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Convection Heat Transfer From Tube Banks in Crossflow: Analytical Approach

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Outline

- Introduction
- Literature Review
- Objectives
- Assumptions
- Modeling
- Results and Comparisons
- Conclusions
- Acknowledgments

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Introduction

- Industrial applications:
  - Heat exchanger devices (like steam generators, preheaters, oil coolers, power condensers)
  - Process industry
  - Air conditioning and refrigeration industry

- Primary interest of thermal engineers:
  - Average heat transfer coefficient for the entire tube bundle

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Available Correlations (Single Cylinder)

Authors	Correlations/ Models	Conditions
Churchill and Bernstein (1977)	$\mathrm{Nu_D} = 0.3 + \dfrac{0.62\mathrm{Re_D^{1/2}Pr^{1/3}}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}} \left[1 + \left(\dfrac{\mathrm{Re_D}}{282000}\right)^{5/8}\right]^{4/5}$	$\mathrm{Re_D\,Pr} > 0.2$
Morgan(1975)	$\mathrm{Nu_D} = 0.795\mathrm{Re_D^{0.384}}$ $\mathrm{Nu_D} = 0.583\mathrm{Re_D^{0.471}}$ $\mathrm{Nu_D} = 0.148\mathrm{Re_D^{0.633}}$ $\mathrm{Nu_D} = 0.0208\mathrm{Re_D^{0.814}}$	$4 \le \mathrm{Re_D} \le 40$ $40 \le \mathrm{Re_D} \le 4\times10^3$ $4\times10^3 \le \mathrm{Re_D} \le 4\times10^4$ $4\times10^4 \le \mathrm{Re_D} \le 4\times10^5$
Zukauskas(1972)	$\mathrm{Nu_D} = 0.75\mathrm{Re_D^{0.4}Pr^{0.37}}$ $\mathrm{Nu_D} = 0.51\mathrm{Re_D^{0.5}Pr^{0.37}}$ $\mathrm{Nu_D} = 0.26\mathrm{Re_D^{0.6}Pr^{0.37}}$ $\mathrm{Nu_D} = 0.076\mathrm{Re_D^{0.7}Pr^{0.37}}$	$1 \le \mathrm{Re_D} \le 40$ $40 \le \mathrm{Re_D} \le 1\times10^3$ $1\times10^3 \le \mathrm{Re_D} \le 2\times10^5$ $2\times10^5 \le \mathrm{Re_D} \le 1\times10^6$
Hilpert (1933)	$\mathrm{Nu_D} = 0.891\mathrm{Re_D^{0.33}}$ $\mathrm{Nu_D} = 0.821\mathrm{Re_D^{0.385}}$ $\mathrm{Nu_D} = 0.615\mathrm{Re_D^{0.466}}$ $\mathrm{Nu_D} = 0.174\mathrm{Re_D^{0.618}}$ $\mathrm{Nu_D} = 0.0239\mathrm{Re_D^{0.805}}$	$1 \le \mathrm{Re_D} \le 4$ $4 \le \mathrm{Re_D} \le 40$ $40 \le \mathrm{Re_D} \le 4\times10^3$ $4\times10^3 \le \mathrm{Re_D} \le 4\times10^4$ $4\times10^4 \le \mathrm{Re_D} \le 4\times10^5$

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Available Correlations (Tube Banks)

- **Zukauskas, 1972:** $\mathrm{Nu_D = FCRe_D^n Pr^m}$

F = Correction factor for $N_L \leq 16$

Geometry	C	n	m	Conditions
In-Line				
	0.9	0.4	0.36	$10 \leq \mathrm{Re_{Dmax}} \leq 100$
	0.52	0.5	0.36	$100 \leq \mathrm{Re_{Dmax}} \leq 10^3$
	0.27	0.63	0.36	$10^3 \leq \mathrm{Re_{Dmax}} \leq 2 \times 10^5$
	0.21	0.84	0.4	$\mathrm{Re_{Dmax}} > 2 \times 10^5$

Available Correlations (Tube Banks)

- **Zukauskas, 1972:** $\mathbf{Nu_D = FCRe_D^n Pr^m}$

Geometry	C	n	m	Conditions
Staggered				
	1.04	0.4	0.36	$\mathbf{10 \leq Re_{Dmax} \leq 500}$
	$\mathbf{0.35(S_T/S_L)^{0.2}}$	0.60	0.36	$\mathbf{S_T/S_L < 2}$ $\mathbf{10^3 \leq Re_{Dmax} \leq 2 \times 10^5}$
	0.40	0.60	0.36	$\mathbf{S_T/S_L > 2}$ $\mathbf{10^3 \leq Re_{Dmax} \leq 2 \times 10^5}$
	0.022	0.84	0.36	$\mathbf{Re_{Dmax} > 2 \times 10^5}$

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Correction Factor F for $N_L \leq 16$

Rows	1	2	3	4	5	7	10	13	16
In-Line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99	1.00
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99	1.00

Available Correlations (Tube Banks)

- **Grimison, 1937:** $\mathrm{Nu_D = FCRe_D^n Pr^{1/3}}$

F = Correction factor for $N_L \leq 10$

$S_T/D \rightarrow$ $S_L/D \downarrow$	1.25		1.5		2		3	
	C	n	C	n	C	n	C	n
Staggered								
0.600	-	-	-	-	-	-	.213	.636
0.900	-	-	-	-	0.446	0.571	0.401	0.581
1.000	-	-	0.497	0.558	-	-	-	-
1.125	-	-	-	-	0.478	0.565	0.518	0.560
1.25	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
1.5	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2.000	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
3.000	0.310	0.592	0.356	0.580	0.440	0.562	0.421	0.574

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Available Correlations (Tube Banks)

- **Grimison, 1937:** $Nu_D = FCRe_D^n Pr^{1/3}$

$S_T/D \rightarrow$ / $S_L/D \downarrow$	1.25		1.5		2		3	
	C	**n**	**C**	**n**	**C**	**n**	**C**	**n**
In-Line								
1.250	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.500	0.367	0.586	0.250	0.620	0.101	0.762	0.0678	0.744
2.000	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3.000	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608

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Correction Factor F for $N_L \leq 10$

Rows→	1	2	3	4	5	6	7	8	9	10
In-Line	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99	1.00
Staggered	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99	1.00

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Objectives

❖ Develop analytical correlations to determine heat transfer for :

- In-line tube banks
- Staggered tube banks

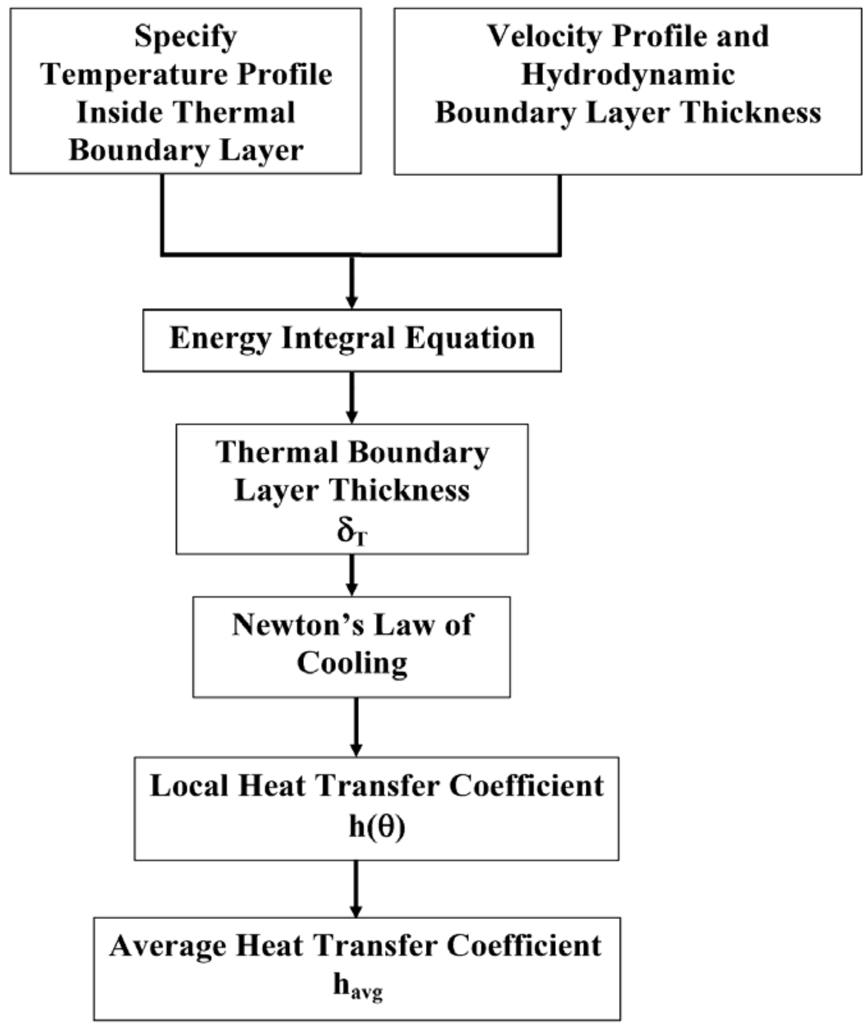
that can be used for a wide range of geometric parameters

❖ Validate developed correlations with experimental data

Assumptions

1. Forced Convection
2. Steady, laminar, fully developed and 2-D flow
3. Incompressible fluid with constant properties
4. Reynolds number is based on D and U_{max}
5. Inviscid flow outside boundary layer
6. Flow normal to tube bank

Heat Transfer Model



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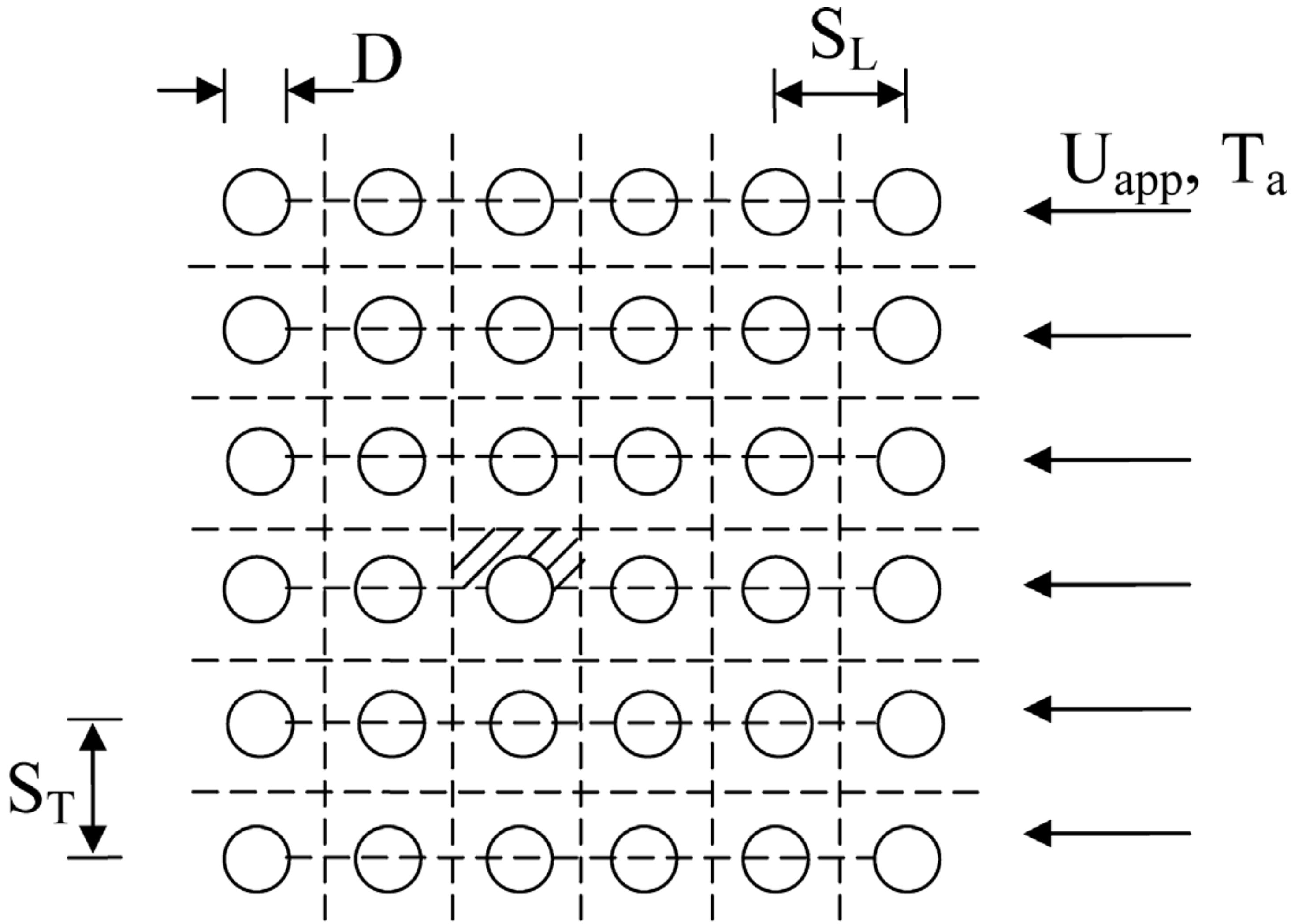
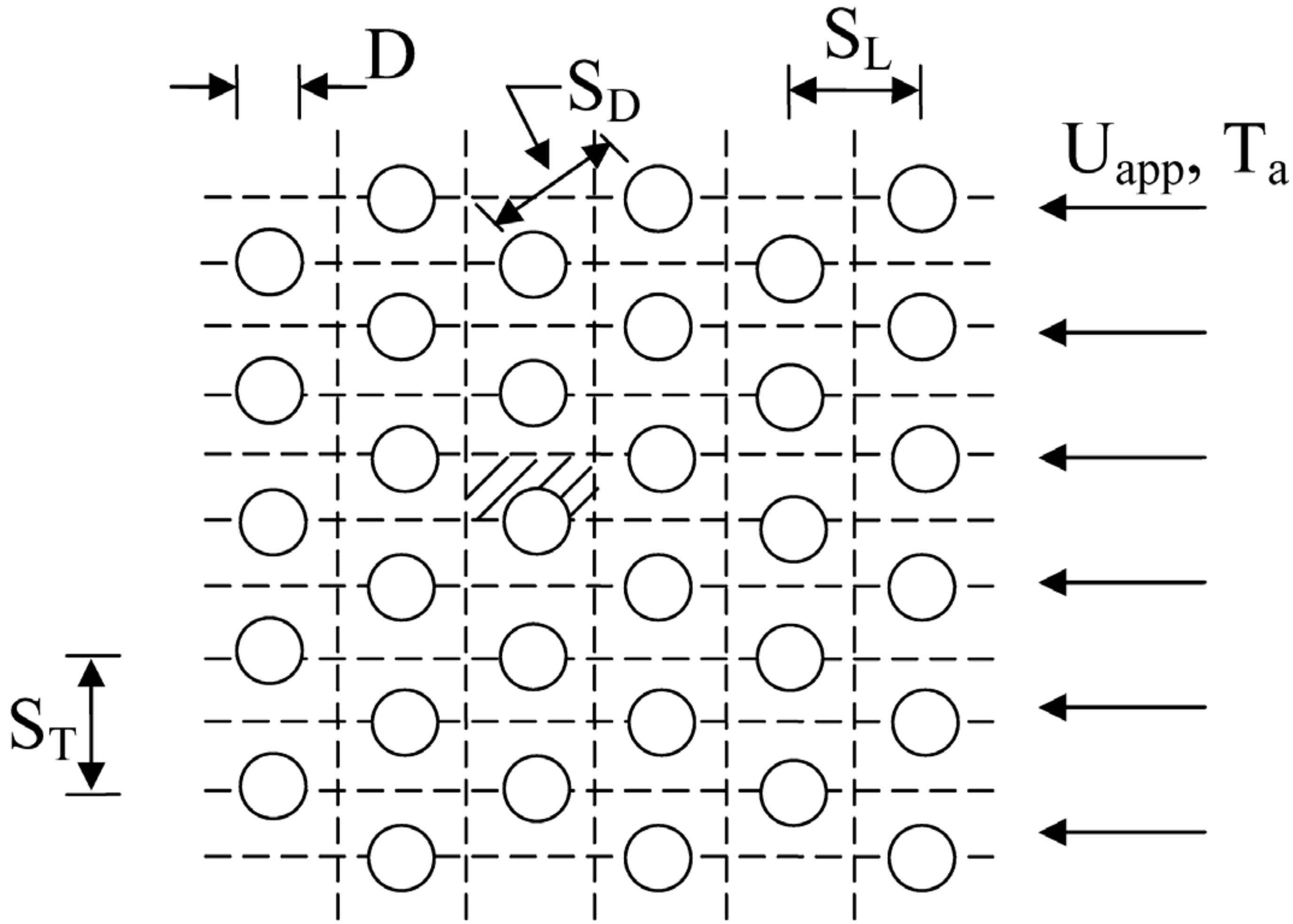
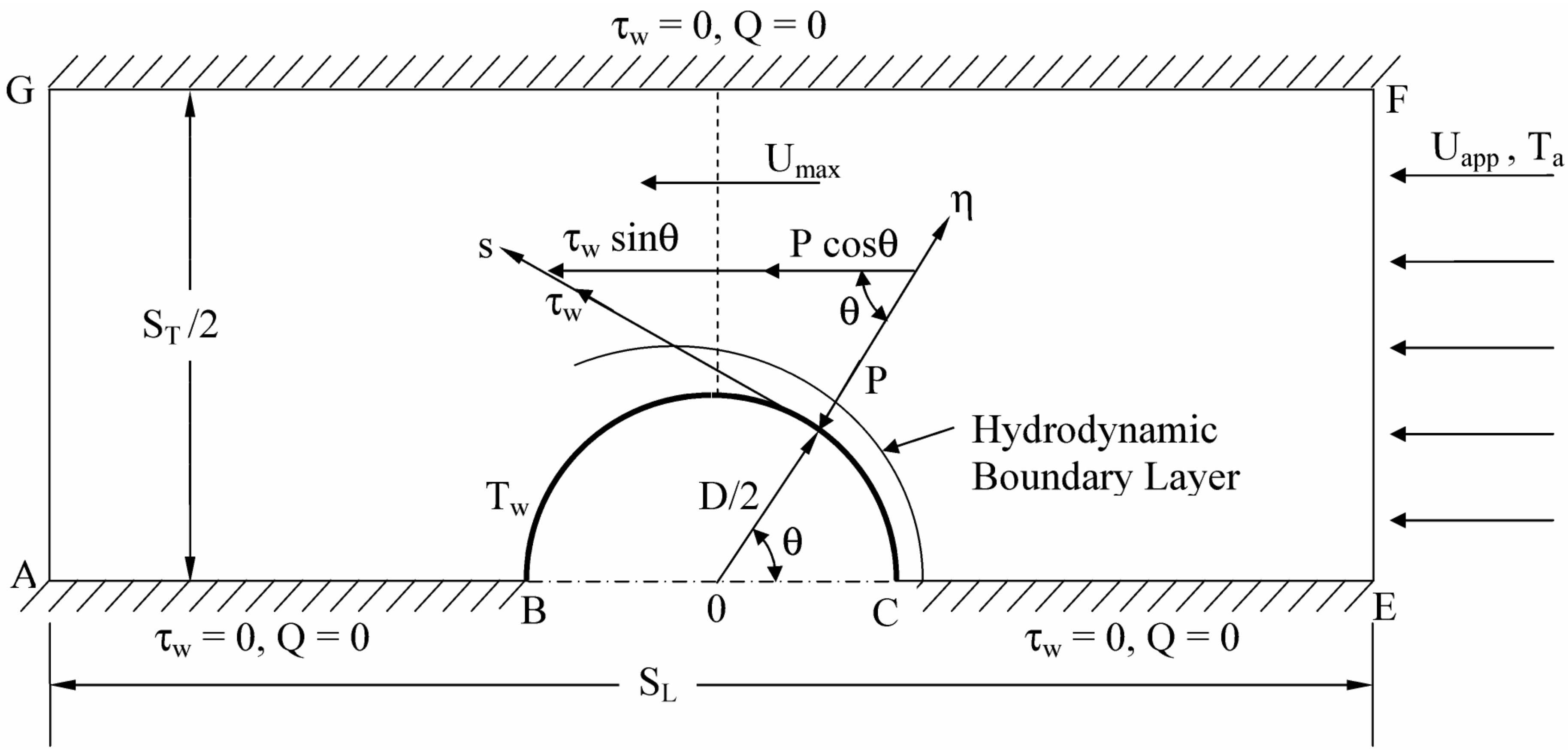
In-Line Arrangement



Staggered Arrangement



Control Volume



Boundary Conditions

1. On curved surfaces of tubes:

$$u = 0 \qquad v = 0 \qquad and \qquad T = T_w$$

2. Along top and bottom of CV:

$$v = 0 \qquad \tau_w = 0 \qquad and \qquad Q = 0$$

3. At large distances upstream of CV:

$$u = U_{app} \qquad and \qquad T = T_a$$

4. Well downstream of tubes:

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = 0 \qquad and \qquad \frac{\partial T}{\partial x} = 0$$

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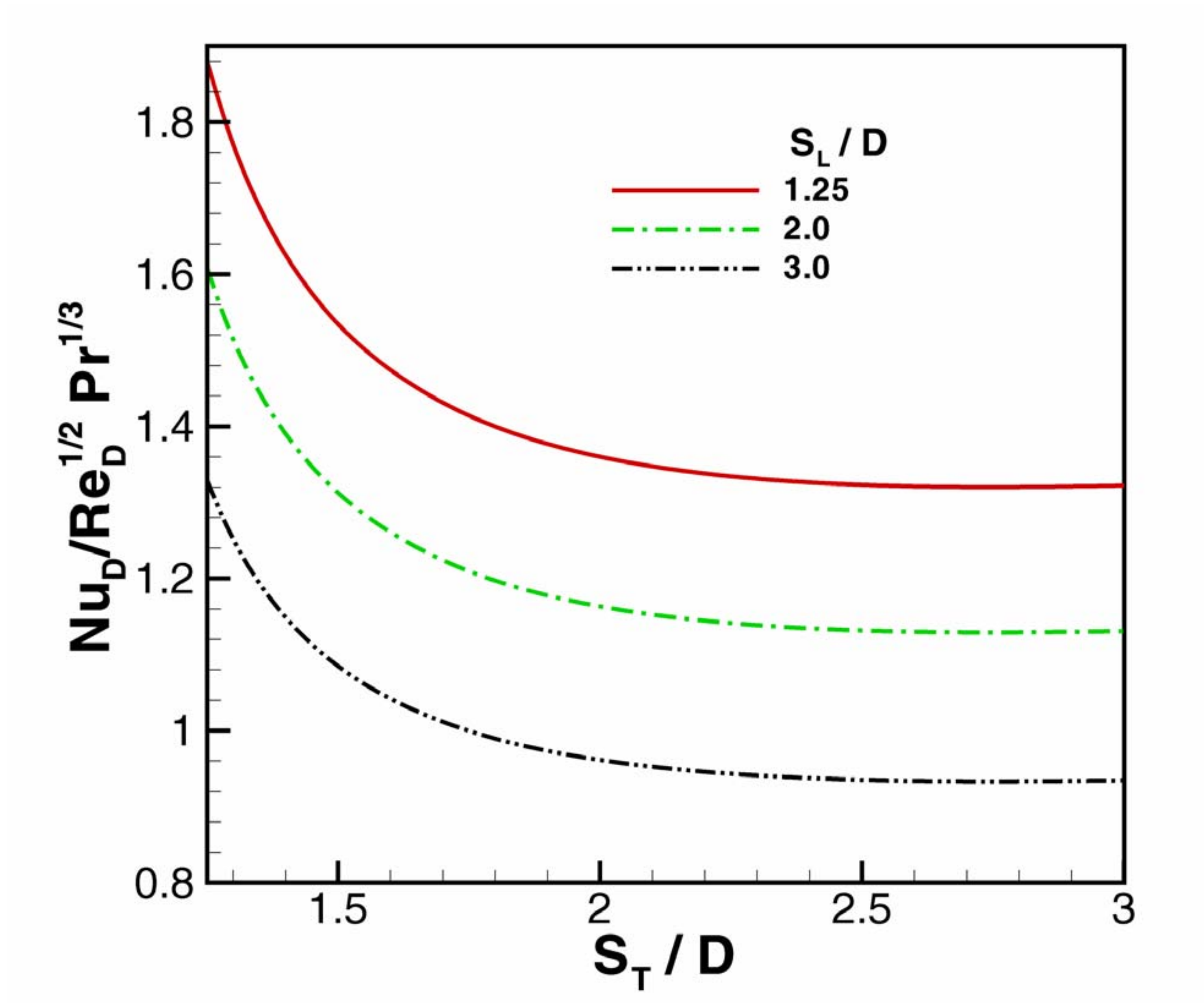
Results

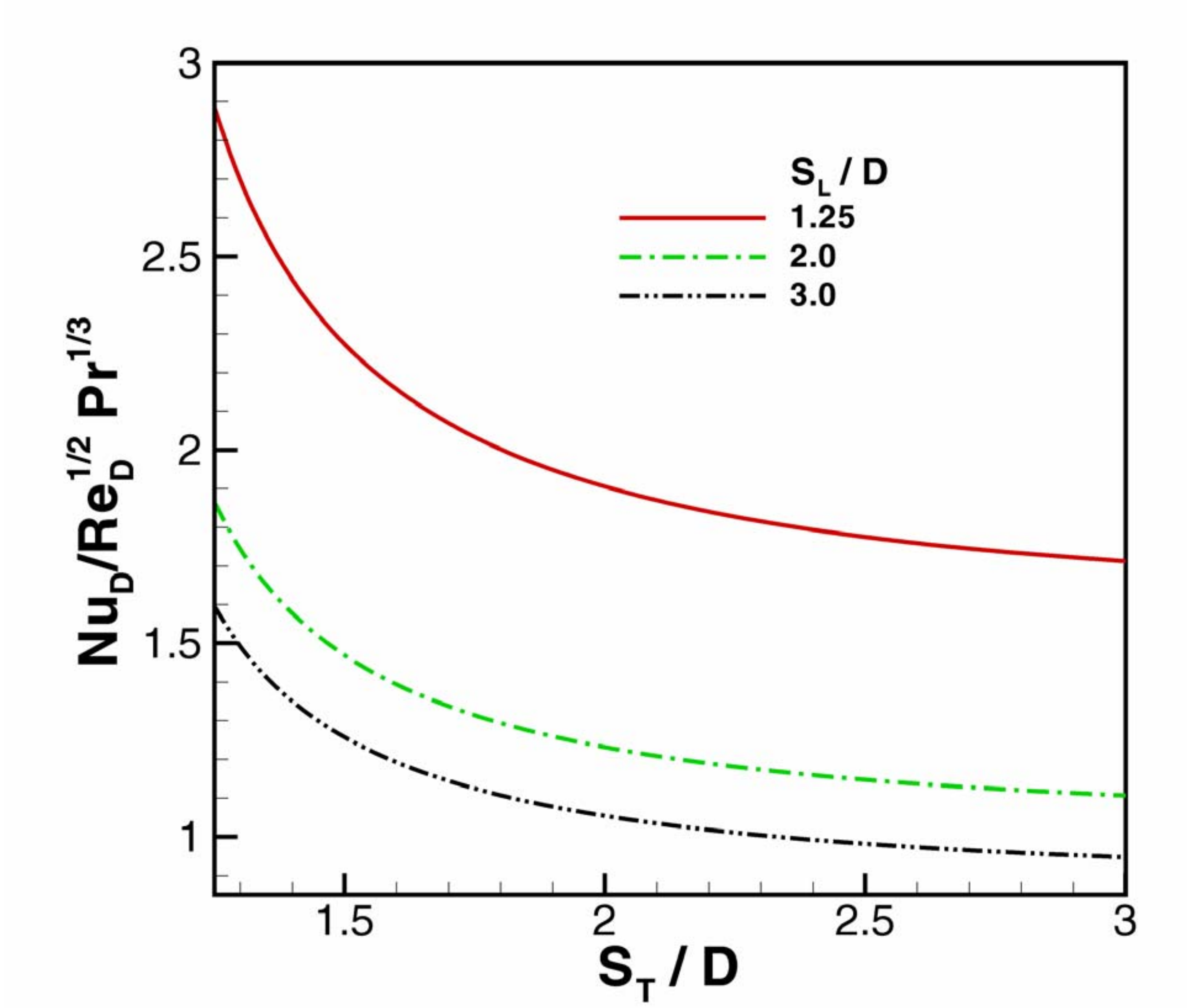
$$Nu_D = C_1 Re_D^{1/2} Pr^{1/3}$$

$$C_1 = \begin{cases} [0.2 + \exp(-0.55\mathcal{S}_T)]\mathcal{S}_T{}^{0.285}\mathcal{S}_L{}^{0.212} & \text{In-Line arrangement} \\ \dfrac{0.61\,\mathcal{S}_T{}^{0.091}\mathcal{S}_L{}^{0.053}}{[1 - 2\exp(-1.09\,\mathcal{S}_T)]} & \text{Staggered arrangement} \end{cases}$$

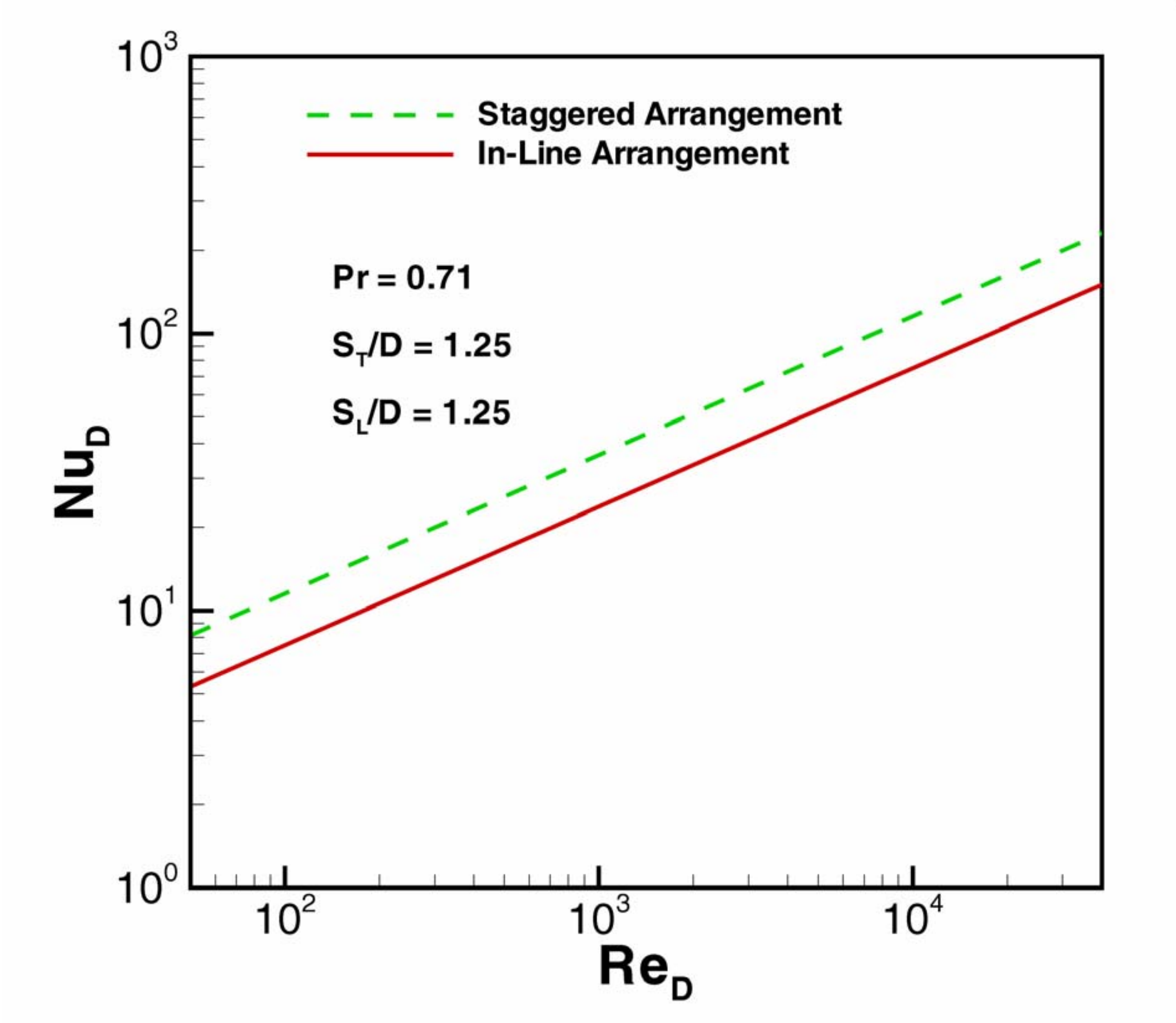
HTP with S_T (In-Line Arrangement)



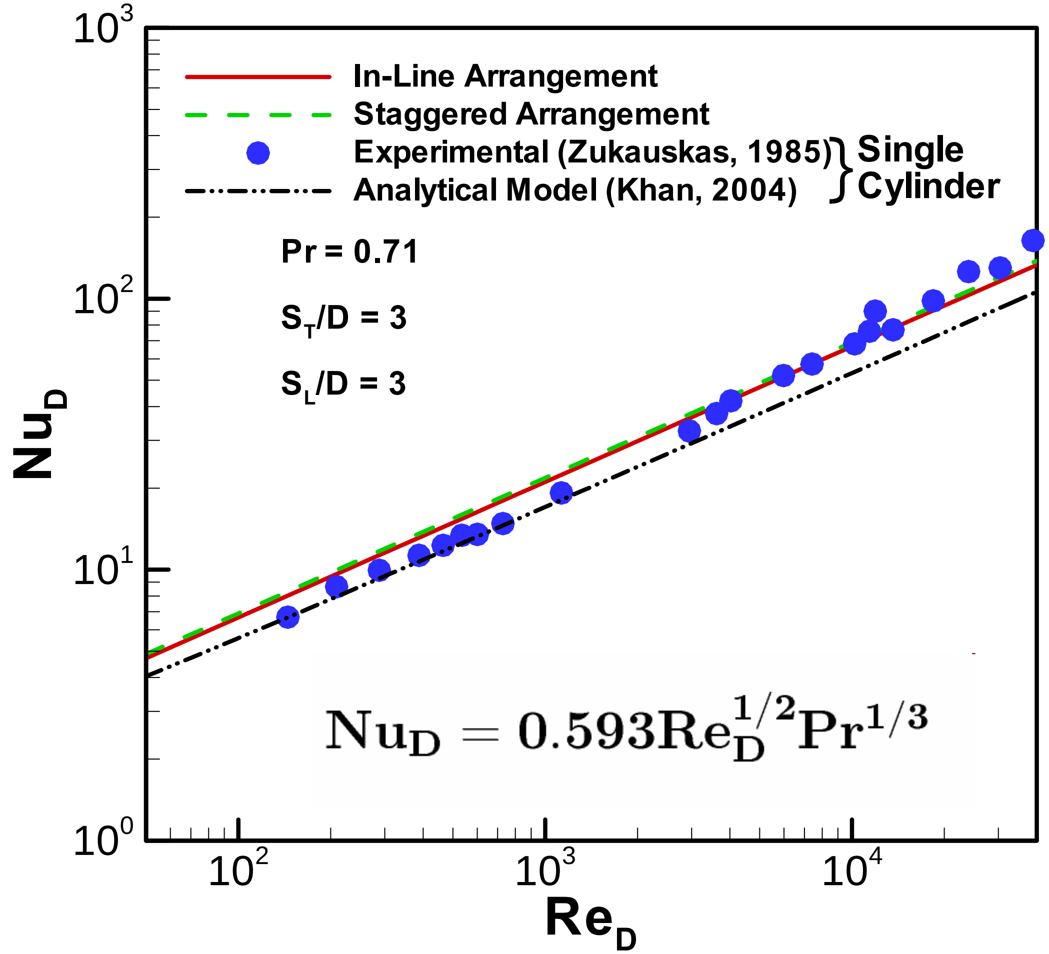
HTP with S_T (Staggered Arrangement)



Effect of Tube Arrangement (1.25 x 1.25)



Effect of Tube Arrangement (3.0 x 3.0)



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Staggered Tube Bank (Incropera and DeWitt)

Quantity	Dimension
Tube Diameter (mm)	16.4
Longitudinal Pitch (mm)	$20.5, 34.3$
Transverse Pitch (mm)	$20.5, 31.3$
Number of Tubes (Staggered)	8×7
Approach Velocity (m/s)	6
Thermal Conductivity of Air ($W/m \cdot K$)	0.0253
Density of Air (kg/m^3)	1.217
Specific Heat of Air ($J/kg \cdot K$)	1007
Kinematic Viscosity (m^2/s)	14.82×10^{-6}
Prandtl Number (Air)	0.701
Ambient Temperature ($^\circ C$)	15
Tube Surface Temperature ($^\circ C$)	70

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Comparisons

(i) Compact Tube Bank (1.25 x 1.25)

	NuD	h $(W/m^2 \cdot K)$	T_o $°C$	Q kW
Incropera and DeWitt[30]	152.0	234.0	38.5	28.4
Present Analysis	196.1	302.5	43.3	34.1

(ii) Wide Tube Bank (2.1 x 2.1)

	NuD	h $(W/m^2 \cdot K)$	T_o $°C$	Q kW
Incropera and DeWitt[30]	87.9	135.6	25.5	19.4
Present Analysis	88.3	136.2	26.9	19.2

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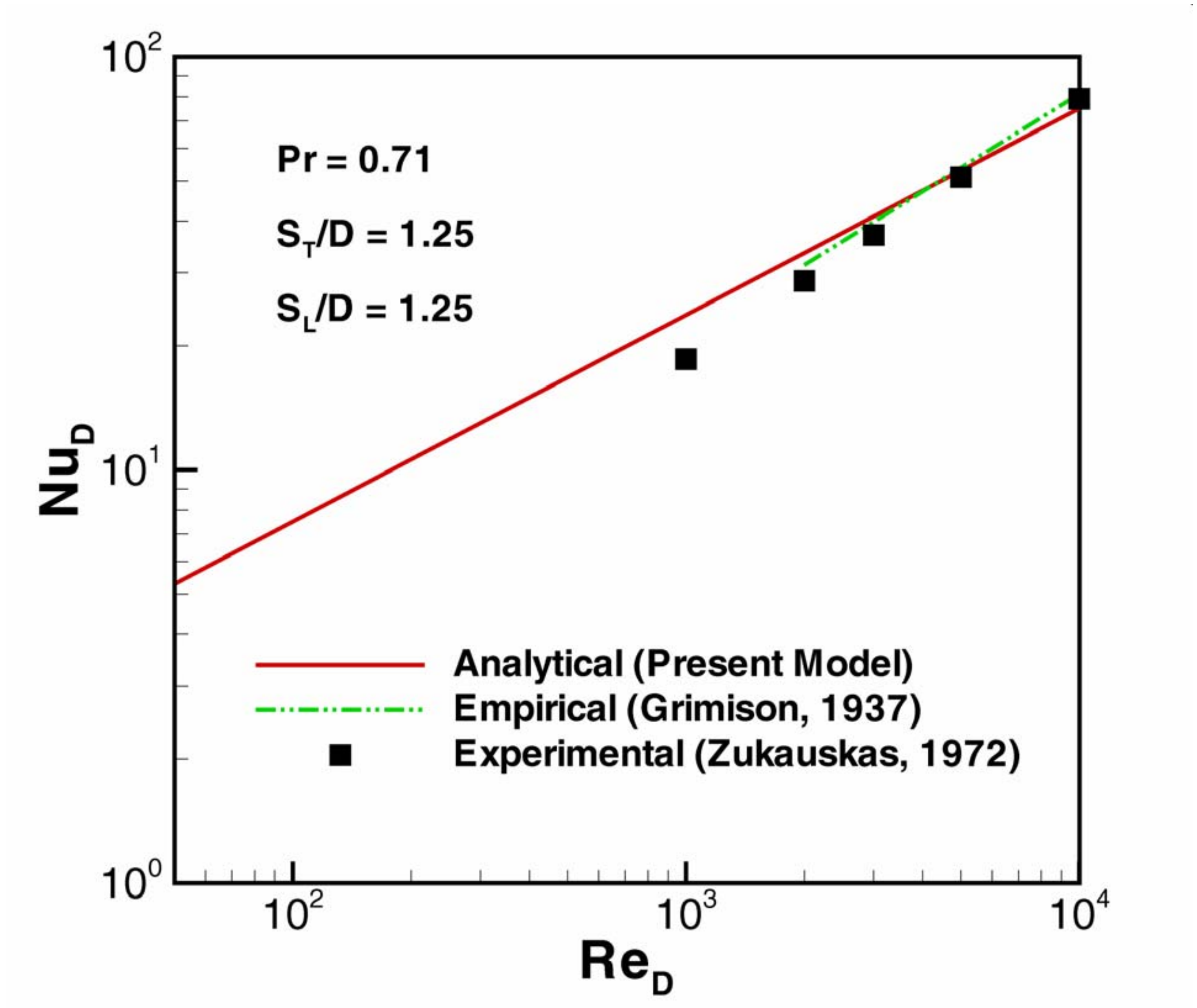
Nu_D vs. Re_D (In-Line Arrangement)



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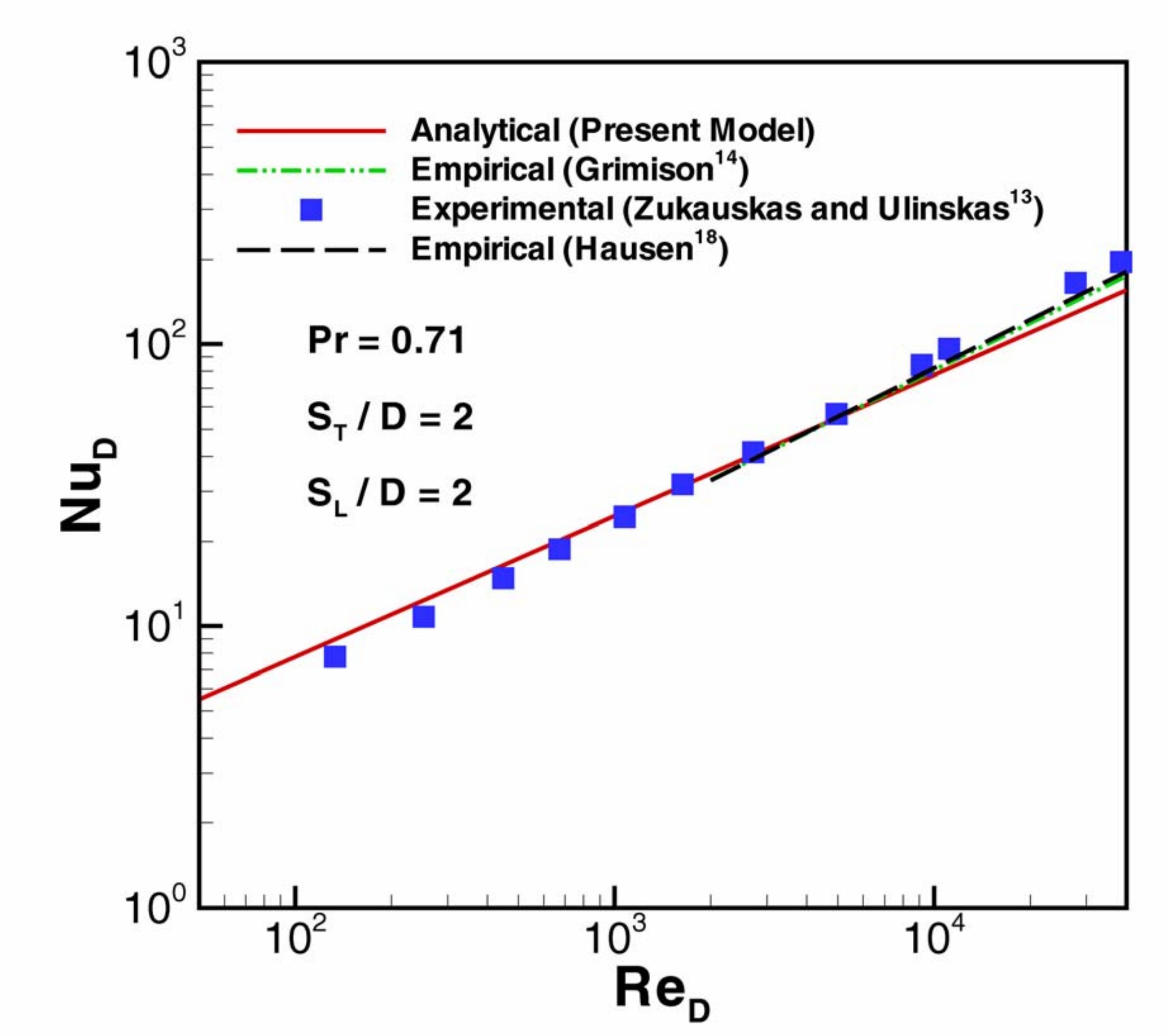
Nu_D vs. Re_D (Staggered Arrangement)



Summary and Conclusions

Comparisons show that higher heat transfer rates are obtained from:

- compact tube banks (any arrangement)
- staggered arrangement (any spacing)

Both In-Line and Staggered models are:

- applicable over a wide range of parameters
- suitable for use in design of tube banks

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