NATURAL CONVECTION FROM HORIZONTAL ISOTHERMAL ELLIPTIC DISKS: MODELS AND EXPERIMENTS

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Abstract

General models are proposed for natural convection from horizontal isothermal thin elliptic disks. The models are based on the linear superposition of the corresponding diffusive limits (shape factors) and the laminar boundary layer asymptotes. The dimensionless shape factor for the elliptic disk is based on a modification of the Smythe solution for the finite circular cylinder. A comprehensive procedure is presented that leads to a complex formulation of the body-gravity function. A simpler procedure based on the method of inscribing and circumscribing circular cylinders within the elliptic cylinder yields a simpler expression for accurate evaluations of the body-gravity function. The Nusselt and Rayleigh numbers, and the dimensionless shape factor and body-gravity function are based on the characteristic body length proposed by Yovanovich, i.e., the square root of the total surface area of the body. The proposed models are compared against air data obtained over eight decades of the Rayleigh number for three thin elliptic disks having a range of aspect ratios. The agreement between theory and experiment is shown to be excellent.

Nomenclature

A	=	surface area of the body; m^2
$\tilde{A}$	=	area fraction
$\tilde{A}_i$	=	area fraction of the ith component
$\sqrt{A}$	=	characteristic length of the body proposed by Yovanovich[2]; m
AR	=	aspect ratio of elliptic disk; $AR = L/\sqrt{ab}$
a, b	=	major and minor axes of ellipse; $a \geq b$
D_{GM}	=	geometric-mean diameter of elliptic disk; $D_{GM} = \sqrt{a\,b}$; m
$E(\kappa)$	=	complete elliptic integral of second kind of modulus $\kappa = \sqrt{1-(b/a)^2}$
$F(Pr)$	=	Prandtl number function[5]; $f\left[0.670/[1+(0.50/Pr)^{9/16}]^{4/9}\right]$
g	=	scalar gravitational acceleration; m/s^2
$G_{\sqrt{A}}$	=	laminar boundary layer body-gravity function based on $\sqrt{A}$
$Gr_{\sqrt{A}}$	=	Grashof number; $g\beta(T_s - T_a)(\sqrt{A})^3/\nu^2$
h	=	heat transfer coefficient; $W/m^2{\cdot}K$
k	=	thermal conductivity; $W/m{\cdot}K$
L	=	elliptic disk thickness; m
$Nu_{\sqrt{A}}$	=	Nusselt number; $h\sqrt{A}/k$
$Nu^{\infty}_{\sqrt{A}}$	=	diffusive limit; $Nu^{\infty}_{\sqrt{A}} = S^{\star}_{\sqrt{A}}$
$P(\theta)$	=	local perimeter; m
Pr	=	Prandtl number; ν/α
$Ra_{\sqrt{A}}$	=	Rayleigh number; $Gr_{\sqrt{A}}Pr$
RMS	=	Root-Mean-Square value

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$S^\star_{\sqrt{A}}$ = dimensionless shape factor
T_a = ambient temperature; K
T_s = surface temperature; K
T_f = film temperature; $(T_s + T_\infty)/2$; K

Subscripts

$\sqrt{A}$ = based on characteristic length $\sqrt{A}$

Superscripts

∞ = estimated at $Ra \to 0$
$\tilde{}$ = dimensionless quantity
$*$ = dimensionless quantity

Greek Symbols

β = volumetric expansion coefficient; K^{-1}
θ = angle between gravity vector and outward normal to surface; rad
ν = kinematic viscosity; μ/ρ; m^2/s
ρ = density; kg/m^3

Miscellaneous

bot = bottom surface area of elliptic disk
side = side surface area of elliptic disk
top = top surface of elliptic disk
total = total surface area of elliptic disk

Introduction

Models to predict natural convection heat transfer over a wide range of the Rayleigh number from thin convex bodies which possess horizontal surfaces facing upward and downward with respect to the gravity vector are presently not available in the open literature. Experimental data are also unavailable to develop correlation equations. The difficulty lies in modeling the contribution of the horizontal surfaces, in particular when they constitute more than 50% of the total active surface area.

The objectives of this paper are: 1) to present a procedure for development of a general model that can accurately predict the Nusselt number Nu for thin horizontal elliptic disks over several decades of the Rayleigh number Ra; 2) to develop simple general correlation equations from the proposed general model; and 3) to compare the correlation equations against the air data of Jafarpur[10].

The proposed thin elliptic disk model should be capable of predicting natural convection from thin horizontal circular disks and other body shapes which are similar such as thin oblate spheroids.

General Model for Isothermal Convex Bodies

The general expression for natural convection heat transfer from three-dimensional isothermal convex bodies

$$Nu_{\sqrt{A}} = Nu^\infty_{\sqrt{A}} + F(Pr)\ G_{\sqrt{A}}\ Ra^{1/4}_{\sqrt{A}} \qquad (1)$$

was first proposed by Yovanovich[21,22,23]. This relationship is based on the linear superposition of the diffusive limit (shape factor) $Nu^\infty_{\sqrt{A}}$ corresponding to $Ra_{\sqrt{A}} = 0$ and the laminar boundary-layer asymptote $F(Pr)\ G_{\sqrt{A}}\ Ra^{1/4}_{\sqrt{A}}$ which is valid for large values of the Rayleigh number.

The laminar boundary-layer asymptote consists of the product of the Prandtl number function $F(Pr)$, the body-gravity function $G_{\sqrt{A}}$, and the Rayleigh number $Ra_{\sqrt{A}}$. The characteristic length in the Nusselt and Rayleigh numbers and the body-gravity function is the square-root of the total active surface area $\sqrt{A}$ which was first proposed by Yovanovich[21,22,23] for natural and forced convection heat transfer from bodies of arbitrary shape. Yovanovich had previously used this characteristic length to nondimensionalize thermal constriction resistance (Yovanovich et al[19]. Yovanovich and Schneider[18] Yovanovich and Burde[20] and conduction shape factors (Yovanovich[17], Chow and Yovanovich[4], Yovanovich and Wang[24].

The laminar Prandtl number function

$$F(Pr) = \frac{0.670}{\left[1 + (0.5/Pr)^{9/16}\right]^{4/9}} \qquad (2)$$

was recommended by Churchill and Churchill[5] as the *universal* function valid for all geometries and all values of the Prandtl number.

The body-gravity function

$$G_{\sqrt{A}} = \left[\frac{1}{A}\iint_A \left(\frac{P(\theta)}{\sqrt{A}}\sin\theta\right)^{1/3} dA\right]^{3/4} \qquad (3)$$

was recommended by Lee, Yovanovich and Jafarpur[11] for axisymmetric and two-dimensional geometries.

The proposed three-dimensional model, Eq. (1) has been experimentally validated (Jafarpur[10]) for a range of body shapes such as i) axisymmetric spheroids (oblate and prolate), sphere, ii) two-dimensional elliptic and circular cylinders, iii) thin circular and square plates in the vertical and horizontal orientation, and iv) other body shapes (cube, cones with apex facing upward and downward).

Composite Body-Gravity Function For Complex Body Shapes

Buoyancy-induced flow over complex body shapes can be modelled by i) partitioning the total body surface into component surfaces corresponding to the fluid flow, and ii) using the general formula, Eq. (3), for each component surface A_i to find the corresponding body-gravity function $G_{\sqrt{A_i}}$.

The overall body-gravity function for the total active body surface is determined by combining the component surfaces A_i and their respective body-gravity functions $G_{\sqrt{A_i}}$ into a composite value. Equation (3) can be used for all surfaces except horizontal surfaces ($\sin\theta = 0$) facing upward or downward. At present semi-empirical methods must be used to model buoyancy-induced flow over horizontal surfaces (Jafarpur[10], and Yovanovich and Jafarpur[25,26]).

There are two important flow patterns for which the composite or overall body-gravity function can be determined with relative ease. These are complex bodies such as a circular cylinder with hemispherical ends which is placed in a large extent of air in either the horizontal (axis perpendicular to the gravity vector) or vertical (axis parallel to the gravity vector) orientations.

In the first orientation the two ends and the horizontal surface are cooled by *different flows* of air and the component surfaces are said to be in the *parallel flow pattern*. In the second orientation the component surfaces are cooled by the same fluid flow which starts at the lower stagnation point, flows over the lower hemispherical end, then over the vertical cylindrical surface, and finally over the top hemispherical end. In this case the component surfaces are said to be in the *series flow pattern*.

The above method of partitioning a complex body shape into parallel or series flow patterns can be applied to many interesting natural convection problems. Some orientations such as inclined short cylinders with flat ends or hemispherical ends, or inclined cuboids are more difficult to model.

If the buoyancy-induced flow over a complex body shape can be partitioned into N component surfaces with area-fractions $\tilde{A}_i$ where $\sum_{i=1}^{i=N} \tilde{A}_i = 1$, and the corresponding body-gravity functions $G_{\sqrt{A_i}}$ can be determined, then the composite body-gravity function for the entire body surface can be evaluated by means of either the parallel flow pattern formula: (Lee-Yovanovich-Jafarpur[11]):

$$G_{\sqrt{A}} = \sum_{i=1}^{N} G_{\sqrt{A}_i} \tilde{A}_i^{7/8} \tag{4}$$

or the series flow pattern formula:

$$G_{\sqrt{A}} = \left[\sum_{i=1}^{N} G_{\sqrt{A}_i}^{4/3} \tilde{A}_i^{7/6} \right]^{3/4} \tag{5}$$

For two-dimensional surfaces, such as vertical disks or plates of arbitrary shape with variable perimeter $P(z)$ the body-gravity function can be easily obtained from the following simple formula which was derived from Eq. (3) after setting $\sin\theta = 1$:

$$G_{\sqrt{A}} = \frac{2}{A^{7/8}} \int_0^{P_{max}/2} S(z)^{3/4} \, dz \tag{6}$$

where $S(z)$ denotes the flow distance from the leading edge to the trailing edge of the differential surface dz and P_{max} is the maximum perimeter of the surface.

These formulas along with the semi-empirical results recommended by Yovanovich and Jafarpur[25,26] for horizontal surfaces facing upward or downward will be used to develop the composite body-gravity function for the horizontal elliptic disks.

Model Development

Body-Gravity Functions

Three methods are proposed for the determination of the composite body-gravity function for horizontal elliptic disks. The first method is comprehensive, and it requires the computation of the complete elliptic integral of the second kind (Abramowitz and Stegun[1]) for calculation of the component body-gravity functions. The second simpler method is based on an approximation of the expression that appears in the determination of the body-gravity functions for horizontal elliptic surfaces that face upward and downward. The third simpler method is based on the approximation recommended by Jafarpur[10] for estimating the composite body-gravity function of vertical cylinders of constant cross-section with active ends. This method is based on inscribing and circumscribing vertical circular cylinders inside and outside the cylinder of interest. Two close estimates of the required composite body-gravity function are obtained by this procedure. Finally some average value is determined by means of the two values so calculated.

The elliptic disks are characterized by three geometric parameters: the major axis a, the minor axis b and its thickness L (see Fig. 1). There are therefore two aspect ratios that define elliptic disks:

$a/b \geq 1$ and $L/\sqrt{ab}$ where $\sqrt{ab}$ represents the geometric mean of the major and minor axes.

Comprehensive Model

The first model is comprehensive and therefore it is more complex. It is based on the following component body-gravity functions. For the horizontal surface facing upward the body-gravity function can be obtained from the general expression (Yovanovich and Jafarpur[25,26]):

$$G_{\rm top} = \frac{5}{6}\left[\frac{P}{\sqrt{A_{\rm top}}}\right]^{1/4} \tag{7}$$

where P is the perimeter of the elliptic disk. Substituting the expressions for the perimeter of an ellipse and its surface area $A_{\rm top} = (\pi/4)\, ab$ gives

$$G_{\rm top} = \frac{5}{6}\left(\frac{16}{\pi}\right)^{1/8}\left[\sqrt{\frac{a}{b}}E\left(\sqrt{1-\left(\frac{b}{a}\right)^2}\right)\right]^{1/4} \tag{8}$$

where $E(\cdot)$ is the complete elliptic integral of the second kind, and the value of the constant before the square bracket is equal to 1.0214. The values of the body-gravity function vary slowly for the aspect ratio range: $1 \leq a/b \leq 10$. For $a/b = 1$ and $a/b = 10$, $G_{\rm top} = 1.143$ and 1.367 respectively. The difference is about 20%. The body-gravity function for horizontal surfaces facing downward is equal to (Yovanovich and Jafarpur[25,26]):

$$G_{\rm bot} = \frac{1}{2}G_{\rm top} \tag{9}$$

For the vertical side surface the body-gravity function can be obtained from the general expression developed by Lee, Yovanovich and Jafarpur[11] based on the disk perimeter and thickness:

$$G_{\rm side} = \left[\frac{P}{L}\right]^{1/8} \tag{10}$$

Substituting the perimeter of the ellipse gives

$$G_{\rm side} = 2^{1/8}\left[\frac{a}{L}E\left(\sqrt{1-\left(\frac{b}{a}\right)^2}\right)\right]^{1/8} \tag{11}$$

Examination of the above results shows that the component body-gravity functions for the horizontal surfaces are dependent on the ellipse aspect ratio, i.e. b/a; while the component body-gravity function for the side surface is dependent on two aspect ratios, i.e. b/a and a/L. The composite body-gravity function is obtained by means of the series flow arrangement expression, Eq. (5), which yields the general formulation:

$$G_{\sqrt{A}} = \left[G_{\rm top}^{4/3}\,\tilde{A}_{\rm top}^{7/6} + G_{\rm bot}^{4/3}\,\tilde{A}_{\rm bot}^{7/6} + G_{\rm side}^{4/3}\,\tilde{A}_{\rm side}^{7/6}\right]^{3/4} \tag{12}$$

This model requires the computation of the complete elliptic integral of the second kind of modulus $\kappa = \sqrt{1-(b/a)^2}$. Accurate numerical values can be obtained by means of Computer Algebra Systems (CAS) such as *Maple*[12] or *Mathematica*[13]. The elliptic integral lies in the range: $1 < E(\cdot) \leq \pi/2$ in the ellipse aspect ratio range: $0 < b/a \leq 1$.

Approximation for Complete Elliptic Integral

Although $E(k)$ and its complement $E\left(\sqrt{1-k^2}\right)$ can be computed by polynomial approximations and series expansions for small and large arguments (Abramowitz and Stegun[1]), for convenience and completeness of this work, the following expressions are recommended for quick, accurate computations:

<u>Complete Elliptic Integral</u>

$$E(k) = \frac{\pi/2}{1+k_1}\left[1 + \frac{k_1^2}{4} + \frac{k_1^4}{64} + \frac{k_1^6}{256}\right] \tag{13}$$

where the parameter k_1 is defined as

$$k_1 = \frac{1-\sqrt{1-k^2}}{1+\sqrt{1-k^2}} \tag{14}$$

This approximation provides 6 digit accuracy everywhere except at $k = 1$ where the error is approximately 0.3%.

<u>Complementary Elliptic Integral</u>

$$E\left(\sqrt{1-k^2}\right) = \frac{\pi}{4}(1+k)\left[1 + \frac{p^2}{4} + \frac{p^4}{64} + \frac{p^6}{256}\right] \tag{15}$$

where $p = (1-k)/(1+k)$. This approximation provides 6 digit accuracy everywhere except at $k = 0$ where the error is approximately 0.3%.

In order to minimize the computational effort, a second model is presented which does not require the computation of the special function $E\left(\sqrt{1-k^2}\right)$.

Approximation of $G_{\rm top}$ and $G_{\rm bot}$

Examination of the range of values for $G_{\rm top}$ for the ellipse aspect ratio range: $1 \leq a/b \leq 10$ suggests that the expression given in Eq. (8) can be approximated with acceptable accuracy by:

$$G_{\text{top}} = 2^{1/8} \left(\frac{a}{b}\right)^{1/8} \qquad (16)$$

and G_{bot} is given by Eq. (9).

Approximate Model

The third method is based on the method of inscribing and circumscribing right circular cylinders inside and outside the elliptic disk. The body-gravity function for the vertical circular cylinder is known (Jafarpur[10]). It depends on the circular cylinder aspect ratio which is defined as the ratio of the cylinder length to cylinder diameter. This approach yields two values of the body-gravity function which bound the value that is sought as demonstrated by Jafarpur[10]. The value of the composite body-gravity function is then obtained by taking some average of the two bounding values. It has been observed by Jafarpur[10] that the arithmetic, harmonic and geometric mean values of the two bounding results are quite close. The geometric mean value was recommended. To simplify the procedure further, the aspect ratio for an elliptic disk of length L, and major and minor axes: a and b is defined as

$$AR = \frac{L}{\sqrt{ab}} \qquad (17)$$

The composite body-gravity function for the horizontal elliptic disk can be estimated by means of the following relationship which was developed by means of *Mathematica*[13]:

$$G_{\sqrt{A}} = \pi^{1/8} \frac{\left(0.2662 + \frac{L}{\sqrt{ab}}\right)^{3/4}}{\left(0.5 + \frac{L}{\sqrt{ab}}\right)^{7/8}} \qquad (18)$$

It will be shown later that for the test elliptic disks that the comprehensive method and the method of estimating G_{top} and G_{bot} give values of $G_{\sqrt{A}}$ that differ by less than 1%; and that the third simple method give values of $G_{\sqrt{A}}$ that differ by less than 4% from the more exact values given by the comprehensive model.

Diffusive Limit

The diffusive limit for elliptic disks is presently unavailable. It can, however, be estimated accurately by the method of inscribing and circumscribing circular cylinders which have accurate analytic solutions (Smythe[14,15]). The diffusive limit of finite circular cylinders based on the square root of the total active surface area depends only on the cylinder aspect ratio which is defined as the ratio of the cylinder length to its diameter. This approach has been demonstrated to yield very tight bounds of the required result (Chow and Yovanovich[4]; Jafarpur[10]; Yovanovich[21,22,23,27]). Jafarpur[10] has demonstrated that the computations can be simplified by the use of the geometric mean diameter defined as $D_{\text{GM}} = \sqrt{ab}$. The elliptic disk aspect ratio is defined as L/D_{GM}.

Inserting the above relationship for the elliptic disk aspect ratio into the diffusive limit for the right circular cylinder gives

$$Nu^{\infty}_{\sqrt{A}} = \frac{1}{\sqrt{2\pi}} \frac{\left[8 + 6.96\left(\frac{L}{\sqrt{ab}}\right)^{0.76}\right]}{\sqrt{1 + 2\frac{L}{\sqrt{ab}}}} \qquad (19)$$

for the approximation of the diffusive limit of elliptic disks. This relation will provide numerical values with errors less than 3 % for $a \geq b$ and $L > 0$. The three proposed methods will be compared against air data obtained for three isothermal horizontal elliptic disks over a wide range of $Ra_{\sqrt{A}}$.

Experimental Results: Thin Elliptic Disks

Elliptic Disk Geometric Characteristics

The three elliptic disks used in the experimental program (Jafarpur[10]) have major and minor axes: a and b, thickness L, aspect ratio $L/\sqrt{ab}$ and $\sqrt{A}$ as reported in Table 1.

Table 1: Geometric Characteristics of Elliptic Disks

disk	a	b	L	$L/\sqrt{ab}$	$\sqrt{A}$
ec1	96	64	6.4	.0815	106.2
ec2	92.6	61.8	9.3	.123	106.1
ec3	89.9	59.8	11.9	.162	106.2

The ellipse aspect ratio a/b was held close to the value 1.5 for the three disks. The elliptic disk aspect ratio $L/\sqrt{ab}$ ranged from about 8.2 % up to about 16 %. The total active surface area was maintained constant to within 1 %, and the characteristic body length was $\sqrt{A} = 0.1062\ m$ for all three disks. This characteristic length is seen to be approximately 11% to 18% greater than the major axes.

The fraction of the total active surface area of the disk ends and the disk side are reported in Table 2. There it is seen that the top and bottom horizontal surfaces contribute about 86 % for the thinnest disk and 75 % for the thickest disk.

Table 2: Surface Area Fractions of Elliptic Disks

cylinder	$\tilde{A}_{\text{ends}}$	$\tilde{A}_{\text{side}}$
ec1	.856	.144
ec2	.798	.202
ec3	.749	.251

The three elliptic disks were constructed from highly polished aluminum alloy 6061 $T6$ to ensure that they will be isothermal during the convective heat transfer tests, and that the radiation contribution will be minimized.

The initial tests were conducted in a special vacuum chamber to ascertain the radiation and conduction losses along the thermocouple and power leads (Jafarpur[10]). These losses were correlated and used in the natural convection data reduction. The air data reported by Jafarpur[10] were obtained in another chamber designed and instrumented for obtaining transient natural convection results over a wide range of $Ra_{\sqrt{A}}$ typically in the range: $1 < Ra_{\sqrt{A}} < 10^{10}$. The test chamber and the test procedure have been described fully in several theses: (Chamberlain[2]; Hassani[8]; Clemes[6]; and Jafarpur[10]), and some publications (Chamberlain et al.[3]; Hassani and Hollands[9]; and Clemes et al[7]). The test procedure has been demonstrated to give accurate results over the full range of $Ra_{\sqrt{A}}$. The uncertainty of the convection data are reported by Jafarpur[10] to be ± 6 % on the Nusselt number and ± 4 % on the Rayleigh number. The data will be compared against the proposed models in the following section.

Comparison of Models and Data

For the three elliptic disks used in the tests the diffusive limit was calculated to have the values: $3.342, 3.364, 3.377$ respectively. The smallest and largest values differ by less than 1 %.

The component and composite body-gravity functions for the top, bottom and side surfaces according to the comprehensive model are reported in Table 3.

Since the ellipse aspect ratio was held nearly constant, the component values for the top and bottom surfaces are almost constant. Even though the elliptic disk aspect ratio varies from about 8.2% to

Table 3: Values of Body-Gravity Functions of Elliptic Disks

cylinder	G_{top}	G_{bot}	G_{side}	$G_{\sqrt{A}}$
ec1	1.152	.5760	1.585	.8612
ec2	1.152	.5759	1.505	.8810
ec3	1.152	.5762	1.454	.8968

about 16%, the component value for the side surface varies by about 9%. The composite value of the body-gravity function $G_{\sqrt{A}}$ varies by only 4.1%. The smallest value corresponds to the thinnest disk and the largest value corresponds to the thickest disk.

The product of the Prandtl number function which has the value $F(0.71) = 0.513$ for air cooling and the body-gravity function $G_{\sqrt{A}}$ has the values: $0.442, 0.452, 0.460$ for the three test elliptic disks according to the comprehensive method, and the values are: $0.440, 0.451, 0.459$ when the approximations for G_{top} and G_{bot} are employed. The differences are clearly negligible for these elliptic disks.

The comparison of the air data and the proposed comprehensive model for the three elliptic disks is shown in Fig. 1. The agreement is seen to be excellent over the full range of $Ra_{\sqrt{A}}$. The root-mean-square percent differences were found to be: $3.85\%, 3.81\%, 3.74\%$ respectively for the three test disks.

The approximate simple model gives the values of $F(Pr)\, G_{\sqrt{A}} = 0.427, 0.438, 0.447$ respectively for the test disks. These values are in close agreement with the values obtained from the comprehensive model and are compared to the experimental results in Fig. 2. For the approximate model the root-mean-square percent differences were found to be: $2.84\%, 4.92\%, 4.79\%$ respectively for the three test elliptic disks.

We observe that the comprehensive model gives consistent RMS percent differences close to 3.8% while the approximate model gives a smaller RMS percent difference for the thinnest disk and larger RMS percent differences for the two thicker disks.

Summary and Discussion

A procedure was presented for the development of a general model for predicting natural convection from isothermal, horizontal, thin elliptic disks over several decades of the Rayleigh number. The proposed model consists of the linear superposition of the diffusive limit (shape factor) applicable for negligible fluid motion and the laminar boundary

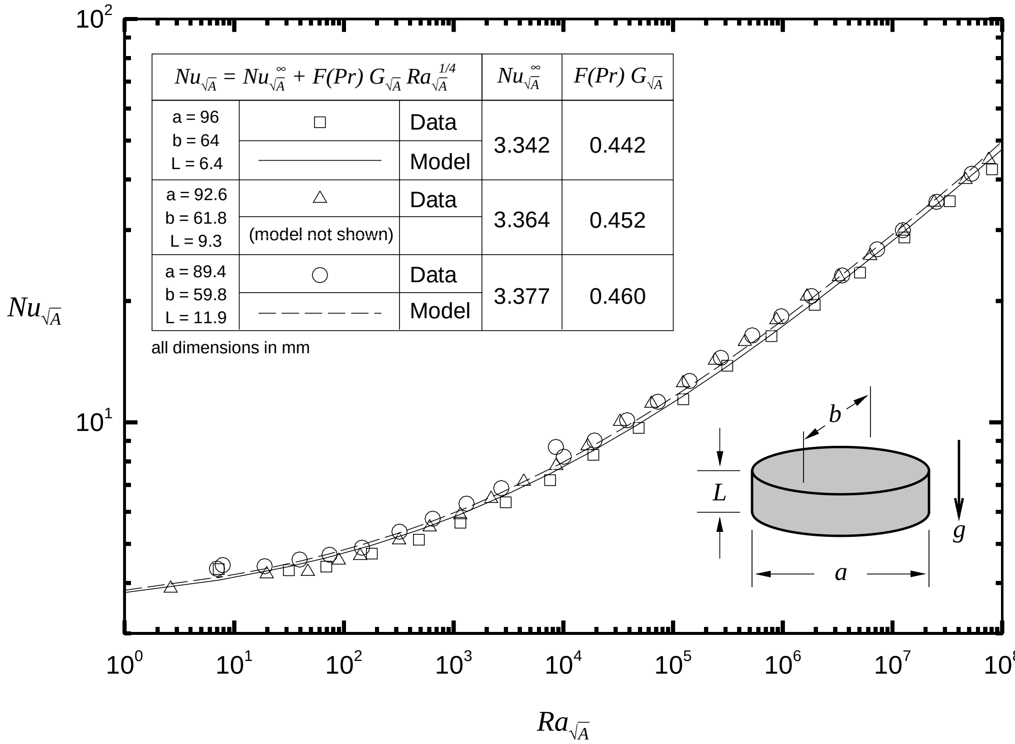

Figure 1: Comparison of Comprehensive Model and Air Data for Three Elliptic Disks

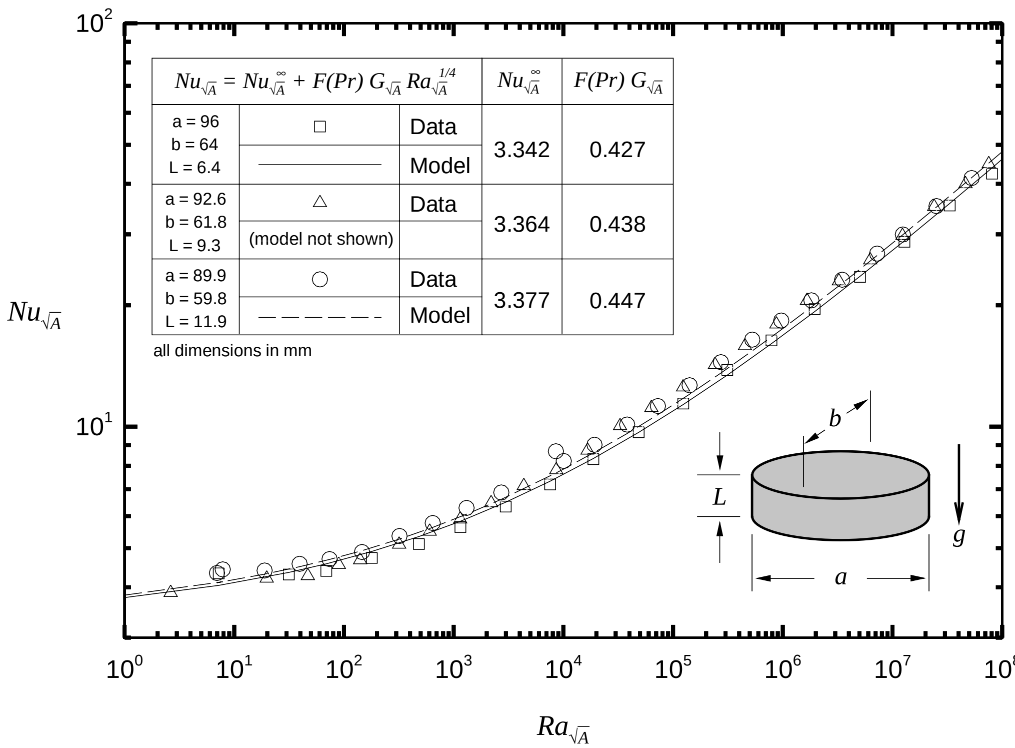

Figure 2: Comparison of Inscribed/Circumscribed Circular Cylinder Model and Air Data for Three Elliptic Disks

layer asymptote. Simple correlation equations were developed for elliptic disks. These simple correlation equations are close approximations of the general, more complex model. The correlation equations were compared against air data obtained over several decades of the Rayleigh number. The root-mean-squared percent difference between the data and the proposed correlation equations was less than 3.9% for the three elliptic disks. The shape factors differed by less than 1.1% and the body-gravity function differed by less than 4.1%. The proposed simple model based on the inscribed-circumscribed circular cylinder model to estimate the diffusive limit and the body-gravity function gave correlation equations which are in acceptable agreement with the data. The root-mean-square percent differences lie in the range: 2.84% to 4.92%. Simple relationships are also presented for the accurate computation of the complete elliptic integral of the second kind and its complement which appear in the comprehensive model.

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