

# Experimental Study of Forced Convection From Isothermal Circular and Square Cylinders and Toroids

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*Experimental studies of forced convection heat transfer from different body shapes were conducted to determine the effects of Reynolds number and different characteristic body lengths on the area-averaged Nusselt number. Although the bodies differed significantly in their shapes, they had approximately the same total surface area,  $A = 11,304 \text{ mm}^2 \pm 5\%$ . This ensured that for a given free stream velocity and total heat transfer rate all bodies had similar trends for the relationship of Nusselt and Reynolds numbers. The experimental program range was conducted in the Reynolds number range  $10^4 \leq Re_{\sqrt{A}} \leq 10^5$  and Prandtl number 0.71. Finally, the empirical models for forced convection heat transfer were developed. These empirical models were valid for a wide range of Reynolds numbers  $0 \leq Re_{\sqrt{A}} \leq 10^5$ . The present experimental correlations were compared with available correlation equations and experimental data. These comparisons show very good agreement.*

## Introduction

Experimental forced convection heat transfer from circular and square cylinders, or toroids, is important in a number of fields, such as heat exchange, boiler design, gas turbine blades, hot wire anemometry and the rating of the electrical conductors. The present study has been initiated to resolve the issue of the effect of the geometric of the body on the area-averaged Nusselt number. Also, this work seeks to provide a simple procedure to predict forced convection heat transfer from bodies of different shapes, based on the characteristic length of the square root of total surface area,  $\sqrt{A}$ , which was first suggested by Yovanovich (1987a, 1987b). The experimental data obtained from the present study will provide the necessary empirical correlations needed.

A schematic diagram of the different body shapes, which will be investigated in the present study, is shown in Fig. 1. These different body shapes will be maintained at  $T_s$  and the ambient will be maintained at  $T_\infty$ . These bodies will be subjected to a uniform crossflow of air.

## Literature Survey

**Circular Cylinder.** Numerous studies have attempted to correlate the area-averaged heat transfer by forced convection from long circular cylinders in crossflow in the form:  $Nu = Nu(Re, Pr)$ . However, all of them have failed to establish this goal in a general form,  $Nu = Nu(L/D, Re, Pr)$ . Table 1 shows a summary of the important previous studies and their correlations. In addition, Table 1 reveals disagreement in the results. These results can be summarized as follows:

- 1 For three-dimensional flow, in general, each  $L/D$  has its own correlation.
- 2 For the same  $L/D$ , it is common to find disagreement in the value of  $Nu$ , e.g., between Ahmad and Qureshi (1992) and Galloway and Sage (1967) for  $L/D = 8$ ,

Burn et al. (1956)<sup>2</sup> and Krujilin (1938)<sup>2</sup> for  $L/D = 50$  ... etc.

- 3 The power of Reynolds number varied between 0.3 and 0.75.
- 4 A few studies considered the diffusive term and it was included in the correlations. However, these studies disagreed on how to estimate this term, for example, King (1914)<sup>2</sup> reported it to be equal to 0.315 ( $1290 < L/D < 1520$ ) and Laurence and Landes (1952)<sup>2</sup> found it to be 0.19 ( $400 < L/D < 2500$ ). Also, Hatton et al. (1970) estimated the diffusive term to be 0.384 ( $96 < L/D < 1190$ ). However, Delleur (1964)<sup>2</sup> evaluated it as 1.148 ( $L/D = 100$ ). On the other hand, most of the other studies neglected the diffusive term.
- 5 There are two-dimensional numerical studies that assumed  $L \gg D$ . However, it is not known when we can conclude that the circular cylinder is two-dimensional.

In addition, Churchill and Bernstein (1977) concluded many reasons for the above limitations. These limitations are: the lack of a comprehensive theory for the dependence on  $Pr$ , even for the boundary layer regime; competitive theories for low  $Re$ ; the influence of natural convection at very low  $Re$ ; discrete transitions in the boundary layer and the wake at high  $Re$ ; the influence and the lack of specification of free stream turbulence; the use of different and undefined thermal boundary conditions; significant variation in physical properties between the surface and free stream around the surface; the incorporation of erroneous physical properties in older tabulated data, for example the work of Hilpert (1933); end effects, particularly at low  $Re$ ; tunnel blockage; significant scatter in most of the data sets; and finally, unresolved discrepancies between the various sets of data. In addition, Morgan (1975) recognized many of these effects especially the blockage of the wind tunnel, turbulence intensity, and thermal properties. Based on that, Morgan (1975) also corrected and correlated many of the previous studies, in particular, the data of Hilpert (1933) as shown in Table 2.

Churchill and Bernstein (1977) developed a correlation for forced convection heat transfer from circular cylinders as follows:

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<sup>2</sup> From Morgan (1975)

and Williams (1959)<sup>2</sup> ( $2070 \leq L/D \leq 8860$ ) and Krall and Eckert (1973) ( $L/D = 6.3$ ). These experimental studies have been done for different  $L/D$ . However, Churchill and Bernstein (1977) developed Eq. (1) by using the following procedure:

$$Nu_D = a \cdot Re_D^{0.5} \frac{Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \quad (2)$$

where  $a$  has been obtained from the numerical studies of Masliyah and Epstein (1973) and Jain and Goel (1976) at  $Re_D = 1$ . They found  $a = 0.62$  in order to provide the best fit for their equation. The diffusive term has been estimated from the experimental and theoretical work of King (1914)<sup>2</sup>.

**Square Cylinder.** A few previous studies investigated forced convection heat transfer from square cylinders. Table 3 shows the summary of the previous studies experimental correlations. Hilpert (1933) and Reiher (1926) examined this problem experimentally in different orientations using opposing flow and assisting flow. Igarashi (1985) also examined forced convection from square cylinders experimentally. He used three square cylinders,  $L/W = 10, 7.5, 5$  and  $L = 0.15$  m, and the turbulence level in the working section was 0.5% where  $5.6 \times 10^3 \leq Re_w \leq 5.6 \times 10^4$ . Recently, Bishop (1987) and Oosthuizen and Bishop (1987) investigated mixed convection from square cylinders experimentally and numerically using opposing flow and assisting flow. Also, they examined this problem using the heat transfer transient technique. One can conclude the following limitations of the previous studies: the influence of free convection was ignored, especially at low Reynolds number; significant variations in physical properties between the surface and the free stream such as Oosthuizen and Bishop (1987),  $\Delta T = 70$  K; the use of erroneous physical properties in older tabulated data, e.g., Hilpert (1933) and Reiher (1926); the effect of free stream turbulence on forced convection results; and finally, significant scatter in some of the data sets.

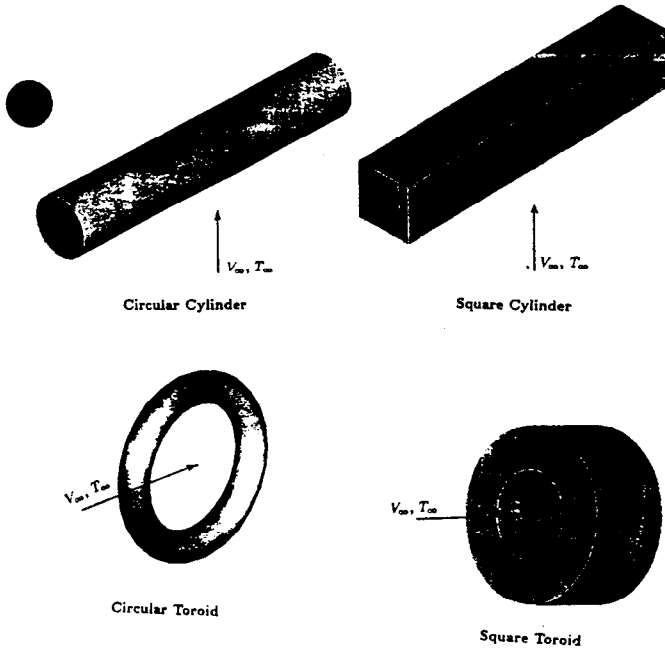


Fig. 1 Schematic diagram of body shapes

$$Nu_D = 0.3 + 0.62 Re_D^{0.5} \frac{Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \times [1 + (Re_D/282000)^{5/8}]^{4/5} 10^2 \leq Re_D \leq 10^7. \quad (1)$$

Equation (1) was developed for "infinite" circular cylinders, i.e.,  $L \gg D$ , but they did not mention the ratio of  $L/D$ . However, they compared their correlation with experimental data for air of Hilpert (1933) ( $0.95 \leq L/D \leq 5120$ ), Collis

## Nomenclature

$A$  = surface area,  $m^2$   
 $a$  = constant in Eq. (2)  
 $c$  = constants in Eq. (16)  
 $C_2$  = constants in Eq. (21)  
 $C_{\bar{A}}$  = body shape parameter in Eq. (32)  
 $C_1$  = constants in Table 1  
 $C_2$  = constants in Tables 1, 2 and 3  
 $C_p$  = specific heat transfer,  $kJ/kg$  K  
 $CR$  = correction factor  
 $D$  = cylinder diameter,  $m$   
 $D_{CS}$  = cross section diameter,  $m$   
 $D_i$  = inner diameter of toroids,  $m$   
 $D_o$  = outer diameter of toroids,  $m$   
 $F$  = view factor  
 $\bar{F} = F \times \epsilon$   
 $G_{\bar{A}}$  = gravity function  
 $h$  = coefficient of convection heat transfer,  $W/m^2$  K  
 $I$  = electric current,  $A$   
 $k$  = thermal conductivity,  $W/m$  K  
 $K$  = constant in Eq. (23)  
 $L$  = length,  $m$   
 $\ell$  = arbitrary scale length,  $m$   
 $l$  = mixing length,  $l = Ky$ ,  $m$   
 $m$  = exponent in Tables 1, 2 and 3  
 $n$  = exponent in Eq. (9)  
 $Nu_x$  = area-averaged Nusselt number,  
 $Nu_x = \ell h/k$

$Nu_x^0$  = area-averaged diffusive Nusselt number,  $Nu_x = \ell h/k$   
 $q$  = heat flux,  $W/m^2$   
 $Q_{FC}$  = forced convection heat transfer,  $W$   
 $Q_{NC}$  = natural convection heat transfer,  $W$   
 $Q_{Rad}$  = radiation heat transfer,  $W$   
 $Q_T$  = total heat transfer,  $W$   
 $Q_w$  = wire losses,  $W$   
 $P$  = perimeter,  $m$   
 $Pr$  = Prandtl number,  $Pr = \nu/\alpha$   
 $Re_x$  = Reynolds number,  $Re_x = \ell V_\infty/\nu$   
 $T_s$  = surface temperature,  $K$   
 $T_\infty$  = ambient temperature,  $K$   
 $Tu$  = turbulence intensity  
 $V$  = voltage,  $V$   
 $\mathcal{V}$  = volume,  $m^3$   
 $V_\infty$  = free stream velocity,  $m/s$   
 $V'_\infty$  = measured free stream velocity,  $m/s$   
 $W$  = width of the square cylinder,  $m$   
 $x'$  = distance from the end of the contraction to the location of the object in the wind tunnel,  $m$

## Greek Letters

$\alpha$  = thermal diffusivity,  $\alpha = k/C_p\rho$ ,  $m^2/s$   
 $\alpha_t$  = turbulent thermal diffusivity,  $m^2/s$   
 $\gamma_{\bar{A}}$  = constant in Eq. (27)  
 $\epsilon$  = emissivity  
 $\Delta T = T_s - T_\infty$   
 $\nu_t$  = turbulent kinematic viscosity,  $m^2/s$   
 $\rho$  = density,  $kg/m^3$   
 $\sigma$  = Stefan-Boltzmann constant,  $W/m^2$   $K^4$   
 $\Upsilon$  = blockage correction

## Abbreviations

Cond = conduction  
 Conv = convection  
 Diff = Difference  
 FC = forced convection  
 MHTL = Microelectronics Heat Transfer Laboratory  
 max = maximum  
 NC = natural convection  
 Rad = radiation  
 T.S. = test section

Table 1 Forced convection Nusselt for circular cylinders in a crossflow for  $Pr = 0.71$ ,  $Nu_D = C_1 + C_2 Re_D^m$

Author	$L/D$	$C_1$	$C_2$	$m$	$Re_D$
King (1914) <sup>†</sup>	1290 - 1520	0.32	0.48	0.5	0.06 - 50.0
King (1932) <sup>†</sup>	-	0.0	0.764	0.41	3 - 300
	-	0.0	0.282	0.585	300 - 4000
Hilpert (1933)	5120	0.0	0.891	0.33	1 - 4
	1625 - 5120	0.0	0.821	0.385	4 - 40
	20 - 3170	0.0	0.615	0.466	40 - 40000
	5.6 - 20	0.0	0.174	0.618	40000 - 400000
Krujilin (1938) <sup>†</sup>	2.7 - 42.4	0.0	0.27	0.6	6000 - 130000
Kramers (1946)	-	0.39	0.52	0.5	9.5 - 1420
Masil-Sherwood (1950) <sup>†</sup>	4 - 2.5	0.0	0.322	0.57	6,000 - 30,000
Eckert-Soehngren (1952) <sup>†</sup>	6 - 18	0.0	0.57	0.473	20 - 600
Laurence-Landes (1952) <sup>†</sup>	400 - 2500	0.19	0.51	0.5	15 - 70
Snyder (1953) <sup>†</sup>	12	0.0	0.278	0.55	8000 - 20000
Burn et al. (1956) <sup>†</sup>	37.5 - 60.0	0.0	0.136	0.65	6000 - 15000
Van der Hegge (1956) <sup>†</sup>	37.5 - 555	0.0	0.68	0.41	5 - 50
	37.5 - 555	0.35	0.43 *	0.5	20 - 80
Collis-Williams (1959) <sup>†</sup>	2070 - 8660	0.24	0.56	0.45	0.02 - 44
	2070 - 8660	0.0	0.48	0.51	44 - 140
Kazabvetich (1959) <sup>†</sup>	11.4	0.0	0.246	0.6	5000 - 35000
Van Mell (1962) <sup>†</sup>	6.9	0.35	0.5	0.5	5000 - 37000
Delleur (1964) <sup>†</sup>	100	1.15	0.726	0.5	1 - 3
Galloway-Sage (1967)	8.0	0.0	0.167	0.637	2700 - 38000
Hatton et al. (1970)	96 - 1190	0.38	0.581	0.439	0.6 - 45
	96 - 1190	0.0	0.95	0.3	0.6 - 4
	96 - 1190	0.0	0.81	0.38	4 - 40
	96 - 1190	0.0	0.786	0.392	16 - 45
Srenivasen-Ramachandram (1971) <sup>†</sup>	17.5	0.0	0.226	0.6	2500 - 15000
Andrews (1972) <sup>†</sup>	24 - 1300	0.34	0.65	0.45	0.015 - 20
Bradbury (1972) <sup>†</sup>	270 - 2000	0.24	0.56	0.45	0.5 - 12
Krall-Eckert (1973) <sup>†</sup>	6.3	0.0	0.93	0.37	5 - 50
	6.3	0.0	0.64	0.46	50 - 5000
Koch (1972) <sup>†</sup>	230	0.72	0.8	0.45	1.2 - 4.1
Petrie-Simpson (1972) <sup>†</sup>	5.7	0.0	0.042	0.75	4000 - 33000
Churchill-Bernstein (1977)	-	0.3	0.485	0.5	-
Zukauskas-Ziugzda (1985)	-	0.0	0.23	0.60	1000 - $2 \times 10^5$
Ahmad-Qureshi (1991)	-	0.0	0.844	0.383	1 - 60
Ahmad-Qureshi (1992)	8.0	0.0	0.0675	0.733	6800 - 21300

<sup>†</sup> from Morgan (1975)

\*  $Nu_D = C_1 + C_2 Re_D^m + 0.001 Re_D$

The circular and square toroids were not considered in previous studies which were available to authors. The objectives of the present investigation can be summarized as follows:

- 1 use experimental techniques to study the forced convection heat transfer from different arbitrary body shapes such as spheres, circular horizontal cylinders, square horizontal cylinders, toroids and square toroid;
- 2 examine different scale lengths such as square root of the total heat transfer area,  $\sqrt{A}$ , diameter,  $D$ , the ratio of surface area to the perimeter length,  $A/p$  and the ratio of the volume to the surface area,  $4\pi V/A$ , etc. (The effect of the characteristic length,  $L$ , will be taken into consideration in the present study.);
- 3 develop a general model based on the experimental results and the approximate analytical solution of Refai Ahmed and Yovanovich (1995) in order to explain the forced convection heat transfer from different body shapes;
- 4 develop design correlations of the experimental results in order to satisfy the needs of various engineering applications. In addition, the design correlations will also consider the conduction limit,  $Re_c = 0$ , for the different body shapes which have been introduced by Jafarpur (1992) and Wang (1993).

### Experimental Program

The purpose of the experimental program is to study the external forced convection heat transfer from isothermal bodies of different shapes. The experiments are necessary because of the analytical difficulties involved in finding a solution for any shape. These experiments can be a starting point in the development of empirical models of the bodies considered. The current experiments are designed to meet the following requirements: maintain the body at uniform temperature; minimize radiation heat transfer; accurately determine effective surface emissivity of the bodies in order to correct for radiation losses; and determine the effect of turbulence intensity.

The body geometries which were tested in the present study (see Fig. 1) are:

- horizontal circular cylinder with  $L/D = 9$  and  $D = 0.02$  m
- horizontal square cylinder with  $L/W = 9.12$  and  $W = 0.017$  m
- circular toroid with  $D_o = 0.078$  m,  $D_i = 0.039$  m and  $D_{cs} = 0.039$  m
- square toroid,  $D_o = 0.068$  m and  $D_i = 0.034$  m and  $L = 0.017$  m

These body shapes are manufactured from 6061-T6 aluminum with a thermal conductivity of 167 W/mK. The main reason to

**Table 2** Recalculation of the Hilpert data (1933) by Morgan (1975),  $Nu_D = C_2 Re_D^m$

$Re_D$	$L/D$	Hilpert (1933)		Morgan (1975)	
		$C_2$	$m$	$C_2$	$m$
1 - 4	5120	0.891	0.33	-	-
4 - 40	1625 - 5120	0.821	0.385	0.795	0.384
40 - 4000	20 - 3170	0.615	0.466	0.583	0.471
4000 - 40000	5.6 - 20	0.174	0.618	0.148	0.633
40000 - 400000	0.9 - 11.4	0.0239	0.805	0.0208	0.814

choose a high thermal conductivity material is to ensure that we maintain an isothermal boundary condition. Furthermore, the polished aluminum will minimize the radiative heat transfer between 5 percent to 2 percent of the total heat transfer rate. In addition, all body shapes have approximately the same surface area ( $A = 11304 \text{ mm}^2 \pm 5.5\%$ ).

**Wind Tunnel.** The present experiment was conducted in a low-speed open suction wind tunnel. Figure 2 shows a sketch of the wind tunnel. The cross section of the test section is 0.3 m by 0.3 m. The present investigation examined the flow quality in the X-direction and the Y-direction at three different fan speeds. It was observed that the maximum difference between the average velocity and the local velocity was approximately 3.8% (more details in Refai Ahmed, 1994). Furthermore, the velocity can be considered constant to within 30 mm of the side wall of the test section.

**Blockage Effect.** One of the main concerns for any forced convection heat transfer experiment inside the wind tunnel is the blockage effect,  $\Upsilon$ , where the velocity of the stream is affected directly by the blockage as shown in the following equation:

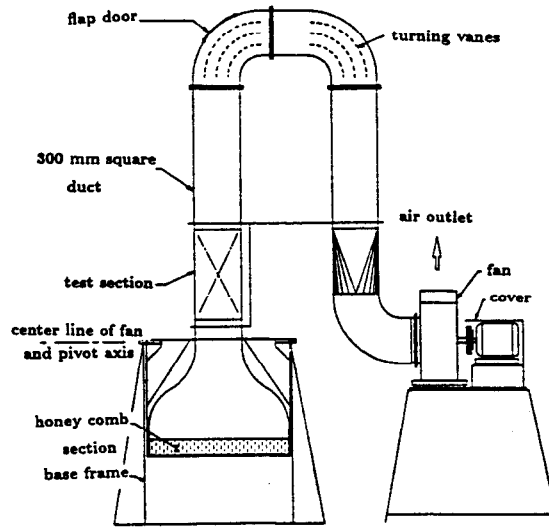
$$V_x = (1 + \Upsilon) \cdot V'_x \quad (3)$$

where  $V'_x$  is the measured free stream velocity which does not account for the blockage effect. Rae and Pope (1984) suggested

**Table 3** Forced convection Nusselt for square cylinders in crossflow for  $Pr = 0.71$ ,  $Nu_w = C_2 Re_w^\dagger$

Author	Orientation*	$C_2$	$m$	$Re_w$
Hilpert (1933)	0.0	0.085	0.675	3900 - 78500
	$\pi/4$	0.201	0.588	3900 - 78500
Reiher (1926)	0.0	0.149	0.691	1960 - 6000
	$\pi/4$	0.238	0.624	1960 - 6000
Igarashi (1985)	0.0	0.14	0.660	5600 - 56000
	$\pi/4$	0.27	0.59	5600 - 56000
Oosthuizen - Bishop (1987)	0	0.281	0.57	300 - 5000
	$\pi/4$	0.414	0.537	300 - 5000

† Correlations were developed for both assisting and opposing flow  
 \* 0 means horizontal square cylinder  
 $\pi/4$  means square horizontal with  $\pi/4$  rotation about the longitudinal axis



**Fig. 2** Sketch of wind tunnel

that the blockage equation,  $\Upsilon$ , for a shape that needs to be tested in a tunnel is

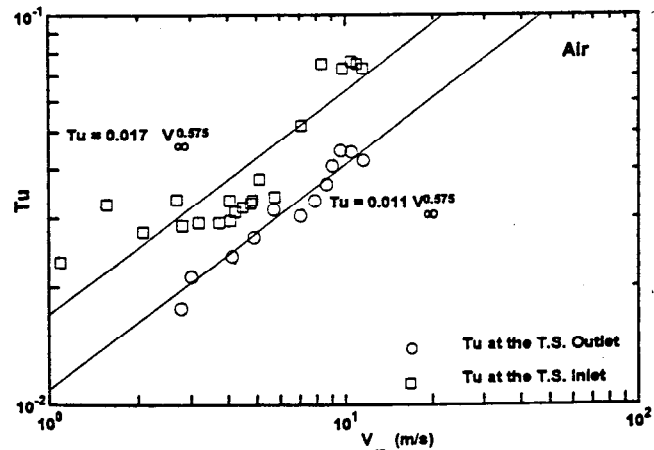
$$\Upsilon = 0.25 \cdot \left[ \frac{\text{Model Frontal Area}}{\text{Test Section Area}} \right] \quad (4)$$

In the present study  $\Upsilon$  will be fixed at a maximum value of 0.01. It is recommended to the reader to review Morgan (1975) for the extensive review of the blockage effect on the area-averaged Nusselt number.

**Turbulence Intensity.** One of the main concerns about the previous forced convection heat transfer studies is the turbulence intensity and its effect on the output results. Therefore, the present study determined the effect of the turbulence intensity. Figure 3 shows the experimental results for different velocities and the corresponding values of the turbulence intensity,  $Tu$ , at the inlet and outlet of the test section (more details in Refai Ahmed, 1994).  $Tu$  is calculated from the following equation:

$$Tu = 6.59 \times 10^{-6} \left( \frac{L}{x'} \right)^{1.15} Re_x^{0.575} \quad (5)$$

where  $L = 2.17 \text{ m}$  and  $x'$  is the distance from the end of the contraction to the location of the object being tested.



**Fig. 3** Relationships between turbulence intensity and free stream velocity at inlet and outlet of MHTL wind tunnel test section

**Emissivity Measurements.** The general experimental method, which is used here, is similar to the approach used by Hassani (1987). The current study is designed to find the thermal emissivity at the steady state condition where

$$Q_T = Q_{Cond} + Q_{NC} + Q_{Rad} + Q_w \quad (6)$$

Hassani (1987) has shown that the heat transfer due to convection and conduction can be ignored if the testing body is placed in a vacuum. However,  $Q_w$ , wire loss is determined by modeling the wire as an infinite circular fin. Therefore,

$$\bar{F} = \frac{VI - Q_w}{\sigma A(T_s^4 - T_\infty^4)} \quad (7)$$

The present study is conducted for different body shapes in a vacuum chamber in which the air pressure is approximately  $10^{-5}$  Torr in order to measure the coefficient  $\bar{F}$  (shape factor and emissivity constant).  $\bar{F}$  was 0.15, 0.117, 0.125 and 0.115 for the circular cylinder, square cylinder, circular toroid, and square toroid, respectively.

### Forced Convection Heat Transfer Calculation

The present experiments will give us the total heat transfer,  $Q_T$ , from the body (more details about the measurement of  $Q_T$  can be found in Refai Ahmed, 1994) where  $Q_T$  at the steady state is the sum of convection heat loss,  $Q_{Conv}$ , the radiation heat loss,  $Q_{Rad}$ , and the wire heat loss,  $Q_w$ . Therefore,

$$Q_{Conv} = Q_T - Q_{Rad} - Q_w \quad (8)$$

The effect of free convection on forced convection phenomena has been investigated under the subject of mixed or combined free and forced convection. Morgan (1975), Jaluria (1980), and Burmeister (1993) provide a detailed review of the many investigations on the subject of mixed convection from isothermal plates, cylinders, and spheres. Most of the previous studies such as Churchill (1977) and McAdams (1942) presented this phenomena in the following form:

$$Nu = (Nu_{FC}^n + Nu_{NC}^n)^{1/n} \quad (9)$$

Yovanovich and Vanoverbeke (1988) developed their mixed convection model based on the forced convection correlation of Yuge (1960) and the free convection correlation of Raithby and Hollands (1975) for sphere and  $Pr = 0.71$ . These correlations are

$$Nu_{D_{FC}} = 0.491 Re_D^{0.5} \quad (10)$$

and

$$Nu_{D_{NC}} = 0.452 Ra_D^{0.25} \quad (11)$$

Yovanovich and Vanoverbeke (1988) also proposed that for forced convection from a sphere, the dimensionless heat transfer rate by convection is the summation of the dimensionless heat transfer by conduction, free convection, forced convection, and a correction factor,  $CR$ , where  $CR$  is  $0.86 - 2.86(Ra_D/Re_D^2)^{1/4}$ . Furthermore, Steinberger and Treybal (1960) proposed a formula similar to Yovanovich and Vanoverbeke (1988) for assisted flow. Churchill (1977) used the same formula for either forced convection or free convection for air, but his constant for forced convection is 0.535 and for free convection is 0.511. For the mixed convection from spheres, this can create some differences in the limits of pure free convection and forced convection. These differences are carried to the mixed convection predication. Therefore, Eq. (9) can not predict  $Nu$  very accurately, unless  $Nu_{NC}$ ,  $Nu_{FC}$ , and  $n$  are well known.

Other previous studies presented the mixed convection as a function of  $(Gr/Re^2)$  such as Sparrow and Gregg (1959), Lloyd and Sparrow (1970), and Eshghy (1964) where at  $Gr/Re^2 \approx 1$ , both free and forced convection are important. In contrast, free convection is dominant when  $Gr/Re^2 \gg 1$  and forced convection is dominant when  $Gr/Re^2 \ll 1$ . Furthermore, Clift et al. (1978) stated that, "consideration of the available data for spheres indicates that forced flow correlations are accurate to about 10% for  $Gr/Re^2 < 0.2$ ." Yovanovich and Vanoverbeke (1988) proved that Clift et al.'s statement is inaccurate for the Rowe et al. (1965) air data. Also, they found that free convection effects in cross flow were observed to be significant for  $Gr/Re^2$  as low as  $10^{-4}$ . In addition, Clift et al. (1978) examined Yuge (1960) sphere's experimental data for mixed convection of aiding flow and cross flow in the ranges of  $3 < Re_D < 300$ ,  $180 < Gr_D < 1800$  and  $Pr = 0.71$ . They concluded that aiding flow data lay slightly above those for crossflow. Similar behavior occurs for cylinders as observed by Hatton et al. (1970) and Oosthuizen and Madan (1971).

The present study will use the Yovanovich and Vanoverbeke (1988) concept to remove the effect of free convection from the data of different body shapes. However, in the experimental range of the present study  $CR$  is negligible. Therefore,  $Q_{FC}$  can be approximated as

$$Q_{FC} = Q_{Conv} - Q_{NC} \quad (12)$$

for assisted flow conditions.

**Radiation Heat Loss.** After the method of how to obtain  $\bar{F}$  is described, the radiation heat loss can be determined as follows:

$$Q_{Rad} = \bar{F}\sigma A(T_s^4 - T_\infty^4) \quad (13)$$

It is estimated from the preliminary experiments that the maximum  $Q_{Rad}$  is approximately 6 percent of the input power when  $\bar{F} = 0.115$  and  $T_s = 310$  K.

**Wire Loss.** The wire loss by heat conduction from the thermocouple wires and the power leads is taken into account by treating them as infinite fins. Mack (1991) reported that the conduction losses were of the order of 0.5% of the total input power which is an order of magnitude less than the radiation loss. However, in the experimental procedure for radiation tests the wire loss is considered.

### Free Convection Heat Loss

The free convection heat loss for the different body shapes will be calculated based on the proposed model of Jafarpur (1992).

$$Q_{NC} = \sqrt{A} \cdot \Delta T \cdot k \cdot F(Pr) \cdot G_{\sqrt{A}} \cdot Ra_{\sqrt{A}}^{1/4} \quad (14)$$

where  $F(Pr)$  is the Prandtl number function, which was proposed by Churchill (1983)

$$F(Pr) = \frac{0.67}{[1 + (0.5/Pr)^{9/16}]^{4/9}} \quad (15)$$

and  $G_{\sqrt{A}}$  is the body gravity function defined as

$$G_{\sqrt{A}} = c \left( \frac{P_{max}}{\sqrt{A}} \right)^{1/4} \quad (16)$$

where  $P_{max}$  is the maximum perimeter and  $\sqrt{A}$  is the surface area, i.e.,  $c$  depends on the body shape and it lies in the range of 0.8 to 1.

**Error Analysis.** The overall uncertainties for  $Re_x$ ,  $Ra_x$ ,  $\epsilon$ , and  $Nu_x$  are investigated, and it is found that the uncertainty of  $Re_x$  is  $\pm 7.2$  percent which occurs at low values of the Reynolds

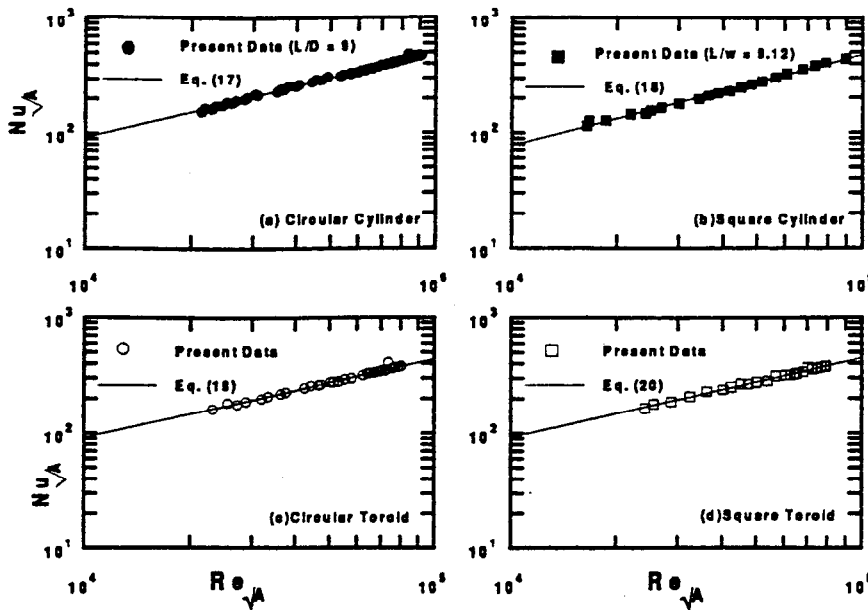


Fig. 4 Relationships between Nusselt and Reynolds number for different body shapes (air)

number. In addition, it is found that the uncertainty in  $Nu_c$  is approximately  $\pm 5.4$  percent (more details in Refai Ahmed, 1994).

### Data Analysis

Figure 4 shows the present experimental results of forced convection heat transfer from the isothermal circular cylinder, square cylinder, circular toroid, and square toroid in the Reynolds number range,  $10^4 \leq Re_{\sqrt{A}} \leq 10^5$  and  $0.012 \leq Tu \leq 0.049$ . These data have been reduced by removing radiation and free convection heat transfer effects. However, the effect of the turbulence intensity has not been removed. The dimensionless parameters,  $Nu_c$  and  $Re_c$ , have been defined with respect to  $L = \sqrt{A}$ .

Two procedures were used to generate the experimental results. The first technique involved fixing the electric power input of the heater at five and ten W and varying the flow velocity. In contrast, the second technique was to fix the temperature difference,  $\Delta T$ , by varying the flow velocity and the electric power simultaneously. Both methods produced similar experimental results.

Figure 4(a) shows the present data of Nusselt and Reynolds numbers for forced convection heat transfer ( $Pr = 0.71$ ) from the isothermal circular cylinder in cross flow with  $L/D = 9$  and  $D = 0.02$  m. The effect of natural convection in the experimental data was between 10 percent and 3.5 percent for the range of Reynolds numbers between 21,000 and 92,000. However, the radiation effect in the same range of Reynolds number was 2 percent to 0.8 percent.

Figure 4(b) presents the present data of Nusselt and Reynolds numbers for forced convection ( $Pr = 0.71$ ) from a horizontal square cylinder with  $L/W = 9.12$  and  $W = 0.017$  m. The maximum effect of radiation in the range of Reynolds number from 24,000 to  $9 \times 10^4$  was 2 percent at  $Re_{\sqrt{A}} \approx 24,000$ . However, the free convection effect was 20 percent at  $Re_{\sqrt{A}} \approx 24,000$  and the minimum effect was 5 percent at  $Re_{\sqrt{A}} \approx 90,000$ .

Figures 4(c) and 4(d) show the present data of the Nusselt and Reynolds numbers for forced convection heat transfer ( $Pr = 0.71$ ) from both the isothermal horizontal circular toroid with  $D_o = 0.078$  m,  $D_i = 0.039$  m, and  $D_{CS} = 0.039$  m and the square toroid with  $D_o = 0.068$  m,  $D_i = 0.034$  m, and  $L = 0.017$  m (see Fig. 1). The effect of free convection was found to be

a maximum of 13.4 percent at  $Re_{\sqrt{A}} = 23153$  and a maximum of 1.6 percent for radiation at  $Re_{\sqrt{A}} = 23153$ . Table 4 shows the relationships between  $Nu_{\sqrt{A}}$  and  $Re_{\sqrt{A}}$  and the maximum and RMS percent differences for the different body shapes.

### Effect of Characteristic Length on $Nu - Re$ Relationship

One of the important issues in a study dealing with different body shapes is how to present the relationships between the Nusselt and Reynolds numbers. Therefore, the choice of an appropriate scale length is a very important step in the development of the final results for this type of study. Yovanovich (1987b) examined a number of different scale lengths, such as the diameter for the sphere and the horizontal circular cylinder, the height of a vertical cylinder and flat plate, and the minor and major axes of oblate spheroids. Yovanovich (1987b) concluded that these scale lengths are not general when they approach zero as the thickness of the body goes to zero. Other scale lengths have also been used in the literature, such as the diameter of a sphere having the same volume as the body,  $L = (6V/\pi)^{1/3}$ ,  $L = 2V/A$  was used by Tsubouchi and Sato (1960). In addition, Churchill and Churchill (1975) used  $L = 4\pi V/A$ . Jafarpur (1991) examined these scale lengths and found that as the thickness goes to zero, the characteristic lengths becomes zero. Therefore, none of these characteristic lengths can be considered as a general scale length. Pasternak and Gauvin (1960) proposed a characteristic length of  $A/P$ . Furthermore, other studies have also used this scale length such as, Goldstein

Table 4 Experimental correlation of forced convection

Body Shape	Correlation Equation of $Nu_{\sqrt{A}}$	RMS Diff. %	Maximum Diff. %	Eq.
Circular Cylinder	$0.508 Tu^{0.174} Re_{\sqrt{A}}^{0.847}$	2.15	6.03	(17)
Square Cylinder	$0.303 Tu^{0.174} Re_{\sqrt{A}}^{0.685}$	1.53	4.73	(18)
Circular Toroid	$0.899 Tu^{0.174} Re_{\sqrt{A}}^{0.584}$	1.41	7.80	(19)
Square Toroid	$0.901 Tu^{0.174} Re_{\sqrt{A}}^{0.585}$	1.98	4.97	(20)

et al. (1973), Weber et al. (1984), and Sahraoui et al. (1990). Jafarpur (1991) stated some criteria for choosing an appropriate characteristic body length. These criteria can be summarized as follows: the characteristic length should be well defined, applicable to all bodies, possible to calculate or measure easily, intrinsic, physically interpretable, related to the orientation of

Recently Refai Ahmed et al. (1996) investigated analytically and experimentally the effect of the turbulence intensity on forced convection heat transfer from isothermal spheres. In addition, they developed an approximate analytical solution in the following form based on the linearization of the thermal energy equation:

$$Nu_{\sqrt{A}} = \left(1 + Pr \frac{\alpha_i}{\nu}\right) \cdot \left[ 3.545 + \frac{1.032 Re_{\sqrt{A}}^{0.5} \left[ \frac{(1/Pr) + (\alpha_i/\nu)}{1 + (\nu_i/\nu)} \right]^{1/6}}{\sqrt{(2\gamma_{\sqrt{A}} + 1) \left( \frac{1}{Pr} + \frac{\nu_i}{\nu} \right) \left( 1 + \left[ \frac{(1/Pr) + (\alpha_i/\nu)}{(2\gamma_{\sqrt{A}} + 1)^3 (1 + (\nu_i/\nu))} \right]^{1/6}} \right)}} \right] \quad (22)$$

the body, able to collapse all data into a single curve. Jafarpur examined both  $\ell = \sqrt{A}$  and  $\ell = A/P$  and based on these criteria he found that  $\sqrt{A}$  is the most appropriate scale length. In addition, the scale length of  $\sqrt{A}$  has also been used by Yovanovich (1987b), (1988) and Yovanovich and Vanoverbeke (1989).

The present study has examined the relationships between  $Nu_i$  and  $Re_i$  based on different characteristic lengths such as  $\sqrt{A}$ ,  $A/P$ ,  $(6\bar{V}/\pi)^{1/3}$  and  $4\pi\bar{V}/A$  (for the circular cylinder, square cylinder, circular toroid, and square toroid). The present investigation concluded that all of the relationships have the same trend and approximately the same slope ( $m \approx 0.7$ ). Therefore,  $Nu_i$  versus  $Re_i$  relationships may be expressed in the following form:

$$Nu_i = Nu_i^0 + C_i Re_i^{0.7} \quad (21)$$

$Nu_i^0$  is the diffusive Nusselt number and  $C_i$  is constant which is dependent on body shape. The maximum percent differences, using various scale lengths such as  $\sqrt{A}$ ,  $A/P$ ,  $(6\bar{V}/\pi)^{1/3}$ , and  $4\pi\bar{V}/A$ , between the coefficients of  $C_i$  for different body shapes are 30 percent, 18 percent, 25 percent and 23 percent and the maximum percent differences between the value of  $Nu_i^0$  for different body shapes are 19 percent, 51 percent, 28 percent and 59 percent, respectively. One can also observe that the characteristic length of  $A/P$ ,  $(6\bar{V}/\pi)^{1/3}$ , and  $4\pi\bar{V}/A$  are better than  $\sqrt{A}$  at high Reynolds number (where the diffusive limit is negligible). However, these scale lengths  $[(6\bar{V}/\pi)^{1/3}$  and  $(4\pi\bar{V}/A)]$  can not be used if the body thickness goes to zero. In contrast, at low Reynolds number (where the diffusive limit is more pronounced) the scale length of  $\sqrt{A}$  is better than the other characteristic body lengths. Therefore, the choice of the characteristic length is limited to either  $A/P$  or  $\sqrt{A}$ . In the present study the scale length of  $\sqrt{A}$  has been selected where this characteristic body length is the appropriate scale length, all over the wide range of Reynolds number, to collapse the experimental data. Also, all the present body shapes have the same surface area.

## Empirical Models

In this section empirical models are developed based on:

- the experimental results for the circular cylinder, the square cylinder, the circular toroid and the square toroid;
- the characteristic body length,  $\sqrt{A}$ , which was shown to have certain advantages in the above section;
- the removal of the effect of the free stream turbulence from  $C_{\sqrt{A}}$  by using Refai Ahmed et al. (1996) method; and
- the separation of  $F(Pr, \gamma_{\sqrt{A}})$  (which was derived in Refai Ahmed and Yovanovich, 1995) from  $C_{\sqrt{A}}$ .

where

$$\frac{\nu_i}{\nu} = [(1.253KTu\sqrt{Re_{\sqrt{A}}})^3 + (0.928K^2Tu^2Re_{\sqrt{A}})^3]^{1/3} \quad (23)$$

and

$$K = 0.05 \quad \text{and} \quad \alpha_i = \nu_i \quad (24)$$

The variable  $\gamma_{\sqrt{A}}$  was defined in Refai Ahmed and Yovanovich (1995). The solution of Refai Ahmed et al. (1996) was found to be in very good agreement with their experimental data and the data of Raithby (1967).

The present study applies the Refai Ahmed et al. (1996) solution to remove the effect of the turbulence intensity from the heat transfer results of the sphere. This solution is now applied to remove the turbulence intensity effect from the circular cylinder, square cylinder, circular toroid, and square toroid results. The relationships between Nusselt and Reynolds numbers based on the square root of the area have the same trend for each of these bodies, as mentioned before ( $m \approx 0.7$ ), after removing the radiation effect and the free convection effect. Therefore, the only effects left are the turbulence intensity and Prandtl number. Furthermore, the magnitude of the turbulence intensity is the same for all body shapes at the same Reynolds number, e.g., at  $Re_{\sqrt{A}} = 22000$ ,  $Tu = 0.022$  and  $Re_{\sqrt{A}} = 80000$ ,  $Tu = 0.046$ .

The present experimental results can be presented in the general form

$$Nu_{\sqrt{A}} = Nu_{\sqrt{A}}^0 + C_{\sqrt{A}} Re_{\sqrt{A}}^{0.5} F(Pr, \gamma_{\sqrt{A}}) \quad 0 \leq Re_{\sqrt{A}} \leq 6 \times 10^6 \quad (25)$$

This general form has been developed through the approximate analytical approach given in Refai Ahmed and Yovanovich (1995). Also, it was shown that  $F(Pr, \gamma_{\sqrt{A}})$  for different body shapes can be defined as follows:

$$F(Pr, \gamma_{\sqrt{A}}) = \frac{Pr^{1/3}}{\left[ (2\gamma_{\sqrt{A}} + 1)^3 + \frac{1}{Pr} \right]^{1/6}} \quad 0 < Pr < \infty \quad (26)$$

where, for the circular cylinder:

$$\gamma_{\sqrt{A}} = \frac{1}{[1 + 0.49 Re_{\sqrt{A}}^{1.25}]^{1/3}} \quad (27)$$

The empirical expression for  $\gamma_{\sqrt{A}}$  for the circular cylinder will be used for the square cylinder, circular toroid, and square toroid. The reduced experimental data from the turbulence intensity effect (using Refai Ahmed et al., 1996) can be correlated for the different body shapes as shown in Table 5.

Table 5 Proposed correlations of forced convection

Body Shape	Correlation Equation of $Nu_{\sqrt{A}}$	RMS Diff. %	Maximum Diff. %	Eq.
Circular Cylinder	$4.15 + 1.71 Re_{\sqrt{A}}^{0.5} F(Pr, \gamma_{\sqrt{A}})$	4.47	9.9	(28a)
	$0.76 + 0.73 Re_D^{0.5} F(Pr, \gamma_D)$	4.47	9.9	(28b)
Square Cylinder	$4.01 + 1.37 Re_{\sqrt{A}}^{0.5} F(Pr, \gamma_{\sqrt{A}})$	1.09	3.69	(29a)
	$0.65 + 0.55 Re_W^{0.5} F(Pr, \gamma_W)$	1.09	3.69	(29b)
Circular Toroid	$3.41 + 1.58 Re_{\sqrt{A}}^{0.5} F(Pr, \gamma_{\sqrt{A}})$	0.92	3.58	(30)
Square Toroid	$3.37 + 1.60 Re_{\sqrt{A}}^{0.5} F(Pr, \gamma_{\sqrt{A}})$	0.87	3.34	(31)

One observes from the correlations in Table 5 that the body shape parameter,  $C_{\sqrt{A}}$ , can be correlated in the general form

$$C_{\sqrt{A}} = 0.84 \left( \frac{P_{\max}}{\sqrt{A}} \right)^{0.5} \quad (32)$$

This semi-empirical form can predict  $C_{\sqrt{A}}$  within  $\pm 9$  percent.

### Comparison of Present Results and Previous Work

This section compares the present experimental model with the available data and correlations from previous studies. These comparisons will be done based on the characteristic length  $D$  for the circular cylinder, and  $W$  for the square cylinder. The main reason is that most previous works were reported based on these scale lengths and there is insufficient information to calculate the surface area.

Figure 5 shows the relationship between  $Re_{\sqrt{A}}$  and  $Nu_{\sqrt{A}} - Nu_{\sqrt{A}}^0 / [0.84(P_{\max}/\sqrt{A})^{0.5}]$  for the reducing experimental data of circular and square toroids and cylinders. Figure 5 also shows that the experimental data have the same trend and all of them together. This relationship concluded that the function  $0.84(P_{\max}/\sqrt{A})^{0.5}$  can estimate the parameter  $C_{\sqrt{A}}$ .

**Circular Cylinder.** Figure 6 shows comparisons between the empirical model, Eq. (28b), the approximate solution of Refai Ahmed and Yovanovich (1995), and the correlations of the previous works in a wide range of Reynolds number,  $1 \leq Re_D \leq 10^5$ . One observes from Fig. 6 at low  $Re_D$  ( $Re_D < 5$ ) that there are significant differences between most of the previous works and the empirical model. This is due to the effect of  $L/D$ , where the correlation equations of the approximate solution, Van der Hegge (1956), Morgan (1975), and Ahmed and Qureshi (1991) are based on  $L/D \gg 1$ , i.e., their Nusselt num-

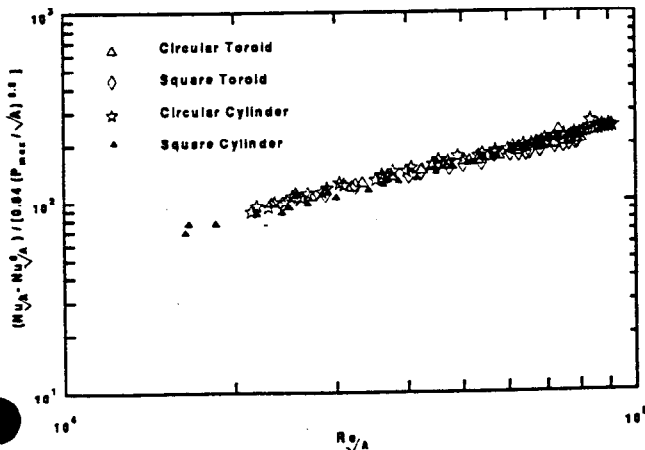


Fig. 5 Relationships between Reynolds number and body shape parameter for different body shapes (air)

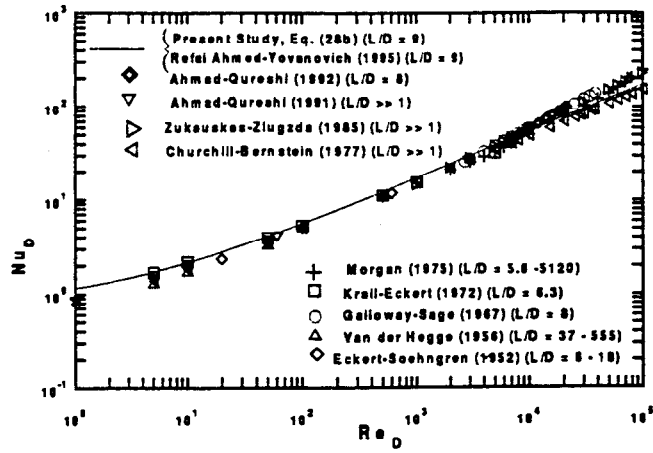


Fig. 6 Comparison between present study and previous works for circular cylinders (air)

bers at the diffusive limit go to zero. At high Reynolds number range,  $Re_D \geq 10^4$ , most of the correlation equations of previous works are slightly higher than the empirical model. This is also due to the effect of  $L/D$ , the free stream turbulence and the free convection. Other correlation such as Ahmed and Qureshi (1992) was higher than the present work due to their boundary condition (isoflux boundary). In contrast, the correlation equation of Churchill and Bernstein (1977) was found lower than both the empirical model and the previous works.

Figure 6 also shows comparisons between the recent forced convection correlations for circular cylinder from Churchill and Bernstein (1977), Morgan (1975), and Zukauskas and Ziugzda (1985) and the present study. Figure 6 shows very good agreement, within three percent, between the present study and Morgan (1975) and the maximum difference is 14 percent. In addition, the present study is higher than the Churchill and Bernstein correlation by 6.8 percent. In contrast, the experimental correlation of Zukauskas and Ziugzda (1985) is higher than the present study, Eq. (28b), by 13 percent. These comparisons show that the present study, Eq. (28b), is in good agreement with most of the previous studies.

**Square Cylinder.** Figure 7 shows comparisons between the experimental model, Eq. (29b), with other available experimental correlations in the literature<sup>3</sup> such as Rieher (1926),

<sup>3</sup>The correlation equations are available in Table 3

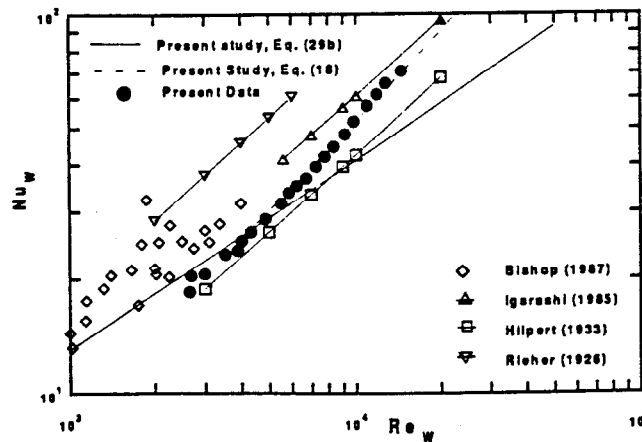


Fig. 7 Comparison between present study and previous works for square cylinders (air)



Helpert (1933), Igarashi (1985), and data of Bishop (1987). Bishop (1987) and Oosthuizen and Bishop (1987) examined this problem and developed an experimental correlation for assisted flow and opposing flow,

$$Nu_w = 0.281 Re_w^{0.57} \quad 7 \leq L/W \leq 12.5$$

The average difference between this correlation and their measurements was  $\pm 22$  percent, which is reflected in the scatter as shown in Fig. 7. Furthermore, the range of Grashof number in the experiments of Bishop was  $74,000 \leq Gr_w \leq 450,000$  which can influence the forced convection results. In addition, the effect of turbulence intensity can be another reason for the scatter in the data. The high temperature difference between the surface temperature and the ambient temperature,  $\Delta T = 75$  K, can also influence the heat transfer results as observed by Hilpert (1933) and Churchill and Brier (1956) in their experimental work. Hilpert (1933) and Churchill and Brier (1956) found that the temperature effect can influence the heat transfer result by  $(T_s/T_\infty)^{0.124}$  and  $(T_s/T_\infty)^{0.12}$ , respectively. In addition, Fig. 7 shows that the Igarashi (1985) correlation is also above the experimental correlation, Eq. (18), by 30 percent. However, he did not discuss the influence of radiation, free convection, and free stream turbulence on his experimental measurements. Furthermore, the experimental error was not reported. Therefore, it is difficult to justify the difference between Eq. (18) and the correlation of Igarashi.

In contrast, Hilpert's (1933) correlation is in good agreement with Eq. (29b). However, concerns about Hilpert's circular cylinder work were reported by Fand and Keswani (1973), Morgan (1975), and Churchill and Bernstein (1977). These concerns include the calculation of the thermal properties, the turbulence intensity effect, and the blockage effect. It is difficult to examine these concerns for the square cylinder since there is insufficient information. Figure 6 also shows the experimental correlation of Reihner (1926) is higher than other previous studies including the present study, Eq. (18). One concludes that there is insufficient information to examine the previous studies and the only available data, Bishop (1987), had a lot of scatter.

## Summary

A series of experiments were conducted for the area mean Nusselt number for forced convection from different body shapes such as isothermal circular cylinders, square cylinders, circular toroids, and square toroids in the range of Reynolds number,  $10^4 \leq Re_{\bar{a}} \leq 10^5$ . The present experiments have been reduced to remove the effects of free convection, radiation, and free stream turbulence. Furthermore, experimental models for heat transfer by forced convection from different body shapes were developed. In addition the present study found that  $C_{f,\bar{a}}$  depends on the geometric of the body shape as shown in Eq. (32). The model, Eq. (28b), of the circular cylinder is compared against approximate solution of Refai Ahmed and Yovanovich (1995), and other available correlations. These comparisons show good agreement between the model and most of the previous studies. However, the present study could not compare the circular toroid and square toroid results because there is insufficient, well documented, data available in the literature.

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