

Spreading Resistance of Isoflux Rectangles and Strips on Compound Flux Channels

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The general expression for the spreading resistance of an isoflux, rectangular source on a two-layer rectangular flux channel with convective or conductive cooling at one boundary was presented. The general expression depends on several dimensionless geometric and thermal parameters. Expressions are given for some two- and three-dimensional spreading resistances for two-layer and isotropic finite and semi-infinite systems. The effect of heat flux distribution over strip sources on two-dimensional spreading resistances was discussed. Tabulated values are presented for three flux distributions, the true isothermal strip and a related non-isoflux, non-isothermal problem. For narrow strips, the effect of the flux distribution becomes relatively small. The dimensionless spreading resistance for an isoflux square source on an isotropic square flux tube was discussed and a correlation equation was reported. The closed-form expression for the dimensionless spreading resistance for an isoflux rectangular source on an isotropic half-space was given.

NOMENCLATURE

a, b	=	half-lengths of source area; m
A	=	channel conduction area; m^2
A_s	=	heat source area; m^2
\sqrt{A}	=	characteristic length of contact area; m
Bi	=	Biot number; $h\mathcal{L}/k_1$
c, d	=	half-lengths of flux channel; m
h	=	contact conductance or film coefficient; $W/m^2 \cdot K$
i	=	index denoting layer 1 and layer 2
$J_\nu(x)$	=	Bessel function of first kind, order ν
k, k_1, k_2	=	thermal conductivities; $W/m \cdot K$
m, n	=	indices for summations
Q	=	heat flow rate; W
q	=	heat flux; W/m^2
R	=	thermal resistance; K/W
R_{1D}	=	one-dimensional resistance; K/W
R_s	=	spreading resistance; K/W
R_{total}	=	total resistance; K/W
t, t_1, t_2	=	total and layer thicknesses; m
T_1, T_2	=	layer temperatures; K
\bar{T}_{source}	=	mean source temperature; K
\bar{T}_{sink}	=	mean sink temperature; K
u	=	relative local position in strip; $u = x/c$
x, y, z	=	Cartesian coordinates; m

Greek Symbols

α	=	conductivity parameter; $\alpha = (1 - \kappa)/(1 + \kappa)$
β	=	eigenvalues; $\sqrt{\delta^2 + \lambda^2}$
Γ	=	Gamma function
δ	=	eigenvalues; $(m\pi/c)$
$\epsilon, \epsilon_1, \epsilon_2$	=	relative contact size; $\epsilon_1 = a/c, \epsilon_2 = b/d$
κ	=	relative conductivity; k_2/k_1
λ	=	eigenvalues; $(n\pi/d)$
μ	=	heat flux shape parameter; $\mu = -1/2, 0, 1/2$
$\phi_{m,n}$	=	three-dimensional spreading function
ϕ_m, ϕ_n	=	two-dimensional spreading functions
ψ	=	dimensionless spreading resistance; $R_s k_1 \mathcal{L}$
ϱ	=	aspect ratio of rectangular source area; $\varrho = a/b \geq 1$
τ, τ_1, τ_2	=	relative layer thickness; $\tau = t/\mathcal{L}$, $\tau_1 = t_1/\mathcal{L}, \tau_2 = t_2/\mathcal{L}$

INTRODUCTION

Thermal spreading resistance occurs whenever heat leaves a heat source of finite dimensions and enters into a larger region as shown in Fig. 1. This shows a planar rectangular heat source situated on one end of a compound heat flux channel which consists of two layers having thicknesses t_1, t_2 and thermal conductivities k_1, k_2 respectively. The heat flux channel is cooled along the bottom surface through a uniform film coefficient or a uniform contact conductance h . The heat source area can be rectangular having dimensions $2a$ by $2b$ or it may be a strip of width $2a$ or $2b$ as shown in Fig. 2. The dimensions of the heat

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flux channel are $2c$ by $2d$ as shown in Fig. 1. The lateral boundaries of the heat flux channel are adiabatic.

The heat flow rate through the heat flux channel Q is related to the mean temperature of the heat source \bar{T}_{source} and the mean heat sink temperature \bar{T}_{sink} and the total system thermal resistance R_{total} through the relationship:

$$QR_{\text{total}} = \bar{T}_{\text{source}} - T_{\text{sink}} \quad (1)$$

The total thermal resistance of the system is defined by the relation:

$$R_{\text{total}} = R_s + R_{1D} \quad (2)$$

where R_s is the thermal spreading resistance of the system and R_{1D} is the one-dimensional thermal resistance defined as:

$$R_{1D} = \frac{t_1}{k_1 A} + \frac{t_2}{k_2 A} + \frac{1}{hA} \quad (3)$$

The conduction area in the previous equation is $A = 4cd$. For an isoflux source area, the heat flow rate through the system is $Q = qA_s$ where q is the uniform heat flux and $A_s = 4ab$ is the heat source area.

For the general case of a rectangular source area on a rectangular heat flux channel as shown in Fig. 1, the spreading resistance will depend on several geometric and thermal parameters such as

$$R_s = f(a, b, c, d, t_1, t_2, k_1, k_2, h) \quad (4)$$

One objective of this work is to obtain a general solution for this problem. A second objective is to report several two- and three-dimensional cases which arise from the general solution. A third objective is to report tabulated values for the strip source on a semi-infinite flux channel, and any available correlation equations.

Several investigators^{1-6,8} have examined spreading resistance in isotropic two- and three-dimensional systems. Sadhal⁵ obtained the solution for an elliptical source on a rectangular flux channel.

PROBLEM STATEMENT

The temperature distributions T_1 and T_2 within the two layers must satisfy the Laplace equation:

$$\nabla^2 T_i = 0, \quad i = 1, 2 \quad (5)$$

where for the rectangular heat source/rectangular flux channel system the three-dimensional Laplacian operator is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Along the common interface $z = t_1$ the two temperatures must satisfy the perfect contact conditions:

$$T_1 = T_2 \quad \text{and} \quad k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z} \quad (6)$$

Along the lateral boundaries $x = \pm c$ and $y = \pm d$ the two temperatures must satisfy the adiabatic conditions:

$$\frac{\partial T_i}{\partial x} = 0 \quad \text{and} \quad \frac{\partial T_i}{\partial y} = 0 \quad i = 1, 2 \quad (7)$$

Along the bottom surface $z = t_1 + t_2$, the Robin boundary condition must be satisfied:

$$\frac{\partial T_2}{\partial z} = -\frac{h}{k_2} (T_2 - T_{\text{sink}}) \quad (8)$$

The parameter h can represent a uniform film coefficient or a uniform contact conductance. Over the top surface $z = 0$, the boundary conditions are (i) the isoflux condition:

$$\frac{\partial T_1}{\partial z} = -\frac{q}{k_1}, \quad -a < x < a, \quad -b < y < b \quad (9)$$

over the heat source area and (ii) the adiabatic condition:

$$\frac{\partial T_1}{\partial z} = 0 \quad (10)$$

for all points which lie outside the heat source area.

METHODOLOGY AND SOLUTIONS

The separation of variables method was employed to find the solutions for T_1 and T_2 . The Computer Algebra System, Maple V Release 4, was used to accomplish all required algebraic manipulations to obtain the two temperatures distributions. The spreading resistance was obtained by means of the definition proposed by Mikic and Rohsenow¹:

$$QR_s = \bar{T}_{\text{source}} - \bar{T}_{\text{contact plane}} \quad (11)$$

The mean temperature of the heat source area is obtained from

$$\bar{T}_{\text{source}} = \frac{1}{4ab} \int_{-a}^a \int_{-b}^b T_1(x, y, 0) dx dy \quad (12)$$

and the mean temperature of the contact plane $z = 0$ is obtained from

$$\bar{T}_{\text{contact plane}} = \frac{1}{4cd} \int_{-c}^c \int_{-d}^d T_1(x, y, 0) dx dy \quad (13)$$

GENERAL SPREADING RESISTANCE EXPRESSION

The methodology described above was used to obtain the solution for the general problem defined previously. The spreading resistance is obtained by the following general expression which shows the explicit and implicit relationships with the geometric and thermal parameters of the system:

$$R_s = \frac{1}{2a^2cdk_1} \sum_{m=1}^{\infty} \frac{\sin(a\delta)^2}{\delta^3} \cdot \phi_m(\delta) + \frac{1}{2b^2cdk_1} \sum_{n=1}^{\infty} \frac{\sin(b\lambda)^2}{\lambda^3} \cdot \phi_n(\lambda) + \frac{1}{a^2b^2cdk_1} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin(a\delta)^2 \sin(b\lambda)^2}{\delta^2 \lambda^2 \beta} \cdot \phi_{m,n}(\beta) \quad (14)$$

The general expression for the spreading resistance consists of three terms. The single summations account for two-dimensional spreading in the x and y directions respectively, and the double summation term accounts for three-dimensional spreading from the rectangular heat source. Figure 3 illustrates the superposition of the two strip solutions and the rectangular solution which yield the general expression.

The eigenvalues: δ , λ and β are given in Table 1. The eigenvalues δ and λ , corresponding to the two strip solutions, depend on the flux channel dimensions and the indices m and n , respectively. The eigenvalues β for the rectangular solution are functions of the other two eigenvalues and both indices as defined in Table 1.

Table 1
Eigenvalues for Eq. (14)

$$\lambda = \frac{n\pi}{d}$$

$$\delta = \frac{m\pi}{c}$$

$$\beta = \sqrt{\delta^2 + \lambda^2}$$

The contributions of the layer thicknesses: t_1, t_2 , the layer conductivities: k_1, k_2 , and the uniform conductance h to the spreading resistance are determined by means of the general expression:

$$\phi(\zeta) = \frac{\alpha(\kappa\zeta\mathcal{L} - Bi)e^{4\zeta t_1} + (\kappa\zeta\mathcal{L} - Bi)e^{2\zeta t_1}}{\alpha(\kappa\zeta\mathcal{L} - Bi)e^{4\zeta t_1} - (\kappa\zeta\mathcal{L} - Bi)e^{2\zeta t_1}} + \frac{(\kappa\zeta\mathcal{L} + Bi)e^{2\zeta(2t_1+t_2)} + \alpha(\kappa\zeta\mathcal{L} + Bi)e^{2\zeta(t_1+t_2)}}{(\kappa\zeta\mathcal{L} + Bi)e^{2\zeta(2t_1+t_2)} - \alpha(\kappa\zeta\mathcal{L} + Bi)e^{2\zeta(t_1+t_2)}} \quad (15)$$

where

$$\alpha = \frac{1 - \kappa}{1 + \kappa}$$

with $\kappa = k_2/k_1$ and $Bi = h\mathcal{L}/k_1$ where \mathcal{L} is an arbitrary length scale employed to define the dimensionless spreading resistance:

$$\psi = R_s k_1 \mathcal{L} \quad (16)$$

which is based on the thermal conductivity of the layer adjacent to the heat source. Various system lengths may be

used and the appropriate choice depends on the system of interest.

In all summations $\phi(\zeta)$ is evaluated in each series using $\zeta = \delta, \lambda$, and β from Table 1.

The general expression for $\phi(\zeta)$ reduces to simpler expressions for two important special cases: i) the semi-infinite flux channel where $t_2 \rightarrow \infty$, see (Fig. 4) and ii) the finite isotropic rectangular flux channel where $\kappa = 1$, (see Fig. 5). The respective expressions are

$$\phi(\zeta) = \frac{(e^{2\zeta t_1} - 1)\kappa + (e^{2\zeta t_1} + 1)}{(e^{2\zeta t_1} + 1)\kappa + (e^{2\zeta t_1} - 1)} \quad (17)$$

where the influence of the contact conductance has vanished, and

$$\phi(\zeta) = \frac{(e^{2\zeta t} + 1)\zeta\mathcal{L} - (1 - e^{2\zeta t})Bi}{(e^{2\zeta t} - 1)\zeta\mathcal{L} + (1 + e^{2\zeta t})Bi} \quad (18)$$

where the influence of κ has vanished.

The dimensionless spreading resistance ψ depends on six independent dimensionless parameters such as (a) the relative size of the rectangular source area ($\epsilon_1 = a/c, \epsilon_2 = b/d$), (b) the layer conductivity ratio ($\kappa = k_2/k_1$), (c) the relative layer thicknesses ($\tau_1 = t_1/\mathcal{L}, \tau_2 = t_2/\mathcal{L}$), and (d) the Biot number, $Bi = h\mathcal{L}/k_1$.

The general solution also reduces to several special cases such as those shown in Figs. 6 and 7 where $k_2 = k_1 = k$, the rectangular area has become a strip and the thickness t of the flux channel is either finite or very large relative to the larger channel dimension. Another special case shown in Fig. 8, is obtained when $t_2 \rightarrow \infty$, while $k_1 \neq k_2$. These and other special cases which will be examined are summarized in Table 2.

The general solution may also be used to obtain the solution for an isoflux square area on the end of a square semi-infinite flux tube⁶.

SPREADING RESISTANCE FOR SOME TWO AND THREE DIMENSIONAL SYSTEMS

Three-Dimensional Spreading Resistances

The first three-dimensional case of interest which arises from the general solution given above is the system which consists of an isoflux rectangular source on an isotropic ($k_2 = k_1 = k$) finite rectangular flux channel which is cooled through a uniform film or contact conductance over the lower boundary. In this case the ϕ function is defined by Eq. (18) which is substituted into Eq. (14). The dimensionless spreading resistance for this case depends on four independent dimensionless parameters where $\psi = \psi(\epsilon_1, \epsilon_2, \tau, Bi)$. With four independent parameters it is not possible to develop correlation equations and to display graphically the general trends of the solution. Numerical values of the dimensionless spreading resistance can, however, be obtained in a straight forward manner by means of Computer Algebra Systems.

Rectangular Area on Semi-Infinite Rectangular Flux Channel

When the relative thickness τ is sufficiently large, $\phi \rightarrow 1$, for the three basic solutions of Eq. (14), then $\psi = \psi(\epsilon_1, \epsilon_2)$, is independent of τ and Bi . This corresponds to the case of a rectangular heat source on a semi-infinite rectangular flux channel, see Fig. 9.

Square Area on Semi-Infinite Square Flux Tube

For the special case of a square heat source on a semi-infinite square, isotropic flux tube, the general solution reduces to a simpler expression which depends on one parameter only. The solution¹ was recast into the form⁶:

$$k\sqrt{A_s} R_s = \frac{2}{\pi^3 \epsilon} \left[\sum_{m=1}^{\infty} \frac{\sin^2(m\pi\epsilon)}{m^3} + \frac{1}{\pi^2 \epsilon^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(m\pi\epsilon) \sin^2(n\pi\epsilon)}{m^2 n^2 \sqrt{m^2 + n^2}} \right] \quad (19)$$

where the characteristic length was selected as $\mathcal{L} = \sqrt{A_s}$. The relative size of the heat source was defined as $\epsilon = \sqrt{A_s}/A_c$ where A_c is the flux tube area. A correlation equation was reported for the above expression⁶:

$$k\sqrt{A_s} R_s = 0.47320 - 0.62075\epsilon + 0.1198\epsilon^3 \quad (20)$$

in the range: $0 \leq \epsilon \leq 0.5$, with a maximum relative error of approximately 0.3%. The constant on the right-hand side of the correlation equation is the value of the dimensionless spreading resistance of an isoflux square source on an isotropic half-space when the square root of the source area is chosen as the characteristic length.

Isoflux Rectangular Source on Isotropic Half-Space

The spreading resistance for an isoflux rectangular source of dimensions: $2a \times 2b$ on an isotropic half-space, shown in Fig. 10, whose thermal conductivity is k has a closed form solution⁷:

$$k\sqrt{A_s} R_s = \frac{\sqrt{\varrho}}{\pi} \left\{ \sinh^{-1} \left(\frac{1}{\varrho} \right) + \frac{1}{\varrho} \sinh^{-1} \varrho + \frac{\varrho}{3} \left[1 + \frac{1}{\varrho^3} - \left(1 + \frac{1}{\varrho^2} \right)^{3/2} \right] \right\} \quad (21)$$

where $\varrho = a/b \geq 1$ is the aspect ratio of the rectangle. When the scale length is $\mathcal{L} = \sqrt{A_s}$, the dimensionless spreading resistance becomes a weak function of ϱ . For a square heat source, the numerical value of the dimensionless spreading resistance is $k\sqrt{A_s} R_s = 0.4732$ which is very close to the numerical value for the isoflux circular source on an isotropic half-space and other singly-connected heat source geometries such as an equilateral triangle and a semi-circular heat source. The solution for

the rectangular heat source on a compound half space, shown in Fig. 11, may be obtained from the general solution for the finite compound flux channel, provided that ($t_2 \rightarrow \infty, c \rightarrow \infty, d \rightarrow \infty$). No closed form solution such as that given by Eq. (21) exists.

Two-Dimensional Spreading Resistances

The first two-dimensional spreading resistance case is shown in Fig. 6. This case consists of an isoflux strip of width, $2a$, on an isotropic rectangular flux channel of finite thickness, t , with uniform conductance over the bottom surface. For this system the appropriate scale length is $\mathcal{L} = c$, the half-width of the flux channel. The general solution reduces to the following expression:

$$kR_s = \frac{1}{\pi^3 \epsilon^2} \sum_{n=1}^{\infty} \frac{\sin^2(n\pi\epsilon)}{n^3} \left[\frac{n\pi + Bi \tanh(n\pi\tau)}{n\pi \tanh(n\pi\tau) + Bi} \right] \quad (22)$$

with $\epsilon = a/c, \tau = t/c$ and $Bi = hc/k$.

When the relative thickness exceeds the critical value, $\tau > 2.65/\pi$, the previous result reduces to the result for the case shown in Fig. 7, an isoflux strip on an isotropic, semi-infinite flux channel for which the spreading resistance is obtained from the expression¹:

$$kR_s = \frac{1}{\pi^3 \epsilon^2} \sum_{n=1}^{\infty} \frac{\sin^2(n\pi\epsilon)}{n^3} \quad (23)$$

which depends on the relative strip size only.

Effect of Heat Source Flux Distribution

The effect of the heat flux distribution on strip sources was examined by Yovanovich⁸. Flux distributions of the form: $f(u) = (1-u)^\mu$ where $u = x/a$ is the arbitrary relative position in the strip source and the flux shape parameter is μ . Yovanovich reported the general result⁸:

$$kR_s = \frac{1}{\pi^2} \Gamma(\mu+3/2) \frac{1}{\epsilon} \sum_{n=1}^{\infty} \frac{\sin(n\pi\epsilon)}{n^2} \left[\frac{2}{n\pi\epsilon} \right]^{\mu+1/2} J_{\mu+1/2}(n\pi\epsilon) \quad (24)$$

where Γ is the gamma function and $J_{\mu+1/2}$ is the Bessel function of the first kind of order $\mu+1/2$. By means of the general expression, Yovanovich obtained results for three flux distributions: (i) equivalent isothermal flux distribution when $\mu = -1/2$, (ii) isoflux strip when $\mu = 0$, and (iii) parabolic flux distribution when $\mu = 1/2$. The general expression with $\mu = -1/2$ for the equivalent isothermal flux distribution reduces to the previously reported result¹:

$$kR_s = \frac{1}{\pi^2 \epsilon} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin(n\pi\epsilon) J_0(n\pi\epsilon) \quad (25)$$

This expression can be compared against the true isothermal closed-form expression^{2,3}:

$$kR_s = \frac{1}{\pi} \ln \left[\left\{ \sin \left(\frac{\pi}{2} \epsilon \right) \right\}^{-1} \right] \quad (26)$$

For $\epsilon < 0.2$, the previous result approaches the asymptote: $kR_s = \pi^{-1} \ln(2/\pi\epsilon)$.

The parabolic flux distribution result⁸ was obtained by setting $\mu = 1/2$:

$$kR_s = \frac{2}{\pi^3 \epsilon^2} \sum_{n=1}^{\infty} \frac{1}{n^3} \sin(n\pi\epsilon) J_1(n\pi\epsilon) \quad (27)$$

For completeness the analytical, closed-form result for the flux channel shown in Fig. 12 is reported here. In this case the flux channel is isotropic, the cross-section changes abruptly from a width of $2a$ to a width of $2b$. The boundary condition over the interface between the upper and lower parts is not known. For the general case, $\epsilon = a/b < 1$, the boundary condition is neither isothermal nor isoflux. The true condition is an unknown variable temperature distribution and an unknown variable flux distribution. When $\epsilon = 1$, the temperature and flux distributions are known, however, the spreading resistance is not present. The spreading resistance can be obtained by means of the closed-form result⁴:

$$kR_s = \frac{1}{2\pi} \left[\left(\epsilon + \frac{1}{\epsilon} \right) \ln \left[\frac{1+\epsilon}{1-\epsilon} \right] + 2 \ln \left[\frac{1-\epsilon^2}{4\epsilon} \right] \right] \quad (28)$$

Numerical values of $\psi = kR_s$ are given in Table 3 for the flux distributions defined by the flux distribution parameter: $\mu = -1/2, 0, 1/2$ and the true isothermal result for a range of the relative strip size parameter ϵ .

We observe that the numerical values for the equivalent isothermal flux distribution, Eq. (25), and the true isothermal, Eq. (26), approach each other as $\epsilon \rightarrow 0$; however, there are large differences in the numerical values for $\epsilon > 0.6$. The numerical values for the parabolic distribution are greater than the isoflux values which are greater than the values for the isothermal strip. For very narrow strips, $\epsilon < 0.02$, the maximum difference between the highest values corresponding to $\mu = 1/2$ and the lowest values corresponding to $\mu = -1/2$ differ by less than 5%. This implies that the spreading resistance for very narrow strips depends weakly on the heat flux distribution.

In Table 4 the numerical values obtained from Eq. (28) are compared against the mean values of Eqs. (23) and (25) for a range of the relative strip size.

The differences are less than 1% for $\epsilon \leq 0.20$ and the differences become negligible for $\epsilon \rightarrow 0$.

SUMMARY

A general expression for the spreading resistance of an isoflux rectangular source on the surface of a finite compound rectangular flux channel was presented. The series solution consists of three summations which correspond to

two strip solutions and a rectangle solution. In the general, the dimensionless spreading resistance depends on several dimensionless geometric and thermal parameters.

Results are presented for isotropic finite and semi-infinite rectangular flux channels for the strip source. Results are also presented for the isoflux rectangular and square source areas on an isotropic half-space.

A correlation equation is reported for the three-dimensional spreading resistance for an isoflux square source on an isotropic semi-infinite square flux tube.

Expressions which show the effect of heat flux distribution over the strip source area are presented. Tabulated values of the dimensionless spreading resistance for various flux distributions are given.

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Table 2
Summary of Solutions for Isoflux Source

Configuration	Limiting Values
Rectangular Heat Source	
Finite Compound Rectangular Flux Channel	$a, b, c, d, t_1, t_2, k_1, k_2, h$
Semi-Infinite Compound Rectangular Flux Channel	$t_2 \rightarrow \infty$
Finite Isotropic Rectangular Flux Channel	$k_1 = k_2$
Semi-Infinite Isotropic Rectangular Flux Channel	$t_1 \rightarrow \infty$
Strip Heat Source	
Finite Compound Rectangular Flux Channel	$a, c, b = d, t_1, t_2, k_1, k_2, h$
Semi-Infinite Compound Rectangular Flux Channel	$t_2 \rightarrow \infty$
Finite Isotropic Rectangular Flux Channel	$k_1 = k_2$
Semi-Infinite Isotropic Rectangular Flux Channel	$t_1 \rightarrow \infty$
Rectangular Source On a Half Space	
Isotropic Half Space	$c \rightarrow \infty, d \rightarrow \infty, t_1 \rightarrow \infty$
Compound Half Space	$c \rightarrow \infty, d \rightarrow \infty, t_2 \rightarrow \infty$

Table 3
Numerical values of ψ for $\mu = -1/2, 0, \text{ and } 1/2$

ϵ		0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60	0.80
$\mu = -1/2$	Eq. (25)	1.1011	0.8808	0.7518	0.6609	0.5902	0.3729	0.1658	0.0607	0.0067
$\mu = 0$	Eq. (23)	1.1377	0.9172	0.7883	0.6970	0.6263	0.4083	0.1984	0.0882	0.0255
$\mu = 1/2$	Eq. (27)	1.1545	0.9340	0.8051	0.7138	0.6430	0.4247	0.2134	0.1007	0.0338
$T = \text{Constant}$	Eq. (26)	1.1015	0.8811	0.7523	0.6611	0.5905	0.3738	0.1691	0.0675	0.0160

Table 4
Typical numerical values of Eq. (28) and the average of Eqs. (23) and (25)

ϵ	0.02	0.20	0.40	0.60	0.80
Eq. (28)	1.122	0.3936	0.1860	0.0794	0.0214
$\frac{Eq.(23) + Eq.(25)}{2}$	1.120	0.3911	0.1838	0.0779	0.0208
% difference	0.24	0.65	1.21	1.95	3.04

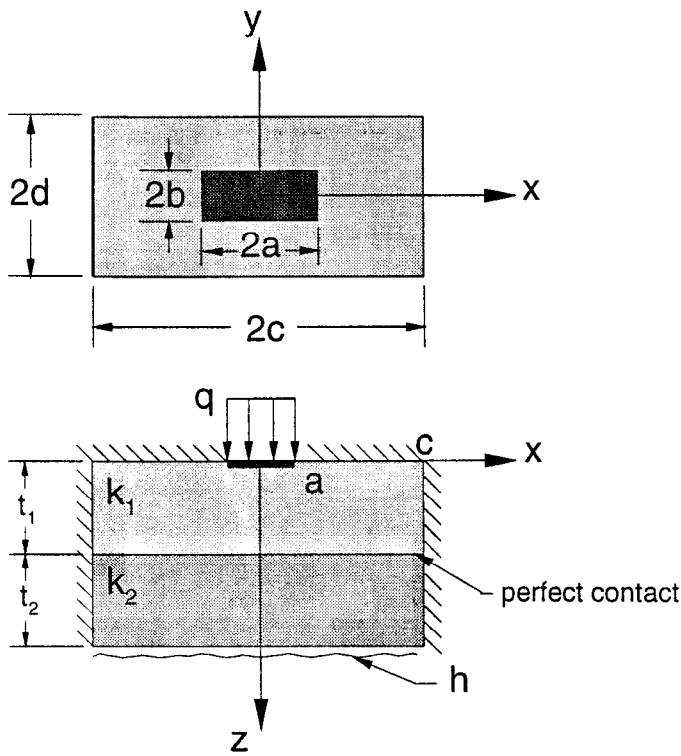


Fig. 1 - Finite Compound Channel with Rectangular Heat Source

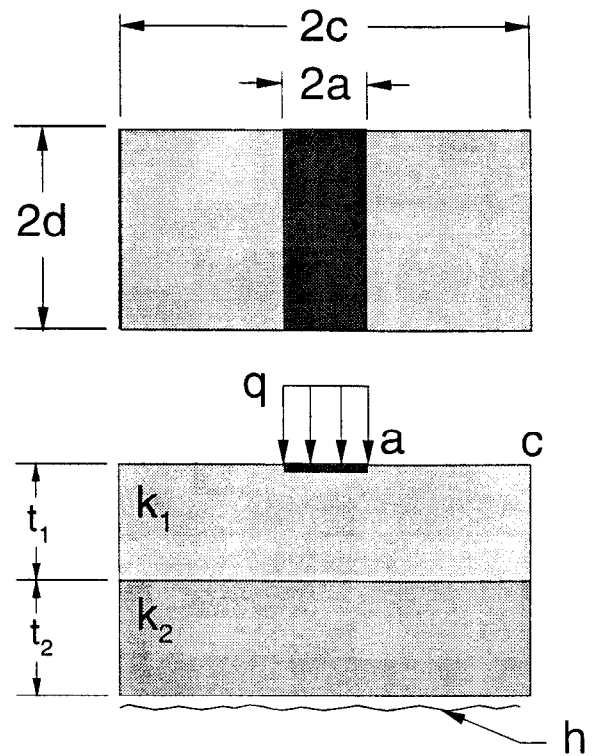


Fig. 2 - Finite Compound Channel with Strip Heat Source

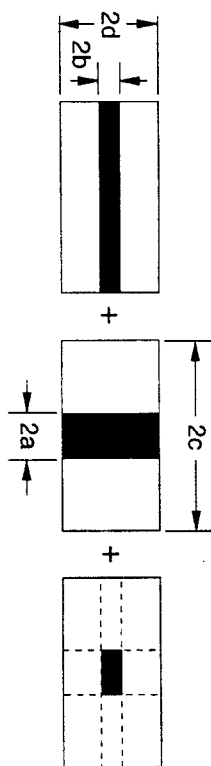


Fig. 3 - Superposition of Solutions

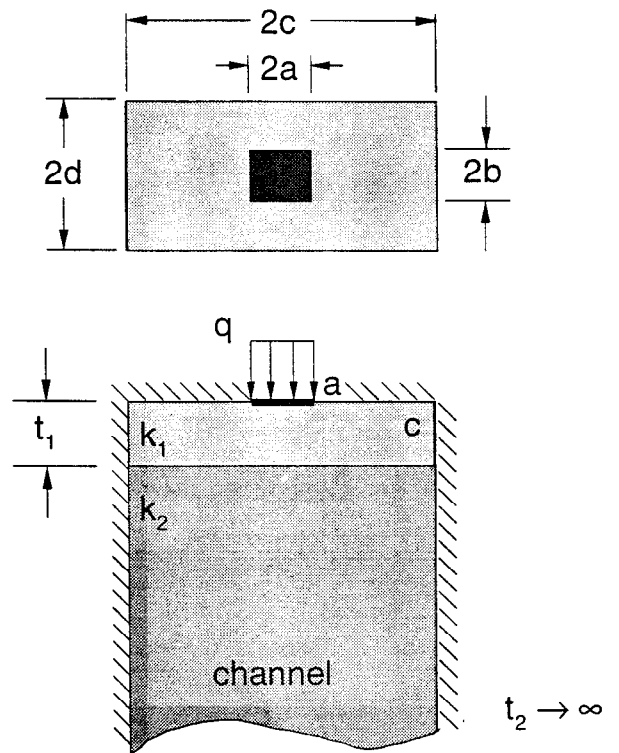


Fig. 4 - Semi-Infinite Coated Channel with Rectangular Heat Source

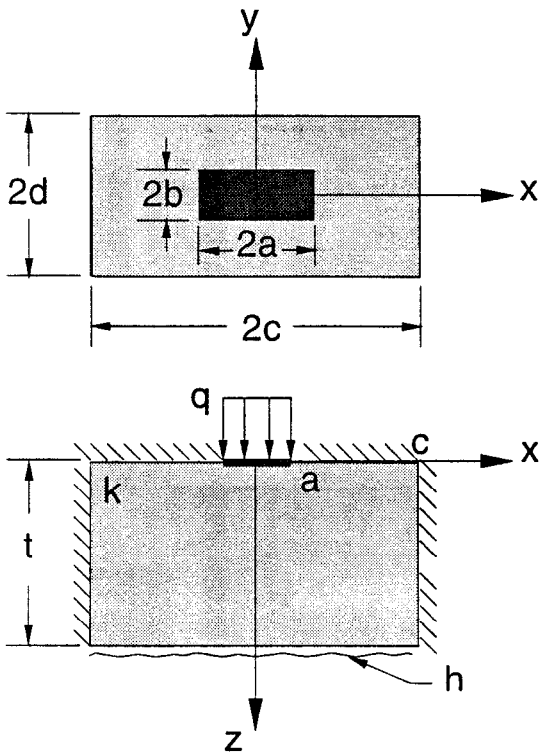


Fig. 5 - Finite Isotropic Channel with Rectangular Heat Source

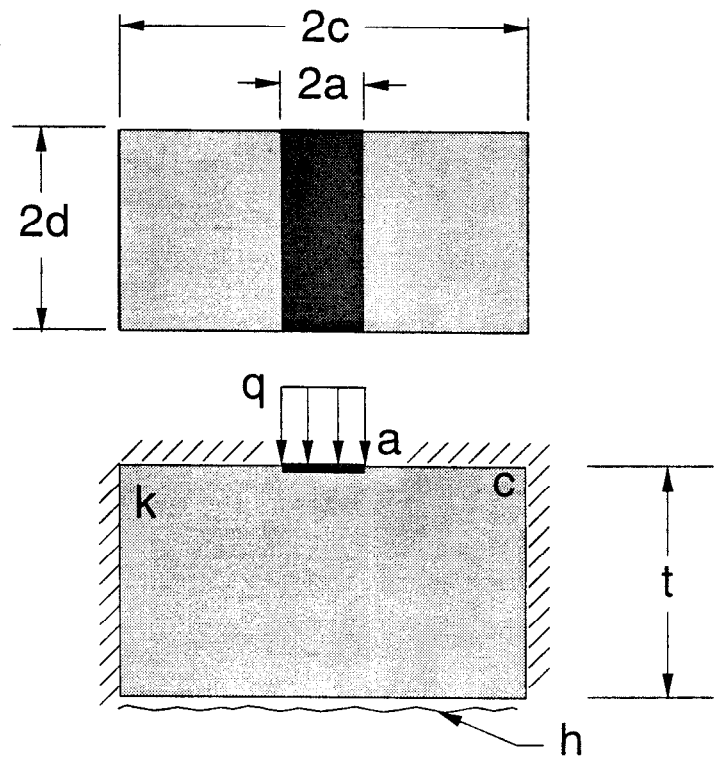


Fig. 6 - Finite Isotropic Channel with Strip Heat Source

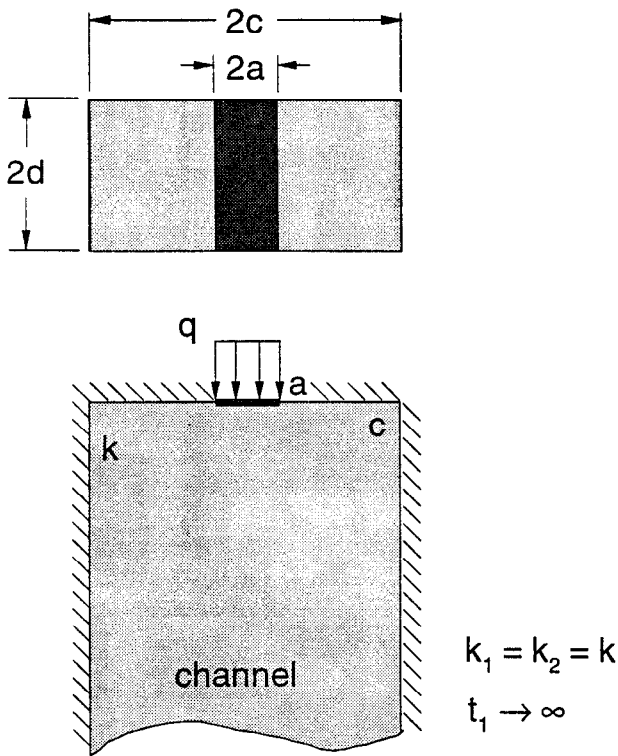


Fig. 7 - Semi-Infinite Isotropic Channel with Strip Heat Source

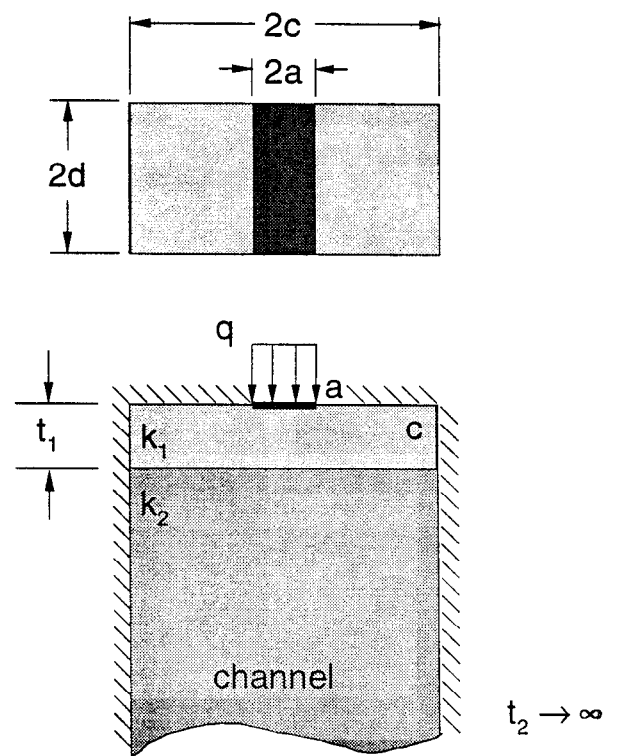


Fig. 8 - Semi-Infinite Compound Channel with Strip Heat Source

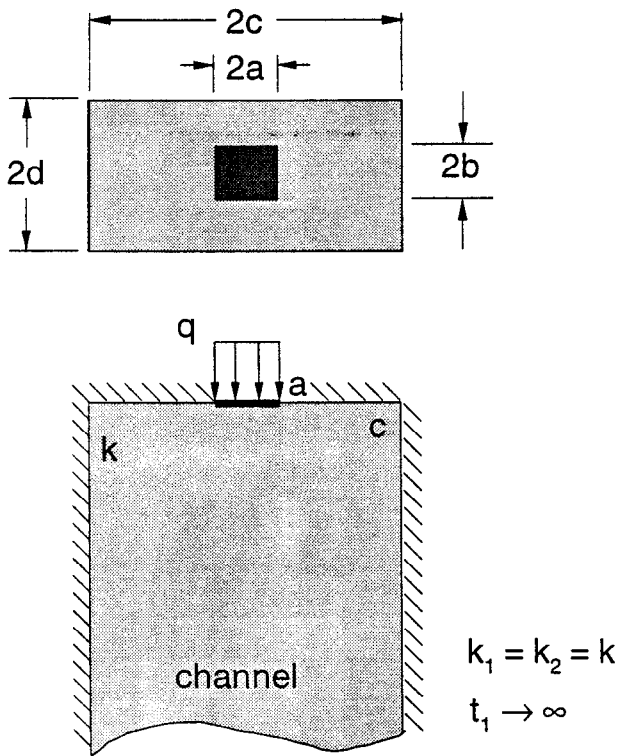


Fig. 9 - Semi-Infinite Isotropic Channel with Rectangular Heat Source

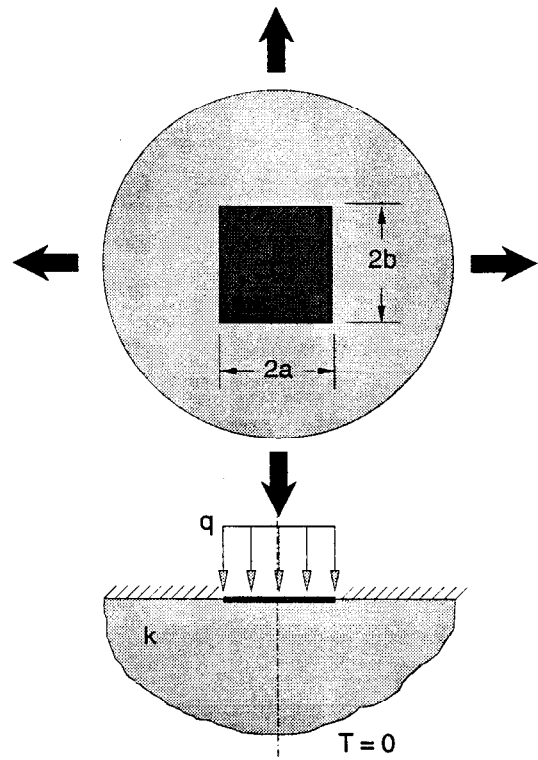


Fig. 10 - Isotropic Half Space with Rectangular Heat Source

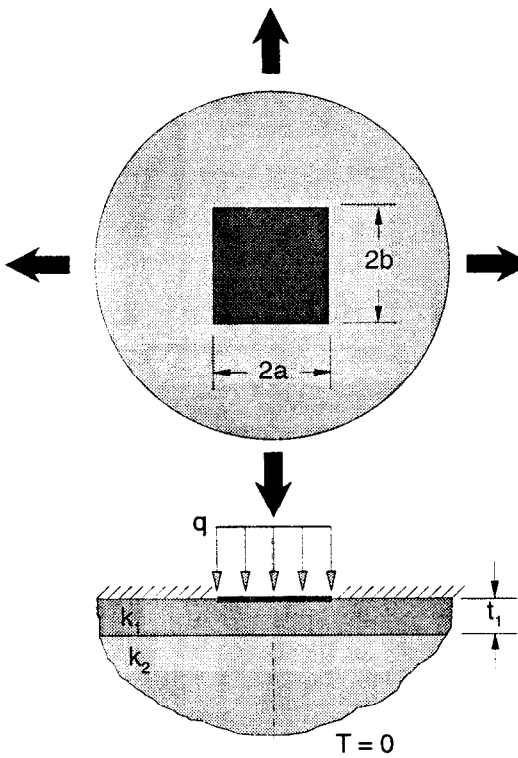


Fig. 11 - Compound Half Space with Rectangular Heat Source

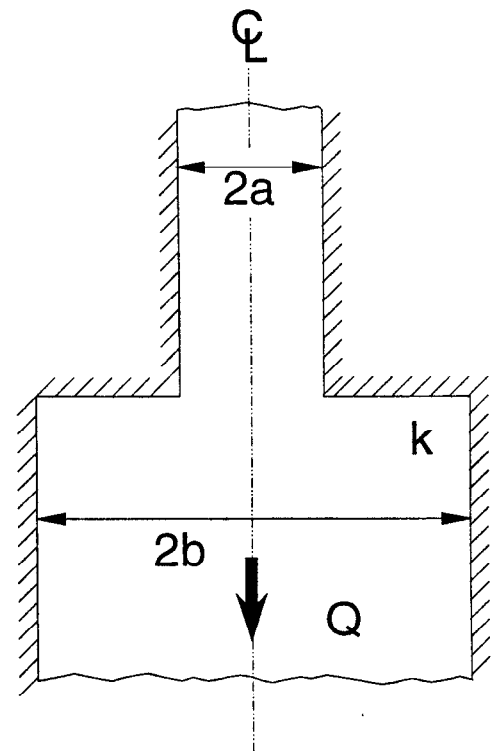


Fig. 12 - Infinite Channel with Abrupt Change in Channel Width