AIAA-97-2458

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32nd Thermophysics Conference
June 23 - 25, 1997 / Atlanta, GA

TRANSIENT SPREADING RESISTANCE OF ARBITRARY ISOFLUX CONTACT AREAS: DEVELOPMENT OF A UNIVERSAL TIME FUNCTION

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Abstract

Dimensionless spreading resistances of isoflux planar contact areas are presented for two families of geometries: regular polygons and the hyperellipse. The spreading resistance is based on the transient centroid temperature rise. Closed-form expressions are presented for the steady-state spreading resistance of regular polygons as a function of the number of sides, and the hyperellipse as a function of the shape parameter and the aspect ratio. Expressions are presented for the centroid based and area-average based spreading resistances which are valid for all dimensionless time for the circular contact area. A closed-form relationship based on the solution for the circular contact area is used to develop a universal time function which is recommended for the accurate calculation of the spreading resistance for all polygons and hyperellipse contact areas having a wide range of aspect ratios. An alternate simpler expression is also presented for quick calculation of the spreading resistance.

Nomenclature

A	=	active area of contact; m^2
$\sqrt{A}$	=	characteristic length of contact area; m
a, b	=	semi-axes of hyperellipse; m
$B(x,y)$	=	Beta function
$\mathrm{erf}(x)$	=	error function
$\mathrm{erfc}(x)$	=	complementary error function
Fo	=	dimensionless time
$Fo_{\sqrt{A}}$	=	dimensionless time based on $\sqrt{A}$
$J_1(x)$	=	Bessel function of first kind, order 1
k	=	thermal conductivity; $W/m{\cdot}K$
N	=	number of sides of regular polygons, $N \geq 3$
n	=	shape parameter of hyperellipse, $(x/a)^n + (y/b)^n = 1$
Q	=	heat flow rate; W
q	=	heat flux; W/m^2
R	=	thermal resistance, θ/Q; K/W
R_0	=	spreading resistance (centroid basis); K/W
$\overline{R}$	=	spreading resistance (area-average basis); K/W
R_0^*	=	dimensionless spreading resistance (centroid basis)
$\overline{R}^*$	=	dimensionless spreading resistance (area-average basis)
r	=	distance from point source to arbitrary point in half-space; m
T	=	temperature; K
T_0	=	centroid temperature; K
T_∞	=	temperature at remote points; K
$\overline{T}$	=	area-average temperature; K

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t	=	time; s
x, y, z	=	cartesian coordinates; m

Greek Symbols

α	=	thermal diffusivity; m^2/s
γ	=	ratio of semi-axes, $b/a \leq 1$
δ	=	perpendicular of right-triangle
η, ζ	=	coordinates of points within contact area
ω, ρ	=	polar coordinates; m
θ	=	temperature rise, $\theta = T - T_\infty$; K
ϑ	=	parameter defined by $\vartheta = \dfrac{1}{2\sqrt{\pi}\sqrt{Fo_{\sqrt{A}}}}$

Introduction

There are several publications[1-8] which reported attempts to find the transient temperature rise or the heat transfer rate within a circular disk (contact area) which is located on the surface of an isotropic half-space. The solutions are presented for (a) the transient heat flow rate from the contact area when the contact area temperature is isothermal, and (b) the temperature rise of the contact area when the area is subjected to a uniform and constant heat flux.

The previous investigations can be divided into three categories: (i) approximate methods[1,2], (ii) analytic methods[3-5] which yield limited information over the entire range of dimensionless time, and (iii) numerical methods[6-8] which provide numerical results which must be correlated to be useful. General numerical methods[9,10] have been used to calculate the dimensionless temperature rise of isoflux contact areas of arbitrary shape. The transient centroid temperature rise values are reported[9,10] for rectangles, ellipses, regular polygons which include the equilateral triangle, square, pentagon, hexagon, etc. Results for the semi-circle and other nonsymmetric geometries are also available[9,10].

The Laplace transform method[11] was employed to obtain analytic solutions of the thermal constriction (spreading) resistance of (i) an isoflux circular contact area on a circular flux tube, and (ii) an isoflux strip on a two-dimensional heat flux channel. The solutions were reported in the form of infinite series and numerical values are given in tabular and graphical form for a range of dimensionless time.

Steady-state solutions[12-14] are available for isoflux contact areas of arbitrary shape located on the surface of an isotropic half-space[12,13], and circular and square isoflux contact areas located on circular or square semi-infinite heat flux tubes[14].

It was demonstrated[9,10,13,14] that when the dimensionless spreading (constriction) resistance and the dimensionless time are based on a length scale which is equal to the square root of the contact area, the results for similar contact areas are very close.

The transient temperature rise within planar contact areas which are subjected to a constant and uniform heat flux is of some interest to the thermal analysts who are involved with the design of fast computer chips. The contact areas are small relative to the substrate dimensions and they can have a variety of shapes, but generally they are rectangular or square.

The available solutions give the centroid or area-average temperature rise of the contact area as a function of time and the given geometry. The temperature rise is used to define the spreading (constriction) resistance of that contact area. The results appear in graphical form or in tables for each geometry.

It is not apparent from the published results whether they are related in some simple manner to the circular contact area for which there is a full analytical solution for the centroid-based and area-average based spreading resistances.

One objective of this paper is to review briefly the integral solution method and to discuss some aspects of the solutions.

The second objective is to present new integral solutions for the spreading resistance for two families of isoflux contact areas: (i) the hyperellipse and (ii) the regular polygons.

The third objective is to develop a simple *universal* time function which can used to calculate accurately the centroid-based spreading resistance of any geometry for a wide range of the dimensionless time.

Problem Statement and Definition of Spreading Resistance

The mathematical problem consists of finding the solution $\theta(r,t)$ for any point $r = \sqrt{x^2 + y^2 + z^2}$ to the three-dimensional diffusion equation

$$\nabla^2 \theta = \frac{1}{\alpha}\frac{\partial \theta}{\partial t} \tag{1}$$

within an isotropic half-space $z > 0$. The initial condition on the temperature rise $\theta = (T(r,t) - T_\infty)$ is $\theta(r,0) = 0$. One boundary condition for all time $t > 0$ is specified within the contact area A where the heat flux is uniform and constant, and the other adiabatic boundary condition is specified for all points outside the contact area. These boundary conditions require:

$$\frac{\partial \theta(x,y,0,t)}{\partial z} = -\frac{q}{k} \quad \text{for points inside A} \tag{2}$$

and

$$\frac{\partial \theta(x,y,0,t)}{\partial z} = 0 \quad \text{for points outside A} \tag{3}$$

The regular condition at points far from the contact area requires

$$\theta(r,t) \to 0, \quad r \to \infty, \qquad t > 0 \tag{4}$$

The spreading resistance $R = \theta/Q$ can be defined with respect to the single point located at the centroid of the contact area $\theta_0 = \theta(0, t)$ or with respect to the area-average temperature rise:

$$\overline{\theta} = \frac{1}{A} \iint_A \theta(x, y, 0, t) dx dy \tag{5}$$

The dimensionless centroid temperature rise and the area-average contact temperature rise are closely related for all geometries provided the characteristic length used to non-dimensionalize the temperature rise and the time is based on the square root of the contact area.[9−14]

The dimensionless spreading resistance is defined as $R_0^* = k\sqrt{A}\ \theta_0/Q$ for the centroid temperature rise and $\overline{R}^* = k\sqrt{A}\ \overline{\theta}/Q$ for the area-average temperature rise where k is the thermal conductivity of the half-space. The dimensionless time is defined as

$$Fo_{\sqrt{A}} = \frac{\alpha t}{A} \tag{6}$$

where α is the thermal diffusivity of the half-space.

During the early stages of heating when $Fo_{\sqrt{A}} \leq 4 \times 10^{-2}$, the centroid temperature rise and the area-average temperature rise are very nearly the same; therefore, $\overline{R}^*/R_0^* = 1$. For very large times $Fo_{\sqrt{A}} \geq 10^3$ the solution is close to steady-state where the maximum difference between the values of the spreading resistance appears, and it was reported[13] that

$$\frac{\overline{R}^*}{R_0^*} = 0.83 \pm 4\% \tag{7}$$

for all geometries whose aspect ratios are close to one. Therefore in the transition from early times to near steady-state, $4 \times 10^{-2} < Fo < 10^3$, the dimensionless spreading resistances are related:

$$1 \geq \frac{\overline{R}^*}{R_0^*} > 0.83 \tag{8}$$

Point Source and Right-Triangle Solutions

The solution method[9,10] is based on the instantaneous point source solution[15] which satisfies the diffusion equation at all points (x, y, z) within the isotropic half-space for all time $t > 0$. The solution method employs the superposition principle to determine the instantaneous temperature rise at the arbitrary point which may lie within the planar contact area or outside the contact area which is subjected to a uniform and constant heat flux q.

Transient Temperature Rise Due to a Point Source

The transient temperature rise at an arbitrary point $r = \sqrt{(x-\eta)^2 + (y-\zeta)^2 + z^2}$ due to a point source of strength qdA which is located at the point (η, ζ) in the surface of the half-space, as shown in Fig. 1, is given by[15]

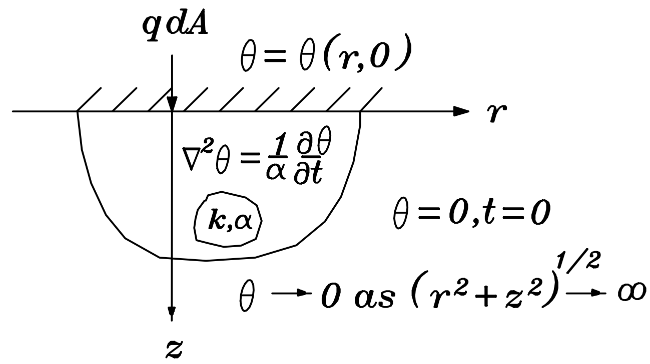

Figure 1: Point Source on Adiabatic Surface of a Semi-Infinite Body

$$\theta(r, t) = \frac{qdA}{2\pi k r} \operatorname{erfc}\left(\frac{r}{2\sqrt{\alpha t}}\right), \qquad t > 0 \tag{9}$$

In the previous equation k and α are the thermal conductivity and diffusivity respectively, and q is the constant heat flux on the differential area dA.

The fundamental solution consists of the product of the steady-state temperature rise $\theta(r) = qdA/(2\pi r k)$ and the space-time function which approaches the value of 1 for all values of $r/(2\sqrt{\alpha t}) < 0.01$.

Transient Temperature Rise at Vertex of a Right-Triangle

The previous point source solution is used to obtain the transient temperature rise at the vertex of the right-triangle defined by the vertex angle ω_0 and the corresponding perpendicular δ (Fig. 2). The instantaneous temperature rise at the vertex of an isoflux right-triangle is obtained from the double integral formulated in polar coordinates with the origin located at the vertex[9,10]:

$$\theta(0, t) = \frac{q}{2\pi k} \int_0^{\omega_0} \int_0^{\frac{\delta}{\cos\omega}} \operatorname{erfc}\left(\frac{\rho}{2\sqrt{\alpha t}}\right) d\rho\, d\omega \tag{10}$$

It was shown[9,10] how the solution for the right-triangle can be used to determine the transient temperature rise at arbitrary points within any area bounded by straight lines such as rectangles (square), triangles, regular polygons, etc. It was also shown how the solution for the right-triangle can be modified to handle areas which are bounded by curves such as ellipses (circle), semi-circle, etc.

Tabulated numerical values[9,10] of the dimensionless transient centroid temperature rise defined[9,10] as

$$T_0^* = \frac{2\pi k T_0}{q\sqrt{A}}$$

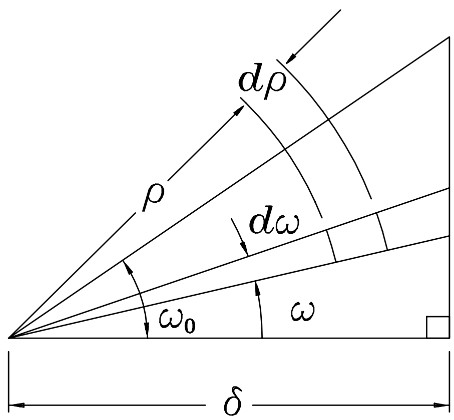

Figure 2: Geometric Parameters of a Right-Triangle

were reported for several values of the dimensionless time defined[9,10] as

$$Fo = \frac{\alpha t}{A}$$

The relationship between the dimensionless spreading resistance based on the centroid temperature and the dimensionless centroid temperature is $R_0^* = T_0^*/(2\pi)$.

Transient Centroid Temperature Rise of a Circular Area

The circular geometry yields an analytic solution for the centroid temperature rise which is valid for all dimensionless time. The transient centroid temperature rise of an isoflux circular area of radius a was given[9,10] as

$$\begin{aligned} T_0^* &= 4\sqrt{\pi}\sqrt{Fo}\Big[\text{-}1 - \exp\left(-1/4\pi Fo\right) \\ &+ \frac{1}{2\sqrt{Fo}} \text{erfc}\left(\frac{1}{2\sqrt{\pi}\sqrt{Fo}}\right)\Big], \quad Fo > 0 \end{aligned} \quad (11)$$

with $Fo = \alpha t/(\pi a^2)$. For $Fo \leq 4 \times 10^{-2}$, $T_0^* \to 4\sqrt{\pi}\sqrt{Fo}$ and for $Fo \geq 10^3$, $T_0^* \to 3.5449$ which is the steady-state value. The transition from small time behavior to near steady-state behavior occurs in the interval $4 \times 10^{-2} \leq Fo \leq 10^3$.

Transient Area-Average Temperature Rise of a Circular Area

The transient spreading resistance defined with respect to the area-average temperature was reported[7]. The following integral solution[7] was obtained for an isoflux circular area:

$$4ka\overline{R} = \frac{8}{\pi}\int_0^\infty J_1^2(\beta)\,\text{erf}(\beta\sqrt{Fo})\frac{d\beta}{\beta^2} \quad (12)$$

where the dimensionless time was defined as $Fo = \alpha t/a^2$. For large dimensionless time the integral solution goes to $4ka\overline{R} = 32/(3\pi^2)$ which is the steady-state value[15]. The special functions $J_1(x)$ and $\text{erf}(x)$ are the Bessel function[17] of the first kind of order 1 and the error function[17].

Approximate solutions for short and long times were reported[7]. For small times ($Fo < 0.6$) the following relationship was recommended[7]:

$$4ka\overline{R} = \frac{8}{\pi}\left[\sqrt{\frac{Fo}{\pi}} - \frac{Fo}{\pi} + \frac{Fo^2}{8\pi} + \frac{Fo^3}{32\pi} + \frac{15Fo^4}{512\pi}\right] \quad (13)$$

and for long times ($Fo \geq 0.6$)

$$\begin{aligned} 4ka\overline{R} &= \frac{32}{3\pi^2} - \frac{2}{\pi^{3/2}\sqrt{Fo}}\Big[1 - \frac{1}{3\,(4Fo)} \\ &+ \frac{1}{6\,(4Fo)^2} - \frac{1}{12\,(4Fo)^3}\Big] \end{aligned} \quad (14)$$

was recommended[7]. The maximum errors of about 0.18% and 0.07% occur at $Fo = 0.6$ for the short and long time expressions respectively. The previous two relationships can be converted to the $\sqrt{A}$ basis by setting $a = \sqrt{A}/\sqrt{\pi}$.

Spreading Resistance of Hyperellipse

The hyperellipse is defined by $(x/a)^n + (y/b)^n = 1$ where n is the shape parameter and a and b are the axes along the $x-$ and $y-$axes respectively as shown in Fig. 3. The hyperellipse reduces to many special cases by setting the values of n and the aspect ratio parameter $\gamma = b/a$ which lies in the range: $0 \leq \gamma \leq 1$. Therefore the solution developed for the hyperellipse can be used to obtain solutions for many other geometries such as: ellipses and circles, rectangles and squares, diamond-like geometries, etc.

The transient dimensionless centroid spreading resistance $R_0^* = k\sqrt{A}R_0$ where $R_0 = T_0/Q$ is given by the double integral solution:

$$R_0^* = \frac{2}{\pi\sqrt{A}}\int_0^{\pi/2}\int_0^{\rho_0} \text{erfc}\left(\frac{\rho}{2\sqrt{A}\sqrt{Fo_{\sqrt{A}}}}\right) d\rho d\omega \quad (15)$$

The dimensionless time is defined as $Fo_{\sqrt{A}} = \alpha t/A$. The area of the hyperellipse is given by $A = \left(4b^2\gamma/n\right) B(1 + 1/n, 1/n)$ and $B(x,y)$ is the Beta function[17]. The upper limit of the radius is given by $\rho_0 = (b\gamma)/\left[(\sin\omega)^n + \gamma^n (\cos\omega)^n\right]^{1/n}$.

The above solution has the following characteristics: i) for small dimensionless times: $Fo_{\sqrt{A}} \leq 4 \times 10^{-2}$, $R_0^* = (2/\sqrt{\pi})\sqrt{Fo_{\sqrt{A}}}$ for all values of n and γ, and ii) for large dimensionless times: $Fo_{\sqrt{A}} \geq 10^3$, the results are within 1% of the steady-state values which are given by the single integral solution:

$$R_0^* = \frac{2\gamma}{\pi\sqrt{A}}\int_0^{\pi/2}\frac{d\omega}{\left[(\sin\omega)^n + \gamma^n (\cos\omega)^n\right]^{1/n}} \quad (16)$$

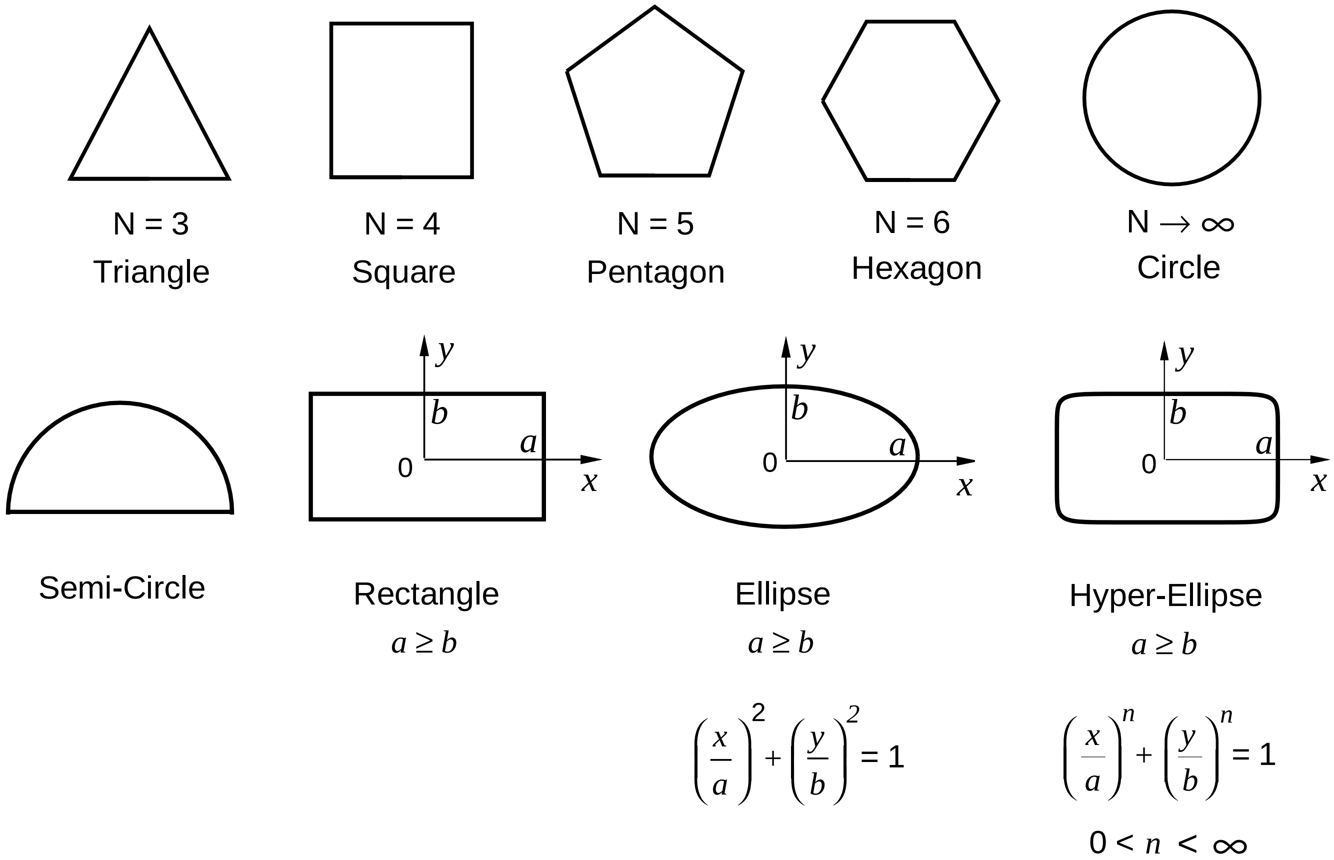

Figure 3: Schematic of Various Geometries

which depends on the aspect ratio γ and the shape parameter n. The parameter R_0^* depends on the three parameters: $\gamma, n, Fo_{\sqrt{A}}$ in the transition region: $4 \times 10^{-2} \leq Fo_{\sqrt{A}} \leq 10^3$ in some complex manner which can be deduced from the solution for the circular area. A procedure for the simple, accurate calculation of R_0^* in the transition will be considered below.

Spreading Resistance of Regular Polygons

All regular polygons (Fig. 3) having sides $N \geq 3$ can be divided into $2N$ right-triangles with their vertices located at the centroid of the polygon. The perpendicular from the vertex to the mid-point of a side is equal to the radius of the inscribed circle which is denoted r_i. The angle of the right-triangle at the vertex is $\omega_0 = \pi/N$. The total area of the regular polygon is therefore $A = Nr_i^2 \tan(\pi/N)$.

The fundamental solution for the right-triangle can be used to obtained the temperature rise at the centroid and the dimensionless spreading resistance based on the centroid temperature rise $R_0 = \theta_0/Q$.

The dimensionless spreading resistance defined as $R_0^* = k\sqrt{A}R_0$ is given by the double integral:

$$R_0^* = 2\sqrt{\frac{N}{\tan\frac{\pi}{N}}}\int_0^{\pi/N}\int_0^{1/\cos\omega} \text{erfc}\left(\frac{\rho}{2\sqrt{N\tan\frac{\pi}{N}}\sqrt{Fo_{\sqrt{A}}}}\right)d\rho\, d\omega \tag{17}$$

where the polygon area is expressed in terms of the number of sides N and for convenience the inscribed radius has been set to unity. This double integral solution has identical characteristics as the double integral solution given above for the hyperellipse, i.e. for small dimensionless time: $Fo_{\sqrt{A}} \leq 4 \times 10^{-2}$, $R_0^* = (2/\sqrt{\pi})\sqrt{Fo_{\sqrt{A}}}$ for all polygons $N \geq 3$, and ii) for large dimensionless times: $Fo_{\sqrt{A}} \geq 10^3$, the values are within 1% of the steady-state values which are given by the closed-form relationship:

$$R_0^* = \frac{1}{\pi}\sqrt{\frac{N}{\tan(\pi/N)}}\ln\left[\frac{1+\sin(\pi/N)}{\cos(\pi/N)}\right] \tag{18}$$

The dimensionless resistance R_0^* depends on the parameters: $N, Fo_{\sqrt{A}}$ in the transition region: $4\times10^{-2} \leq Fo_{\sqrt{A}} \leq 10^3$ in some complex manner which can be deduced from

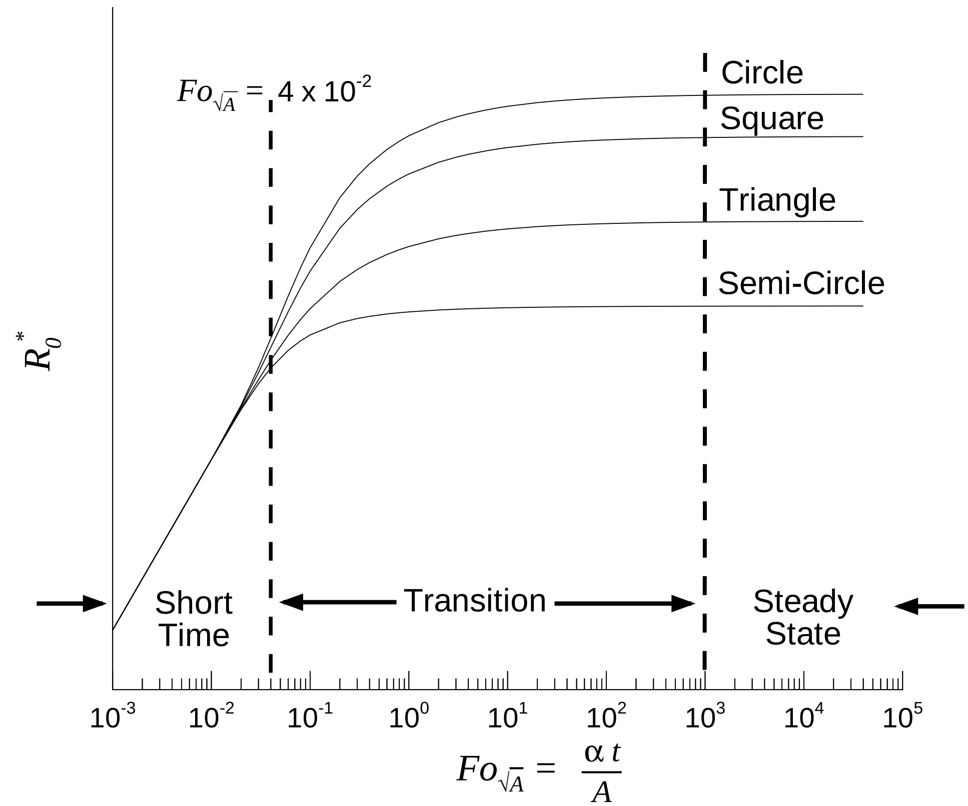

Figure 4: Diagram of Universal Time Function for Various Contact Shapes

the solution for the circular area. The steady-state solution yields the values: $R_0^* = 0.5617, 0.5611, 0.5642$ for the equilateral triangle ($N = 3$), the square ($N = 4$) and the circle ($N \to \infty$). The difference between the values for the triangle and the circle is approximately 2.2%, whereas the difference between the values for the square and the circle is less than 0.6%.

Universal Time Function for Regular Polygons

In this section a procedure will be proposed for simple, quick, and accurate computation of the centroid-based transient spreading resistance for the range: $4 \times 10^{-2} \le Fo_{\sqrt{A}} \le 10^6$, Fig. 4.

The closed form solution for the circle is the basis of the proposed method. For any planar, singly-connected, contact area subjected to a uniform heat flux take

$$\frac{R_0^*}{R_0^*(Fo_{\sqrt{A}} \to \infty)} = 2\sqrt{Fo_{\sqrt{A}}}\left[1 - \exp\left(\frac{-1}{(4\pi Fo_{\sqrt{A}})}\right) + \frac{1}{2\sqrt{Fo_{\sqrt{A}}}}\mathrm{erfc}\left(\frac{1}{2\sqrt{\pi}\sqrt{Fo_{\sqrt{A}}}}\right)\right] \quad (19)$$

The left-hand side is the ratio of the instantaneous value divided by the steady-state value. The right-hand side of the above equation can be considered to be a *universal* dimensionless time function that accounts for the transition from small times to near steady-state. The proposed procedure provides accurate results for all regular polygons. The largest error of approximately 2.2% was found for the triangle at $\sqrt{Fo_{\sqrt{A}}} = 10^{-2}$. For all other dimensionless times the error is less than 1%.

For other planar, singly-connected areas with aspect ratios greater than 0.2, the proposed procedure will give values of R_0^* with an error of approximately 2%. The error decreases as the aspect ratio approaches 1.

A simpler expression which is based on the Greene[16] approximation of the complementary error function is proposed:

$$\frac{R_0^*}{R_0^*(Fo_{\sqrt{A}} \to \infty)} = \frac{1}{\vartheta\sqrt{\pi}}\left[1 - e^{-\vartheta^2} + a_1\sqrt{\pi}\vartheta e^{-a_2(\vartheta + a_3)^2}\right] \quad (20)$$

where $\vartheta = 1/(2\sqrt{\pi}\sqrt{Fo_{\sqrt{A}}})$ and the three correlation coefficients are: $a_1 = 1.5577, a_2 = 0.7182, a_3 = 0.7856$. This approximation will provide values of R_0^* with maximum errors less than 0.5% for $Fo_{\sqrt{A}} \ge 4 \times 10^{-2}$.

Summary and Discussion

Integral solutions are presented for the transient dimensionless spreading resistance which is based on the centroid temperature rise of the hyperellipse and regular polygon contact areas. An integral solution is presented for the steady-state spreading resistance for the hyperellipse and a closed-form solution is given for the regular polygon.

The transient and steady-state solutions for the hyperellipse depends on the shape parameter, the aspect ratio and the dimensionless time. The transient and steady-state solutions for the regular polygon depend on the number of sides and the dimensionless time.

The dimensionless spreading resistance and the dimensionless time are based on the square root of the contact area.

For very small times the dimensionless spreading resistances for all contact areas follow the half-space solution which is independent of shape and aspect ratio. For very large times the differences between the dimensionless spreading resistances for all shapes is less than 3.5% when the aspect ratio is close to 1. The steady-state values of the dimensionless spreading resistance for the elliptical and rectangular contact areas differ by less than 1% when the aspect ratio is 1 and they differ by less than 4.5% when the aspect ratio is 0.1. The spreading resistance of the elliptical contact area is larger than that of the rectangular contact area.

A procedure is proposed for the development of a universal time function which is applicable to all contact areas for dimensionless times in the transition range: $4 \times 10^{-2} - 10^{6}$. The universal time function is based on the solution for the isoflux circular contact area which has an analytic solution for all dimensionless time.

An alternate form of the universal time function which does not require the calculation of the complementary error function is proposed for quick calculations.

Acknowledgments

The financial support of the National Sciences and Engineering Research Council of Canada under Grant No. A7455 is gratefully acknowledged. The assistance of P. Teertstra and M. Stevanovic in the preparation of the figures and plots is appreciated.

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