

Comprehensive Review of Natural Convection in Horizontal Circular Annuli

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1 Abstract

A review of the currently available experimental models, numerical results, and analytical models and correlations for natural convection in a horizontal circular annulus with isothermal boundary conditions is presented. These results have direct application to air-layer insulation for pipelines, nuclear reactor design, and should provide an effective starting point for the analysis of heat transfer in sealed electronic enclosures. This chronological review begins with the well know work of Beckmann (1931) and describes all subsequent data and correlations for both concentric and eccentric cylinders. Through a series of comparisons between the available data and models, specific recommendations concerning the accuracy and application of each of the previous studies are presented.

2 Nomenclature

C	=	correlation coefficient
D_o, D_i	=	outer, inner cylinder diameters (m)
g	=	gravitational acceleration (m/s^2)
Gr	=	Grashof number, Eq. (2)
k	=	thermal conductivity (W/mK)
k_e	=	effective thermal conductivity, Eq. (6) (W/mK)
Nu	=	Nusselt number, Eq. (7)
Pr	=	Prandtl number, $\equiv \nu/\alpha$
Q	=	heat flow rate per unit length (W/m)
Ra	=	Rayleigh number, Eq. (3)

S = conduction shape factor
 T_o, T_i = outer, inner cylinder temperatures (K)

Greek Symbols

α = thermal diffusivity (m^2/s)
 β = thermal expansion coefficient ($1/K$)
 δ = gap spacing, Eq. (1) (m)
 ϵ = eccentricity (m)
 ν = kinematic viscosity (m^2/s)

Subscripts

$cond$ = conduction limit
 $conv$ = convection limit
 D_i = based on inner cylinder diameter
 δ = based on gap spacing
 i = inner cylinder
 o = outer cylinder

3 Introduction

Natural convection heat transfer from a heated body to its surrounding enclosure is currently of interest to designers of microelectronic systems and cabinets. The rapid growth of wireless and cellular communications has led to a more widespread use of “outside plant” applications, products designed to withstand harsh, outdoor environments. The sensitive electronics contained in these products require a sealed enclosure to protect them from moisture and contaminants in their surroundings, and cannot rely on a supply of fresh, ambient air to dissipate the heat produced during typical operation. Analytical models that can be used to predict natural convection from electronics packages and circuit boards to their surrounding enclosure are of interest to engineers for trade-off and “what-if” studies during preliminary product design.

Although the current literature does not contain analytical models for the specific problem of circuit boards in a sealed enclosure, a number of researchers have examined problems involving more simple geometries. One of these configurations, the horizontal circular annulus, has been studied extensively, and these references provide a wide range of experimental and numerical results, as well as analytically-based correlations and models. The information provided by these studies should provide an effective starting point for the analysis of the sealed electronic enclosure, and it also has direct application to air-layer insulation for pipelines (Gröber et al., 1961), underground electric transmission cables and nuclear reactor design (Kuehn and Goldstein, 1976a).

Because of the large time span over which these studies were performed, over 65 years, and the

large number of correlations and data available, some confusion may arise concerning the proper choice of analytical model or correlation for any particular configuration. The accuracy and application of the available correlations and data are in many cases limited to particular ranges of the independent parameters, such as the fluid type, geometry, or the Rayleigh or Grashof numbers. The following paper will present a comprehensive review and comparison of all models and data currently available for natural convection in the horizontal circular annulus. This review will include recommendations for which correlations are appropriate for any particular configuration, and discussion of the expected accuracy of the predicted values.

General heat transfer texts and the handbooks typically present correlations from a single source, and typically do not compare the results with the available data or comment on application of these correlations beyond the specified range. No comprehensive review of this type is currently available in the literature.

4 Background

The problem of interest, as shown in Fig. 1, involves natural convection in an annular region with uniform temperature boundary conditions T_i and T_o on the inner and outer surfaces, respectively. In all previous studies, the geometry of the annulus is expressed in one of two ways. The first uses the aspect ratio, a non-dimensional quantity defined as D_o/D_i , with values for all configurations limited to the range:

$$1 < \frac{D_o}{D_i} < \infty$$

The second method used to express the annulus geometry is the gap spacing δ , defined by:

$$\delta = \frac{D_o - D_i}{2} \quad 0 < \delta < \infty \quad (1)$$

In all cases, the annulus length L is assumed to be large compared to the diameter, such that all heat flow in the axial direction can be neglected. Therefore the heat flow rate, expressed as Q in this work, is the total heat flow rate per unit length in the axial direction, with units W/m .

Two of the models presented in this review include an eccentricity term, where ϵ is defined as the distance between the centers of the inner and outer cylinders as shown in Fig. 1. This dimensional quantity is limited to the range:

$$0 \leq \epsilon < \frac{D_o - D_i}{2}$$

The remaining independent parameter used in many of the previous studies, particularly in air-filled annuli, is the Grashof number, defined as:

$$Gr_{\mathcal{L}} = \frac{g\beta(T_i - T_o)\mathcal{L}^3}{\nu^2} \quad (2)$$

where \mathcal{L} represents a general characteristic length. The Rayleigh number is also widely used, especially for fluids with Pr larger than one, such as water or various oils:

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} \cdot Pr = \frac{g\beta(T_i - T_o)\mathcal{L}^3}{\nu\alpha} \quad (3)$$

Two different characteristic lengths are commonly used in the previous work: the inner cylinder diameter D_i and the gap spacing δ . The gap spacing δ was first proposed as a characteristic length by Kraussold (1934), who noted that dependence of the solution on aspect ratio could be reduced and a single curve could be used to fit the results for all D_o/D_i . However, for the limiting case of large aspect ratio $D_o \gg D_i$, where the solution is anticipated to approach that of a single, isothermal cylinder, a Rayleigh number based on the gap spacing becomes undefined. This limiting case is treated correctly when the diameter of the inner cylinder is used as the characteristic length, but the solution remains a strong function of the aspect ratio.

Average heat transfer rate in the annulus is non-dimensionalized using one of two methods: the effective conductivity ratio and the Nusselt number. The dimensionless effective conductivity ratio, first proposed by Beckmann (1931), is based on the formulation for conduction shape factor in a circular annulus:

$$S = \frac{2\pi}{\ln(D_o/D_i)} \quad (4)$$

Through the use of an effective conductivity, the effects of convection on heat transfer in the annulus are included in this conduction expression:

$$Q = S k_e (T_i - T_o) \quad (5)$$

Solving for k_e and normalizing using the actual thermal conductivity of the medium yields:

$$\frac{k_e}{k} = \frac{Q}{k(T_i - T_o)} \frac{\ln(D_o/D_i)}{2\pi} \quad (6)$$

where $k_e/k \geq 1$ in all cases.

The second method for non-dimensionalizing the results uses the Nusselt number, defined as:

$$Nu_{\mathcal{L}} = \frac{\bar{h}\mathcal{L}}{k} = \frac{Q\mathcal{L}}{\pi D_i k (T_i - T_o)} \quad (7)$$

where both the inner cylinder diameter D_i and the gap spacing δ have been used as the characteristic dimension in previous studies.

The dimensionless effective conductivity ratio can be related to the Nusselt number based on inner diameter using the following relationship:

$$Nu_{D_i} = \frac{k_e}{k} \frac{2}{\ln(D_o/D_i)} \quad (8)$$

For the purposes of the proposed review, a single non-dimensionalization and length scale should be chosen to allow comparison of the available models and data. All results will be converted in

terms of Nusselt and Grashof numbers, or Rayleigh numbers for fluids other than air. All non-dimensionalized values will use the inner cylinder diameter D_i as characteristic length, and the annulus geometry will be reported in terms of the aspect ratio D_o/D_i .

Figure 2 is a plot of typical results anticipated for this problem, and it clearly demonstrates three distinctive regions in the solution. The conduction limit, characterized by small Ra or D_o/D_i , is a strong function of the aspect ratio and is independent of the Rayleigh number. The boundary layer limit occurs at large Ra and large aspect ratios, where the difference in surface area between the inner and outer boundaries leads to the inner cylinder controlling the heat transfer in the annulus. It is anticipated that at this limit the solution will be a strong function of Ra , but a somewhat weaker function of the aspect ratio because of the relative unimportance of the outer boundary for large D_o/D_i .

In the intermediate region between the conduction-dominated and boundary layer regions, the solution is a strong function of both Ra and D_o/D_i as it moves smoothly from one limiting case to the other. The location of this transition on a log-log plot, such as that shown in Fig. 2, is strongly dependent on the aspect ratio and may vary over several decades of Rayleigh number.

The following sections present a brief, chronological review of the previous studies with a description of the available data, models and correlations. Although some of these works include local temperature, velocity and heat flux distributions as a function of angular or radial position, these local effects are beyond the scope of this presentation and will not be considered here.

5 Review of Previous Studies

One of the first, well documented studies of heat transfer in horizontal annular enclosures is presented by Beckmann (1931). He performed experimental measurements for three different gases, air, H_2 and CO_2 , for the ranges of aspect ratio and Gr_{D_i} listed in Table 1. An additional contribution by this author is his fundamental analysis of this problem, resulting in the definition of the dimensionless effective conductivity ratio k_e/k used in many subsequent works. He also proposed a simple correlation for his results, as presented in Table 2. Values for the correlation coefficient C are determined as a function of the aspect ratio using a plot provided in the paper. Beckmann also includes a correlation for C as a function of D_o/D_i , but its predictions vary significantly from those shown in the plot and lead to substantial errors in the results. The following updated correlation predicts coefficient values to within $\pm 1\%$ of those presented by the plot:

$$C = \left(\ln \left[(D_o/D_i)^{0.207} \right] \right)^{1.22} \quad (9)$$

Subsequent publications have raised questions concerning the accuracy of Beckmann's (1931) measurements, particularly the air data for $D_o/D_i > 2.1$. Liu, Mueller and Landis (1961) attribute the large axial conduction errors along the tube walls to Beckmann's failure to employ guard heaters