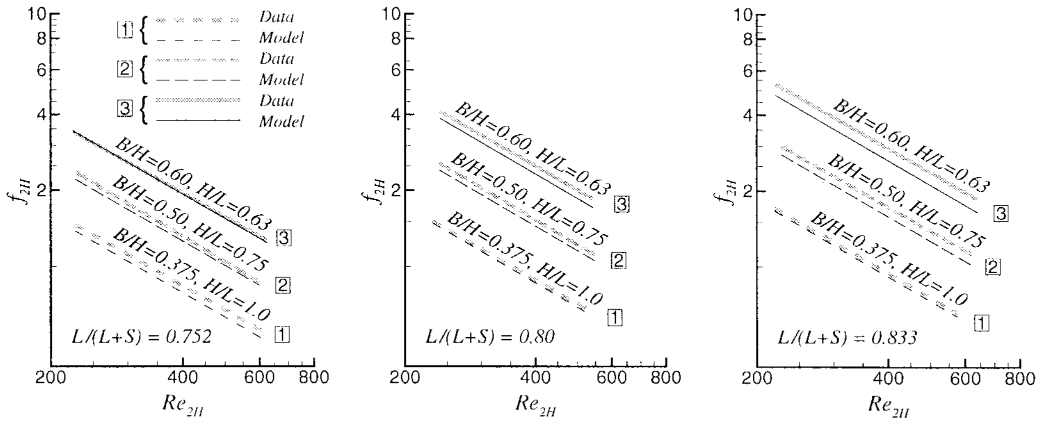


Fig. 5. Comparison of model with numerical data from Asako and Faghri [5].

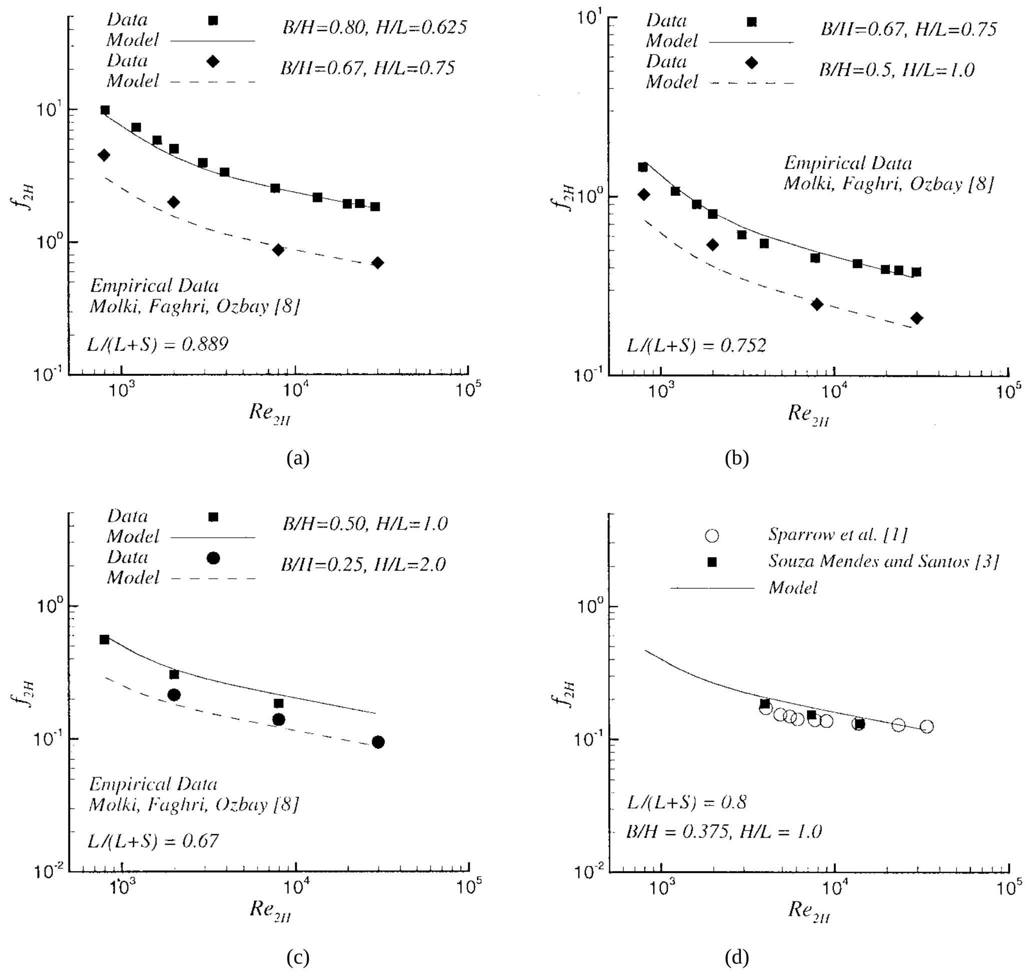


Fig. 6. Comparison of model with experimental data: (a)–(c) Molki, Faghri, and Ozbay [8]; (d) Sparrow *et al.* [1] and Souza-Mendes and Santos [3].

The friction factor model presented in this paper is constrained by the following two limitations. First, it assumes that an inline array of uniformly sized and spaced cuboid blocks can be found which accurately represents the average dimensions of the actual packages on the board. It is left to the reader to determine the block size and spacing that best characterizes the "real" values. The model does not include any local effects caused by large obstructions, such as transformers, heat sinks or card guides. Also, the model predicts friction factor for the fully developed portion of the channel flow only, and does not include any pressure loss due to entrance effects in the channel or developing flow in the first rows of the array.

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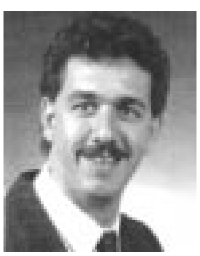

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