FACTORS AFFECTING THE CALCULATION OF EFFECTIVE CONDUCTIVITY IN PRINTED CIRCUIT BOARDS

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ABSTRACT

A comparative study is presented that demonstrates the importance of including both the material resistance and the spreading resistance in the calculation of effective conductivity for printed circuit board applications. An analytically based Fourier series model is used to calculate effective conductivity in three dimensional test coupons.

Results show that models based exclusively on cross-plane and in-plane resistive networks are not adequate for predicting effective conductivity in multilayer, laminated printed circuit boards. The mixed boundary conditions found in most microelectronic applications accentuate the importance of spreading resistance between heat sources and the convective boundaries. Discrepancies between commonly used methods of calculating effective conductivity and methods incorporating both the bulk material and spreading resistance can be greater than 100% for geometries typically found in microelectronics applications.

NOMENCLATURE

a = source half length, m
A = cross sectional flow area, m^2
b = source half width, m
Bi = Biot number, $\equiv h\mathcal{L}/k_1$
c = substrate half length, m
d = substrate half width, m
h = heat transfer coefficient, W/m^2K
k = thermal conductivity, W/mK
L = flow length, m
$\mathcal{L}$ = characteristic length, m
m, n = counters
N = total number of layers
t = layer thickness, m
R = thermal resistance, $°C/W$

Greek Symbols

β = composite eigenvalue in length and width
δ = eigenvalue associated with the length variable
λ = eigenvalue associated with the width variable
κ = conductivity ratio, $\equiv k_2/k_1$
ϕ = functional value used in spreading resistance

Subscripts

$1, 2$ = top and bottom layers, respectively
e = effective
f = fluid
p = parallel
s = series

INTRODUCTION

Thermal modeling of multilayered printed circuit boards is sometimes simplified through the use of an effective conductivity that combines the influence of individual layer conductivities into a unique property value that can be applied as a single, homogeneous value throughout the domain of the PCB. Several schemes have been proposed for calculating effective conductivity, including what is generally considered to be the lower and upper bounds of effective conductivity, namely, the series or "cross-plane conductivity" and the parallel or "in-plane conductivity", respectively. A simple average of the upper and lower bounds on thermal resistance can provide an accurate measure of effective conductivity when the difference between the bounding solutions is small, but this is generally not the case in PCBs where the ratio between the effective conductivity based on the parallel and series paths is typically of order 25:1 or greater. Other averaging schemes, such as the harmonic or geometric mean of the two limiting solutions can be used but these methods tend to favor one of the limiting conditions more than the other and the calculated effective thermal conductivity may be appropriate for certain design conditions but not for others.

Many attempts have been made to characterize an effective conductivity in composite structures ranging from systems of parallel cylinders, as described in the original works of Rayleigh [1] and later detailed in the work of Keller [2], to embedded spheroidal inclusions in a surrounding binder, as described by Benveniste and Miloh [3]. Gautesen [4] in his modeling of a periodic rectangular geometry, proposes the use of the geometric mean of the conductivities based on a series and parallel path resistor

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network. In order to satisfy the limiting conditions as the overall thickness of the rectangular geometry approaches an infinitely thin or infinitely thick body, Gautesen modifies the effective conductivity as follows:

$$k_e = \sqrt{k_s \cdot k_p} \cdot \frac{t_T}{w} \qquad (1)$$

where t_T is the total thickness of the composite and w is the overall width. Lemczyk et. al. [5] propose the use of the harmonic mean of the series and parallel path conductivities based on analytical studies where the overall resistance of composite structures was used to find the effective conductivity necessary to match the equivalent in a single layer structure.

Conventional schemes used to calculate effective conductivity are only applicable if boundary conditions are uniformly applied and spreading resistance can be neglected. Consequently the calculated value of effective conductivity is based only on the resistance of the bulk materials and all other intrinsic resistances between the source and the sink are excluded. Unfortunately, the mixed boundary value problems typical of printed circuit board applications do not lend themselves to these idealized boundary conditions. The spreading resistance, which is often of similar or greater magnitude than the bulk resistance, must be accounted for in the calculation of effective conductivity, and any estimates of conductivity based solely on series and parallel resistor networks can lead to significant miscalculations of temperatures in PCBs.

This study will assess the importance of several design options and examine their significance in the calculation of an effective conductivity that can be used to represent the true thermophysical behavior of a multilayer board with convection cooled boundaries and a heat source at one boundary. The design options examined are:

1. **Relative location of high and low conductivity layers in relation to the heat source:** A significant portion of the heat dissipated from electronic packages mounted on the surface of PCBs flows through the laminated structure of the board and into the surrounding boundary layer. The heat spreading within the PCB and in turn the effective conductivity can be strongly influenced by the proximity of the high and low conductivity layers relative to the location of the heat source.

2. **Heat source size and location:** The manner in which heat moves from the heat source into the layers of the PCB and finally into the surrounding ambient air is influenced by the heat spreading associated with both the size and the relative location of the heat source.

3. **Convective boundary conditions:** Although the film resistance is not directly included in the calculation of effective conductivity, it must be considered indirectly through the calculation of the spreading resistance and consequently the effective conductivity.

The factors described above will be examined for test coupons containing a single heat source. The calculated value of effective conductivity will be compared to limiting cases for in-plane and cross-plane conductivities. Results will be summarized to clearly show which design parameters have the greatest influence on the calculation of effective conductivity.

AVERAGING TECHNIQUES

A laminated plate or printed circuit board has clearly defined upper and lower bounds on thermal conductivity of the bulk material, given by the cross-plane conductivity (series path) and the in-plane conductivity (parallel path), respectively, as defined in Eqs. 2 and 3.

$$k_s = \sum_{i=1}^{N} t_i \Big/ \sum_{i=1}^{N} t_i/k_i \qquad (2)$$

$$k_p = \sum_{i=1}^{N} k_i t_i \Big/ \sum_{i=1}^{N} t_i \qquad (3)$$

For many electronics applications these bounds are widely spaced as a result of the three to four orders of magnitude difference between the bulk conductivities of copper and FR4. As an example, a five layer board with two signal planes and three dielectric planes of FR4 would have an in-plane conductivity of 18 W/mK and a through-board conductivity of 0.42 W/mK. In these cases, neither of the two limiting values appears to be the obvious choice for use as an effective conductivity and an average of the two limiting values is typically used . Three averaging schemes are used to calculate effective conductivity; i) the arithmetic mean, ii) the geometric mean or iii) the harmonic mean, defined as follows:

$$\textbf{arithmetic mean} = \frac{k_s + k_p}{2} \qquad (4)$$

$$\text{geometric mean} = \sqrt{k_s \cdot k_p} \qquad (5)$$

$$\text{harmonic mean} = \frac{2 \cdot (k_s \cdot k_p)}{k_s + k_p} \qquad (6)$$

Each of these averaging procedures weight the upper and lower bound in a different manner, such that the arithmetic mean provides the highest estimate of the average and the harmonic mean provides the lowest estimate of the average. Given these three averaging schemes, and assuming that the effective thermal conductivity could take on any value between the upper and lower bounds, the geometric mean would appear to be the preferred choice because of its more neutral weighting scheme. It can also be shown from Eqs. 4-6 that the geometric mean can be derived from:

$$\text{geometric mean} = \sqrt{\text{arithmetic mean} \cdot \text{harmonic mean}} \tag{7}$$

Although the geometric mean appears to weight the two extremes in a more neutral manner, the question remains; does the geometric mean provide a good estimate of the effective thermal conductivity for the thermophysical properties and the geometry being considered?

Effective Conductivity

The approach chosen to estimate effective conductivity in a composite structure is based on an accurate calculation of the total thermal resistance between the source and boundaries of the solid at the fluid/solid interfaces. The real structure of the composite is then replaced with a homogeneous structure of similar overall dimensions and boundary conditions, where a value for the conductivity is determined that produces the same solid-body thermal resistance.

The solid-body thermal resistance is the accumulation of thermal resistance encountered between a heat source and the fluid/solid interface. The solid-body thermal resistance can be calculated as:

$$R_{solid} = \frac{(\overline{T}_{source} - \overline{T}_{walls})}{Q_T} \tag{8}$$

The thermal resistance within the solid consists of two components, the bulk material resistance and the spreading resistance. Therefore the total resistance between source and the fluid/solid boundaries can be written as:

$$R_{solid} = R_{bulk} + R_{spread} \tag{9}$$

While Eqs. 2 - 6 can be used to calculate the thermal conductivity of the bulk materials, the geometric and thermophysical property values used in these calculations of resistance cannot be expected to capture the influence of the spreading resistance. The spreading resistance is strongly influenced by the size of the source, the relative position of layers in relation to the source and the convective boundary conditions. Yovanovich et al. [6], gives a detailed method for calculating spreading resistance in two layer, circular annular systems. The procedure clearly demonstrates that the spreading resistance is an important component in calculating the solid-side resistance of a body. This procedure is extended in Yovanovich et al. [7], to include rectangular bodies with a single, concentrically located heat source and a convective boundary condition on the boundary opposite the source.

Yovanovich et al. [7] have derived an analytical expression for both the bulk resistance and the spreading resistance in a two-layer, square substrate with a concentric, square heat source. The example chosen by Yovanovich is insulated on five sides of the cuboid-shaped substrate, with a convective boundary condition over the surface opposite the heat source. In this instance, the bulk material resistance of the layers is identical to the cross-plane or series resistance of the cuboid. The spreading resistance is calculated based on the sum of two strip source solutions and a rectangular source solution.

$$\begin{aligned} R_{spread} = & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\sin(b\lambda)^3 \sin(a\delta)^2}{a^2\lambda^2 b^2 \delta^2 \beta c d k_1} \cdot \phi_{m,n} \\ & + \sum_{n=1}^{\infty} \frac{1}{2} \frac{\sin(a\delta)^2}{a^2\delta^2 c d k_1} \cdot \phi_n \\ & + \sum_{m=1}^{\infty} \frac{1}{2} \frac{\sin(b\lambda)^2}{b^2\lambda^2 c d k_1} \cdot \phi_m \end{aligned} \tag{10}$$

where

a, b - heat source half length and width
c, d - substrate half length and width
δ, λ, β - eigenvalues (length, width and composite)
k_1 - conductivity of the first layer
t_1, t_2 - layer thicknesses
Bi - Biot number, $\equiv h\mathcal{L}/k_1$
h - film coefficient
$\mathcal{L}$ - characteristic length
$\phi_m, \phi_n, \phi_{m,n}$ - functional with dependence on $f(\kappa, \beta, \delta, \lambda, Bi, t_1, t_2, \mathcal{L})$

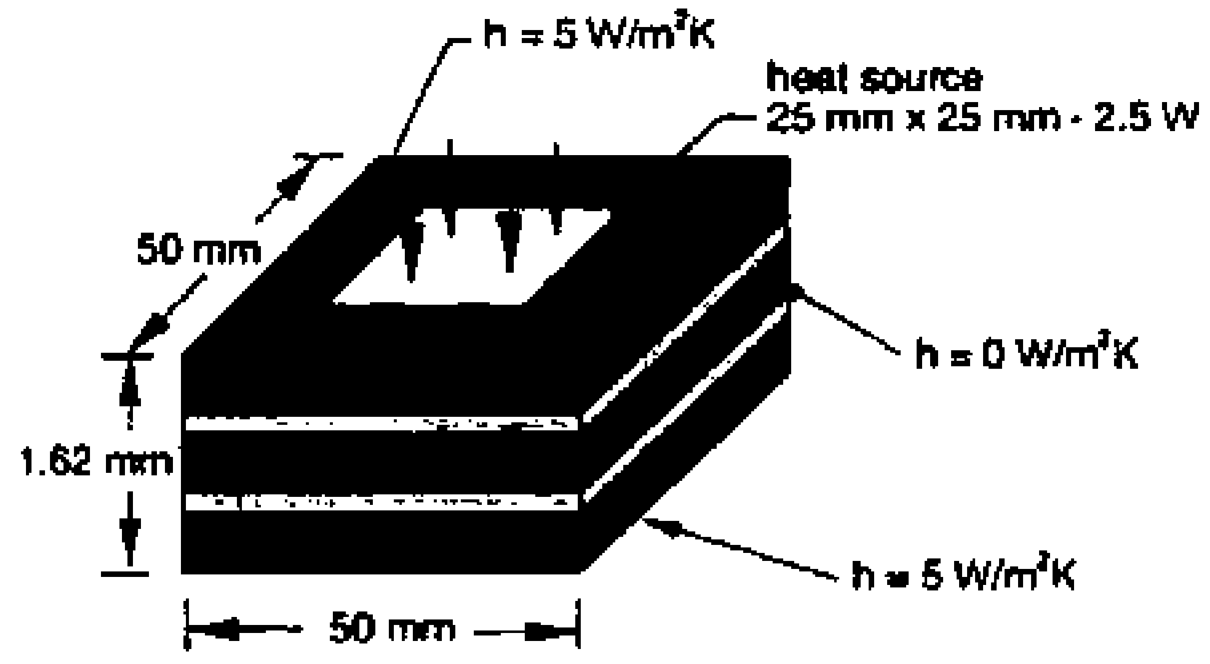
The functional dependence of Eq. 10 clearly shows that the resistance between the source and the sink cannot be based solely on in-plane or cross-plane board resistances or a combination of the two resistances. The calculation of spreading resistance mandates that conditions imposed at the boundaries, such as source size and convective conditions be incorporated into the calculation of a solid-body resistance and in turn an effective conductivity. In addition, the source location must be considered for non-concentric applications.

DISCUSSION

Modeling

All simulations performed in this study were conducted using an analytical model based on a general three-dimensional Fourier series solution applied to laminated substrates with arbitrarily specified boundary conditions, as detailed in Culham and Yovanovich [8]. The solution procedure provides a convenient means of calculating the total resistance to heat flow between a heat source on a multilayer stack and the fluid/solid interface over which the convective boundary conditions are applied. The Fourier series model provides local values of temperature and heat flow rate throughout the composite body that can be integrated over any surface to determine the mean temperatures used in Eq. 8.

Once the total solid-body resistance between the heat source and the convection cooled surfaces is calculated, an iterative procedure is used to find the total solid resistance for a single layer substrate with the same total thickness and boundary conditions. The thermal conductivity required to obtain an equivalent resistance is defined as the effective conductivity of the body



Layer	Material	Thickness (mm)	Conductivity (W/mK)
1	FR4	0.5263	0.4
2	Cu	0.0356	400.0
3	FR4	0.5263	0.4
4	Cu	0.0356	400.0
5	FR4	0.5263	0.4

Figure 1: Test Coupon Used in Effective Conductivity Studies

Base Case

A typical five layer PCB with two signal planes and three dielectric planes is used as the base case for comparative study. The test coupon for the base case, as shown in Fig. 1, has overall dimensions 50 mm × 50 mm × 1.62 mm. This test coupon has a concentrically located 25 mm x 25 mm heat source that is assumed insulated on the exposed surface to assure that the 2.5 W of heat input is directed into the top layer of the multilayer stack.

A convective boundary condition of 5 W/m^2K is assumed on both the upper and lower exposed surfaces. Since the coupon is designed to represent an isolated section of a larger PCB, the side walls of the coupon are treated as adiabatic, representing a plane of symmetry within the PCB.

In each of the parametric studies presented, the overall dimensions of the test coupon and the total heat input to the source are preserved. In addition, the volume fractions of copper and FR4 are preserved in each test case. As a result, the effective conductivity calculated using the averaging techniques given in Eqs. 4-6, provide the same numerical results for all cases studied, as shown in Table 1.

Layer Placement

The calculation of the series and parallel path effective conductivities, as defined in Eqs. 2 and 3, respectively, do not account for the position of individual layers in a multilayer stack. These equations can be used to calculate a representative conductivity for the bulk material but they do not consider the effect of spreading resistance encountered in the heat flow path between the source and the sink. The combined influence of the resistance in the bulk material and the spreading must be included in any calculation for effective conductivity if an accurate representation of the thermal conductivity in a multilayer stack is to be captured by a single value. The spreading resistance, as given in Eq. 10, demonstrates a strong dependence on the proximity of individuals layers to the source and the sink.

Table 1: Conductivity Values Calculated Using Eqs. 4 - 6

Method	Conductivity W/mK
in-plane	17.62
cross-plane	0.42
arithmetic mean	9.02
geometric mean	2.71
harmonic mean	0.82

A five layer test coupon, as shown in Fig. 1, is used as the base case for parametric study. The overall thickness of the copper layers and the FR4 layers are preserved but the position of the layers in the stack is varied, as shown in Fig. 2. The laminate construction for the five cases examined are listed in Table 2.

Both copper layers are placed directly under the heat source in Case 1. In this configuration, the spreading resistance is minimized and the calculated value of effective conductivity approaches the in-plane value of conductivity. By minimizing the spreading resistance the heat flows essentially unrestricted within the copper layers. Since the side walls are insulated, heat must turn to flow to both the upper and lower surfaces. The added resistance associated with the turning assures that the calculated value of effective conductivity will always be less than the upper limit set by the true in-plane conductivity.

Cases 2, 3 and 4 demonstrate configurations in which the spreading resistance plays a proportionately increasing role. The calculated values of effective conductivity lie midway between the limiting cases set by the in-plane and cross-plane conductivities. In each case the constriction to heat flow under the heat source is affected by the laminate structure in the PCB.

The final example given in Case 5 offers a maximum spreading resistance and in turn the lowest calculated value of effective conductivity. The effective conductivity is 5.0 W/mK, more that 80% higher than the estimated

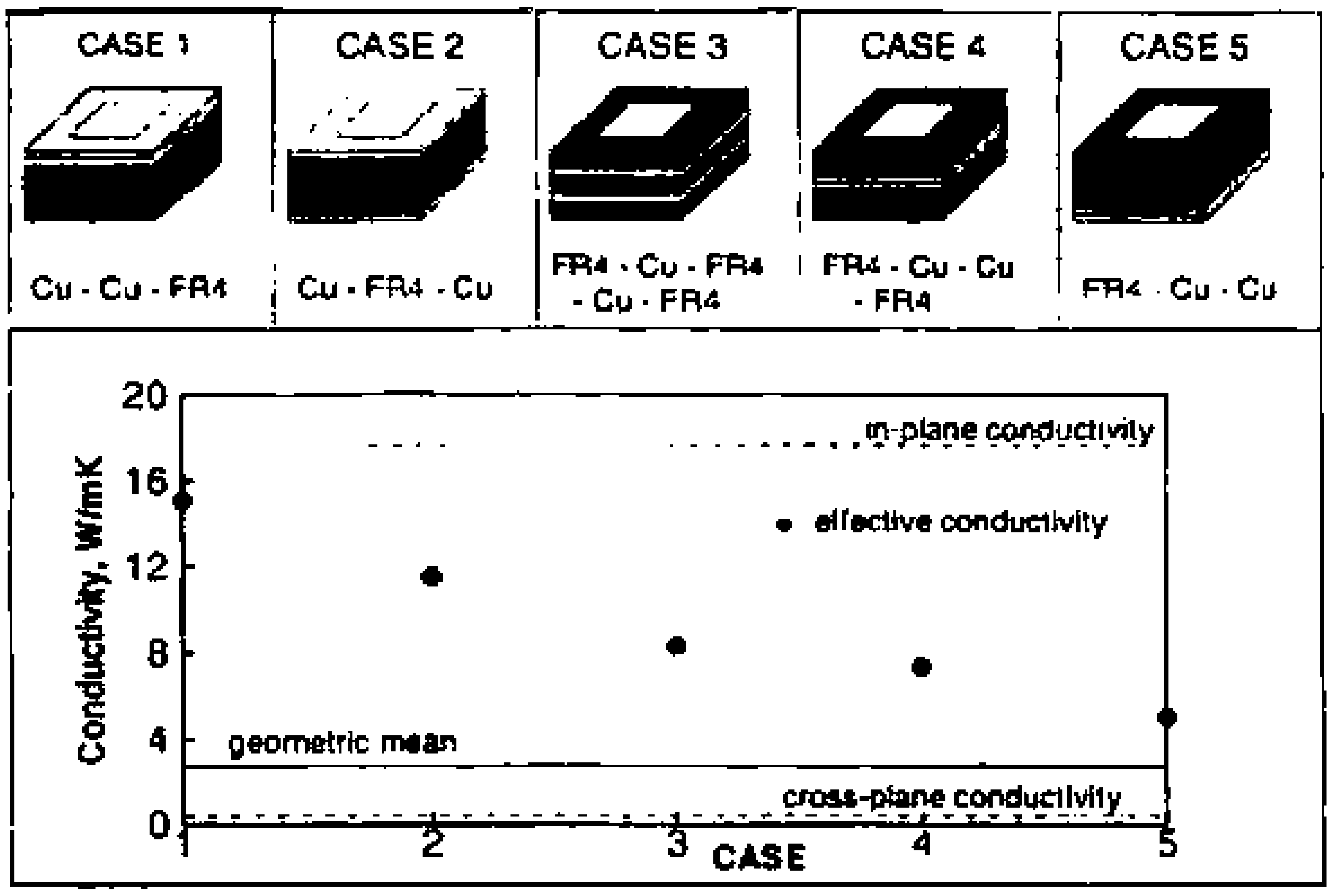


Figure 2: Influence of Layer Placement in the Calculation of Effective Conductivity

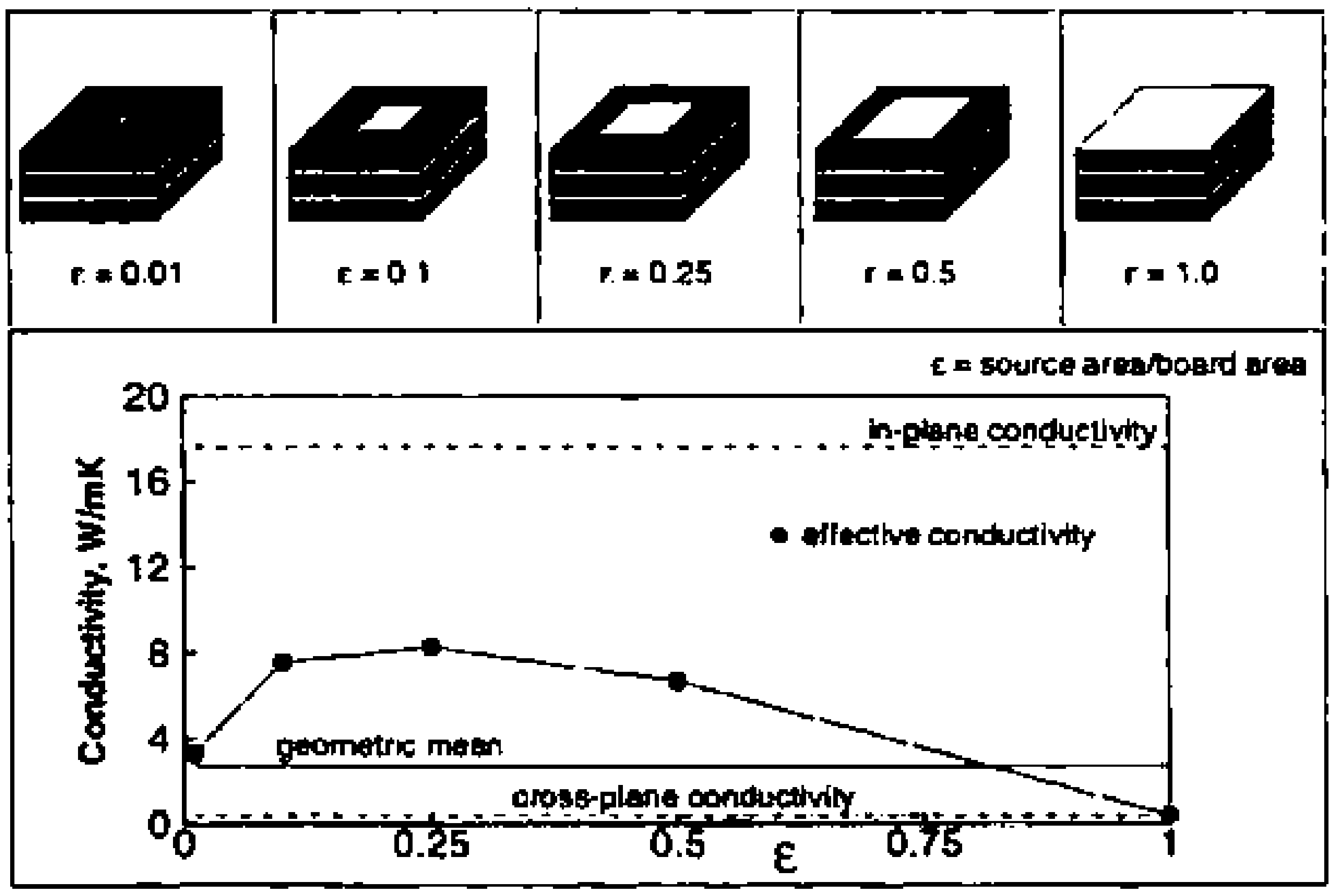


Figure 3: Influence of Source Size in the Calculation of Effective Conductivity

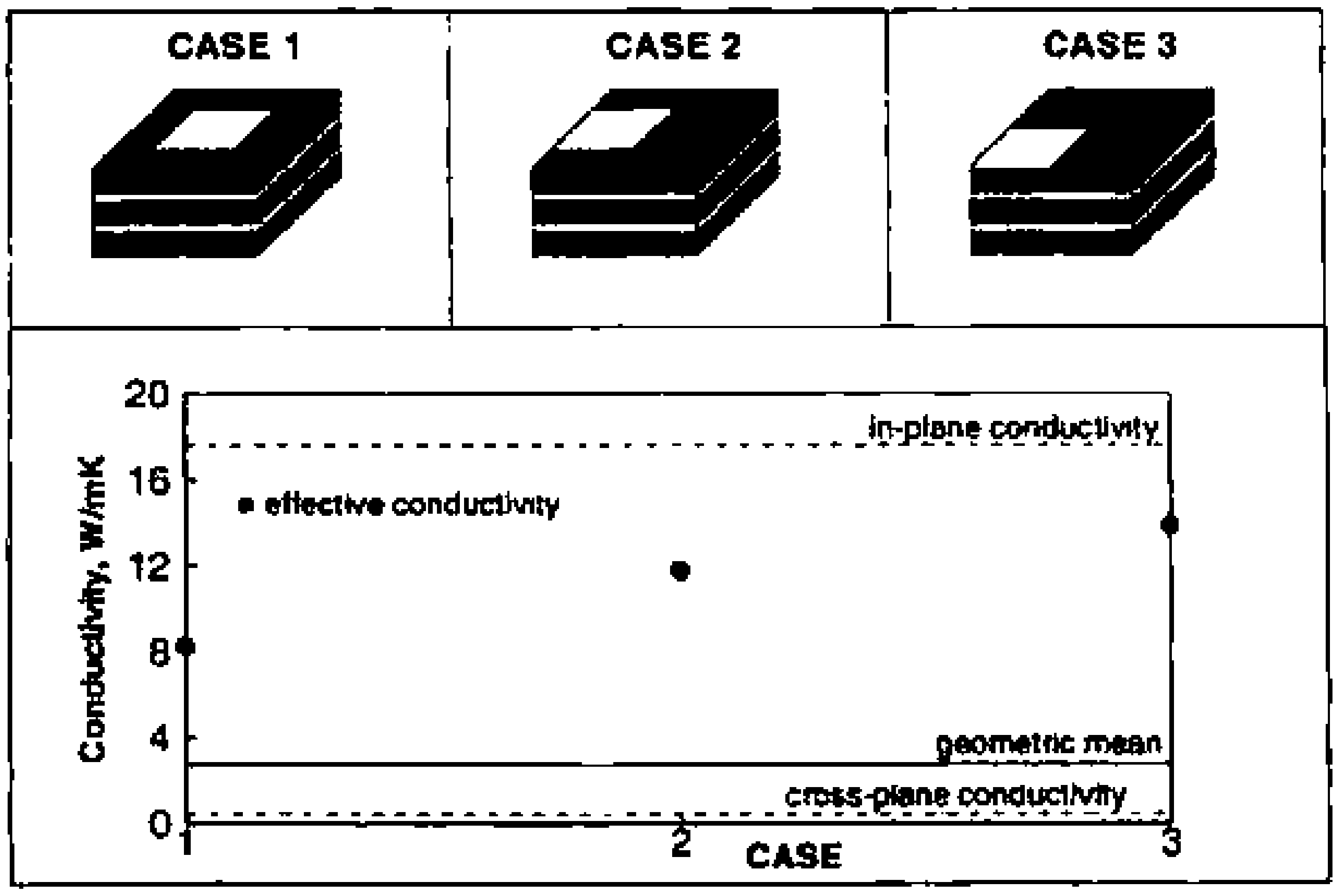


Figure 4: Influence of Source Location in the Calculation of Effective Conductivity

Table 2: Material and Thicknesses for Layer Position Study (all dimensions in mm)

LAYER	CASE 1	2	3	4	5
1	Cu 0.0356	Cu 0.0356	FR4 0.5263	FR4 0.7895	FR4 0.5263
2	Cu 0.0356	FR4 0.5263	Cu 0.0356	Cu 0.0356	FR4 0.5263
3	FR4 0.5263	FR4 0.5263	FR4 0.5263	Cu 0.0356	FR4 0.5263
4	FR4 0.5263	FR4 0.5263	Cu 0.0356	FR4 0.7895	Cu 0.0356
5	FR4 0.5263	Cu 0.0356	FR4 0.5263	— —	Cu 0.0356

effective conductivity obtained using the geometric mean.

As shown in Fig. 2, the effective conductivity can vary significantly as a result of the placement of the various layers in the PCB. Neither the limiting conditions nor the averaging schemes provide a means to account for the effect of spreading resistance. For the five cases shown the effective conductivity can vary between 15.02 and 5.00 W/mK. The arithmetic mean provides the most neutral estimate of effective thermal conductivity in this case but errors of up to 80% are obtained.

Source Size

The relative source area (ϵ = source area/board area) is varied between 0.01 and 1.0 while preserving the laminate structure and the boundary conditions shown in Fig. 1. This study is analogous to examining the packaging density on a populated PCB to determine its effect on the resistance to heat flow in the board and consequently, the estimation of an effective conductivity.

The example of a fully populated board ($\epsilon = 1$) with adiabatic sidewalls leads to a situation where spreading is not a factor and the heat flow lines run in a perpendicular direction between the heat source and the sink. The bulk resistance of the laminate structure is based on a series path resistor network and the calculated value of the effective conductivity is equal to the cross-plane conductivity.

As the relative size of the source is reduced, the role of spreading resistance becomes more significant but at the same time the exposed surface for convection cooling increases and the overall resistance tends to decrease, resulting in a higher effective conductivity, as shown in Fig. 3. However, as the relative source area shrinks below ϵ = 0.1, the spreading resistance increases significantly and the calculated value of effective conductivity starts to decrease.

The geometric mean could be used as an average value for PCBs where the percentage of source coverage can vary, however, errors of 70% can arise for typical values of ϵ.

The variance in the calculation of effective conductivity associated with changes in relative source area is a clear demonstration of the interaction of the bulk resistance and the spreading resistance. As the source area changes, the spreading resistance is affected by both the change in the area of the convection cooled boundary and the constriction of heat flow in the vicinity of the heat source. The interaction of these effects leads to a specific value of ϵ where the effective conductivity takes on a maximum value.

Source Location

The previous studies all pertained to concentrically located heat sources on a test coupon. However, in some instances the heat source may not be centrally located on a substrate, thus influencing the heat flow path established between the source and the sink.

Case 1 is the base case example, shown in Fig. 1, where the effective conductivity is calculated as 8.28 W/mK. Case 2 demonstrates a translation of the source along one axis while Case 3 is a result of a translation along two axes. The effect on effective conductivity is significant in both instances, as shown in Fig. 4, with a 42% and 67% increase, respectively.

Preliminary inspection of the configurations selected in Case 2 and 3 would indicate that the spreading resistance should be significantly higher than with the concentrically located heat source used in Case 1. This in itself would result in a decrease in the calculated effective conductivity for Case 2 and 3. However, the effective conductivity is based on the combined effect of the resistance of the solid and a ratio of the cross sectional flow area to the flow length, as shown in Eq. 11.

$$k_e = 1/\left(R_{solid} \cdot \frac{A}{L}\right) \tag{11}$$

When the heat flow is one-dimensional, the cross sectional flow area and the flow length are clearly defined but in two-dimensional flow these values are difficult to quantify. Even without having the ability to quantify the length of the flow path or the cross sectional flow area, we can see that both Case 2 and 3 will provide an increase in the heat flow path between the source and the sink, which leads to a higher effective conductivity.

Convective Boundary Condition

The calculation of effective conductivity should be based on the resistance to heat flow in the solid and should

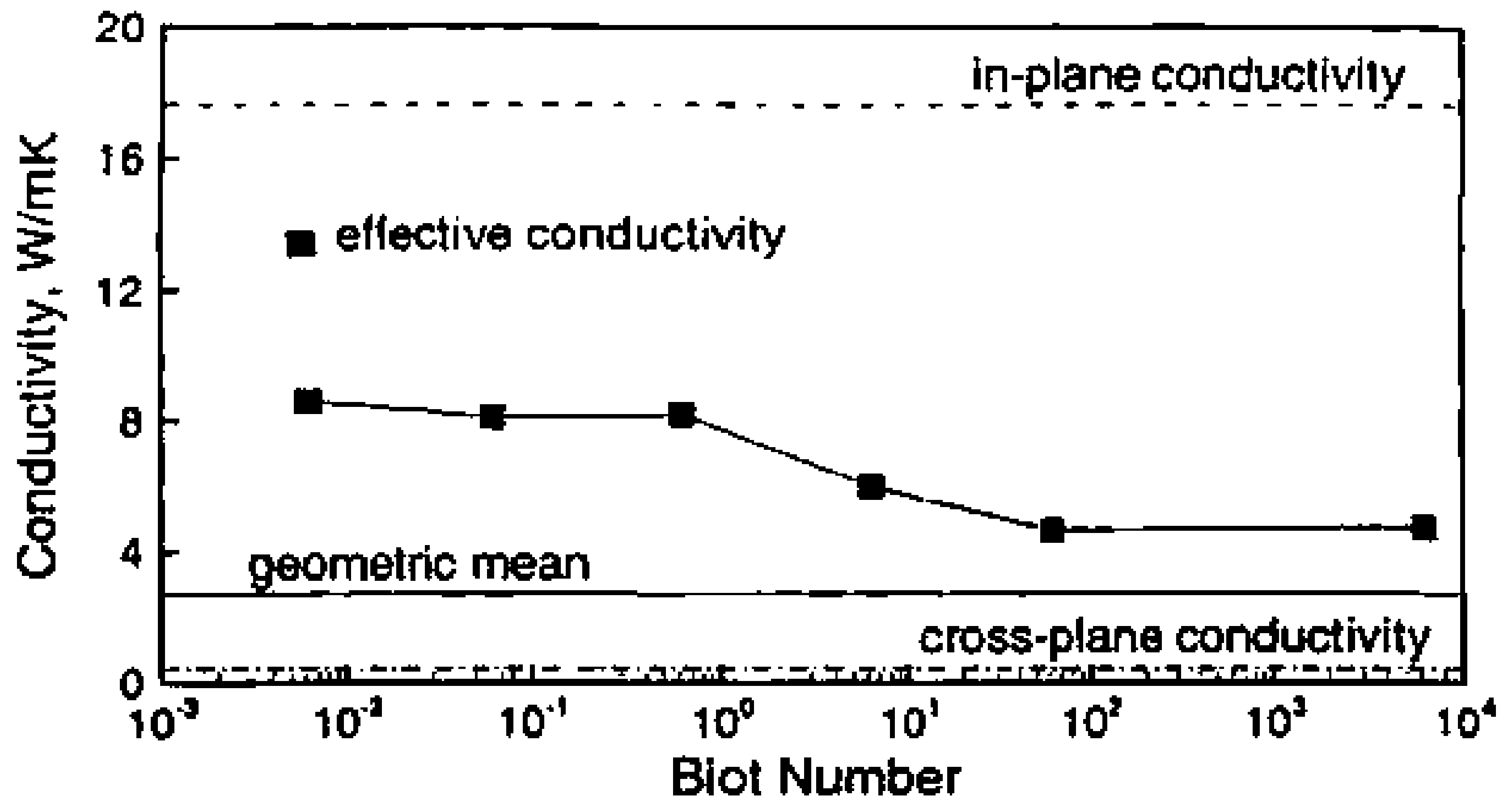


Figure 5: Influence of Heat Transfer Coefficient in the Calculation of Effective Conductivity

not extend beyond the boundary of the solid. The resistance associated with the boundary layer is factored out of the calculation of the solid resistance by defining Eq. 8 with the mean temperature of the wall at the interface between the solid and the surrounding fluid. However, as shown in the functional dependence of the spreading resistance, given in Eq. 10, the spreading resistance is a function of the Biot number and consequently, the heat transfer coefficient. While the fluid-side resistance ($R_f = 1/hA$) is not directly included in the calculation of the effective conductivity, the effect of the fluid boundary condition on spreading resistance must be considered.

The geometry shown in Fig. 1 is used to determine the effect of the heat transfer coefficient on spreading resistance and the calculation of the effective conductivity. The convective boundary condition on the upper and lower surfaces is varied between 10^{-1} and 10^{6} W/m^2K in 10 fold increments. The data are presented in the form of a Biot number, which is defined as $h \cdot \mathcal{L}/k_1$. The characteristic length, $\mathcal{L}$ is chosen to be the square root of the source area which is equivalent to the side length of the source.

The effective conductivity approaches a constant value below $Bi = 1$ and above $Bi = 100$ with a quasi-linear transition between these two values, as shown in Fig. 5. As Bi approaches zero, the spreading resistance becomes independent of the heat transfer coefficient and the spreading is only a function of the geometry and thermophysical properties, which leads to a constant value for effective conductivity. Similarly in the other limit, as Bi approaches infinity, the spreading resistance solution again becomes independent of Bi and a constant value of effective conductivity is obtained for all large values of Biot number.

Typical electronics application rarely see Biot numbers greater that 10 and most applications will be in the range $10^{-2} \leq Bi \leq 10^{0}$. As a result the effect of heat transfer coefficient on effective conductivity is minimal for typical electronics applications.

CONCLUSIONS

The comparative study to ascertain the role of spreading resistance in the calculation of effective conductivity has revealed that both the material resistance and the spreading resistance must be accounted for if a representative, effective conductivity is to be determined for multilayer printed circuit boards with mixed boundary conditions. The in-plane or parallel path conductivity provides an upper bound on effective conductivity while the cross-plane or series path conductivity provides the lower bound. None of the averaging schemes proposed appear to provide a good estimate for the range of design conditions examined.

Although series and parallel resistance paths are commonly used to calculate effective conductivity, these methods do not provide any mechanisms for including the effects of layer location in a multilayer stack, packaging density, heat source placement or Biot number. Effective conductivity can vary by as much as 300% when the conductive signal planes are relocated from adjacent to the source of heat to the opposite boundary. The effective conductivity can range from the lower bound i.e. the cross-plane conductivity, for a full populated board to a maximum value at roughly 25% density for the cases examined in this study. The placement of heat sources near adiabatic boundaries can have a major influence on the calculation of effective conductivity but the effect on the heat flow path length probably has a more significant impact in the calculation of effective conductivity. The boundary layer resistance does not play a direct role in the calculation of effective conductivity but it does introduce a secondary effect through the dependence of spreading resistance on Biot number. For the range of Biot numbers found in most microelectronic applications, changes in the convective boundary conditions do not introduce significant changes in the effective conductivity.

Any procedure for calculating effective conductivity that does not include both the material and spreading resistance should be avoided when calculating temperature or heat flux distribution in multilayer printed circuit boards.

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