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Micro and Macro Hardness Measurements, Correlations, and Contact Models

M. M. Yovanovich, Fellow AIAA

Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo

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Outline of Presentation

- ✓ Introduction
- ✓ Review of Contact, Gap, Radiation, and Joint Conductances
- ✓ Contact Conductance Model (Cooper, Mikic and Yovanovich, 1969)
- ✓ Gap Conductance Model (DeVaal and Yovanovich, 1982)
- ✓ Micro and Macrohardness Indenters, Tests and Correlations
- ✓ Correlation Equations for Vickers Microhardness

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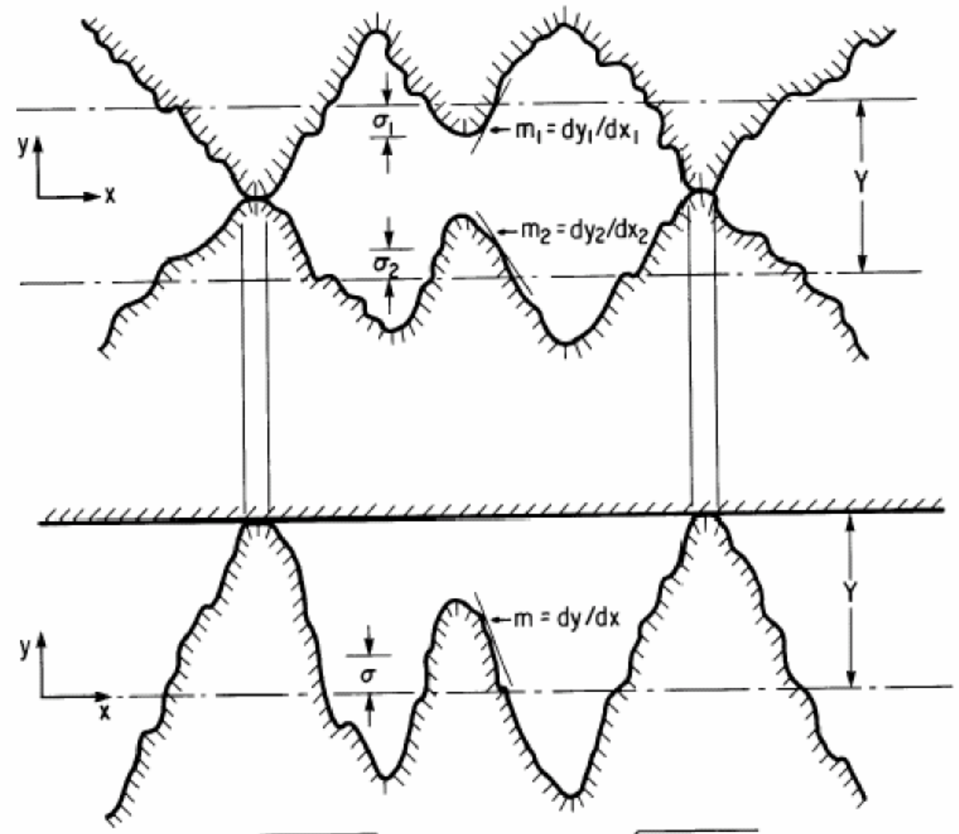
Outline of Presentation

- ✓ Iterative Model for Calculation of Contact Microhardness
- ✓ Direct Approximate Model (DAM) for Calculation of Contact Microhardness
- ✓ Temperature Effects on Yield Strength, Brinell and Vickers Hardness
- ✓ Nanoindentation Indenter and Tests
- ✓ Summary and Conclusions
- ✓ Acknowledgments

Typical Joints Between Conforming Rough Surfaces



$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \quad , \quad m = \sqrt{m_1^2 + m_2^2}$$

Contact Conductance Model: Conforming Rough Surfaces

$$h_c = \frac{2nak_s}{\psi(\epsilon)}$$

$$k_s = \frac{2k_1k_2}{k_1 + k_2}$$

$$\psi(\epsilon) = (1 - \epsilon)^{1.5}$$

with $$\epsilon = \sqrt{A_r/A_a}$$

Contact Conductance Model: Conforming Rough Surfaces

$$na = \frac{1}{4\sqrt{2\pi}}\left(\frac{m}{\sigma}\right)\exp\left(-\frac{\lambda^2}{2}\right)$$

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \qquad m = \sqrt{m_1^2 + m_2^2}$$

$$\lambda = \frac{Y}{\sigma} \qquad \frac{P}{H_c} = \frac{A_r}{A_a} = \frac{1}{2}\mathrm{erfc}\left(\frac{\lambda}{\sqrt{2}}\right)$$

Contact Conductance Model: Conforming Rough Surfaces

$$\lambda = \sqrt{2}\,\mathrm{erfc}^{-1}\left(\frac{2P}{H_c}\right)$$

$$C_c = \frac{\sigma}{m}\frac{h_c}{k_s} = f(\lambda) = \frac{1}{4\sqrt{2\pi}}\frac{\exp(-\lambda^2/2)}{\left[1-\sqrt{\frac{1}{2}\mathrm{erfc}(\lambda/\sqrt{2})}\right]^{1.5}}$$

$$C_c = 1.25\left(\frac{P}{H_c}\right)^{0.95}$$

Gap Conductance Model: Conforming Rough Surfaces

$$h_g = \frac{k_g}{\sigma} \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{\exp\left[-(\lambda - u)^2/2\right]}{u + M/\sigma} du = \frac{k_g}{\sigma} I_g$$

$$M = \alpha\beta\Lambda$$

$$\begin{aligned} \alpha &= \frac{2-\alpha_1}{\alpha_1} + \frac{2-\alpha_2}{\alpha_2} \\ \beta &= \frac{2\gamma}{(\gamma+1)Pr} \\ \Lambda &= \Lambda_0 \left(\frac{T_g}{T_{g,0}}\right)\left(\frac{P_{g,0}}{P_g}\right) \end{aligned}$$

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Gap Conductance Model: Conforming Rough Surfaces

$$I_g = \frac{f_g}{\lambda + M/\sigma}$$

In the range $2 \le \lambda \le 4$:

$$f_g = 1.063 + 0.0471\,(4-\lambda)^{1.68}\,[\ln(\sigma/M)]^{0.84} \quad \text{for} \quad 0.01 \le M/\sigma \le 1$$

$$f_g = 1 + 0.06\,(\sigma/M)^{0.8} \quad \text{for} \quad 1 \le M/\sigma < \infty$$

Relative Mean Planes Separation

Yovanovich (1981)

$$\lambda = 1.184 \left[-\ln \left(3.132 \, \frac{P}{H_c} \right) \right]^{0.547}$$

Song-Yovanovich (1988)

$$\lambda = 1.363 \left[-\ln \left(5.589 \, \frac{P}{H_c} \right) \right]^{0.5}$$

Relative Mean Planes Separation

Ranges of applications are

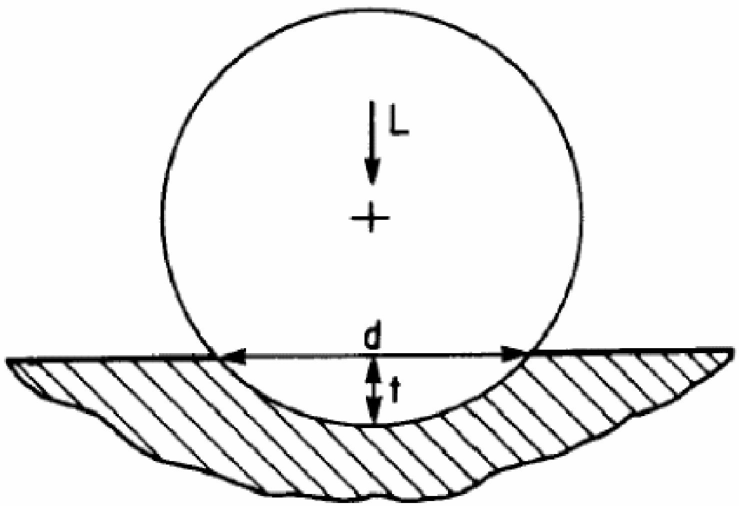
$$2 \leq \lambda \leq 4.75$$

$$10^{-6} \leq \frac{P}{H_c} \leq 2 \times 10^{-2}$$

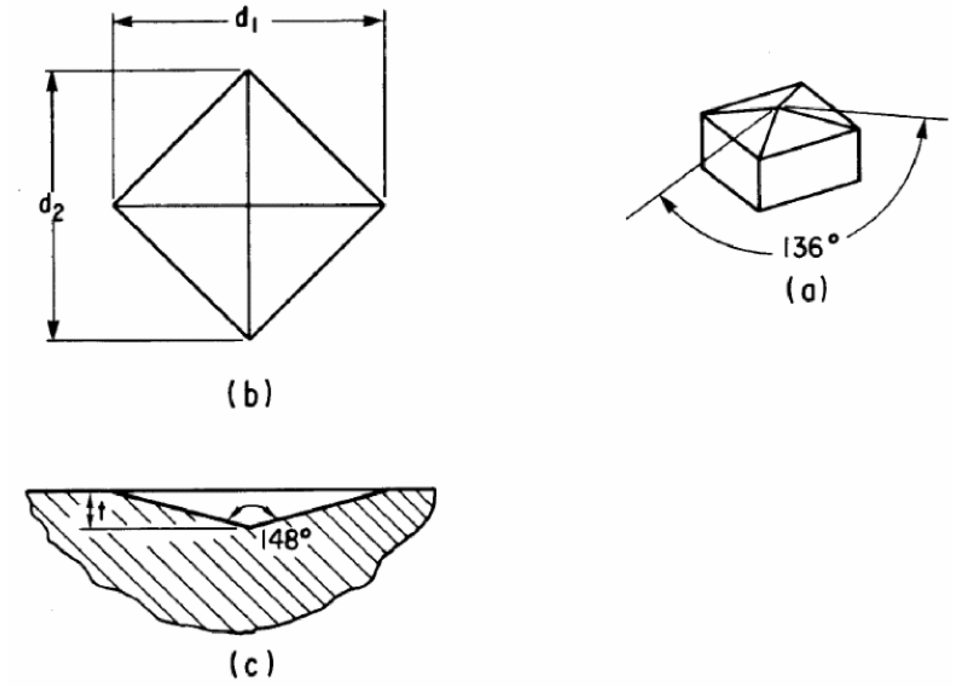
Antonetti Power-Law (1983)

$$\lambda = 1.53 \left(\frac{P}{H_c} \right)^{-0.097}$$

Vickers and Brinell Indenters for Microhardness



$$BH = \frac{2L}{\pi D\left(D - \sqrt{D^2 - d^2}\right)} = \frac{L}{\pi D t}$$



$d_v = (d_1 + d_2)/2$, $d_v/t = 7$

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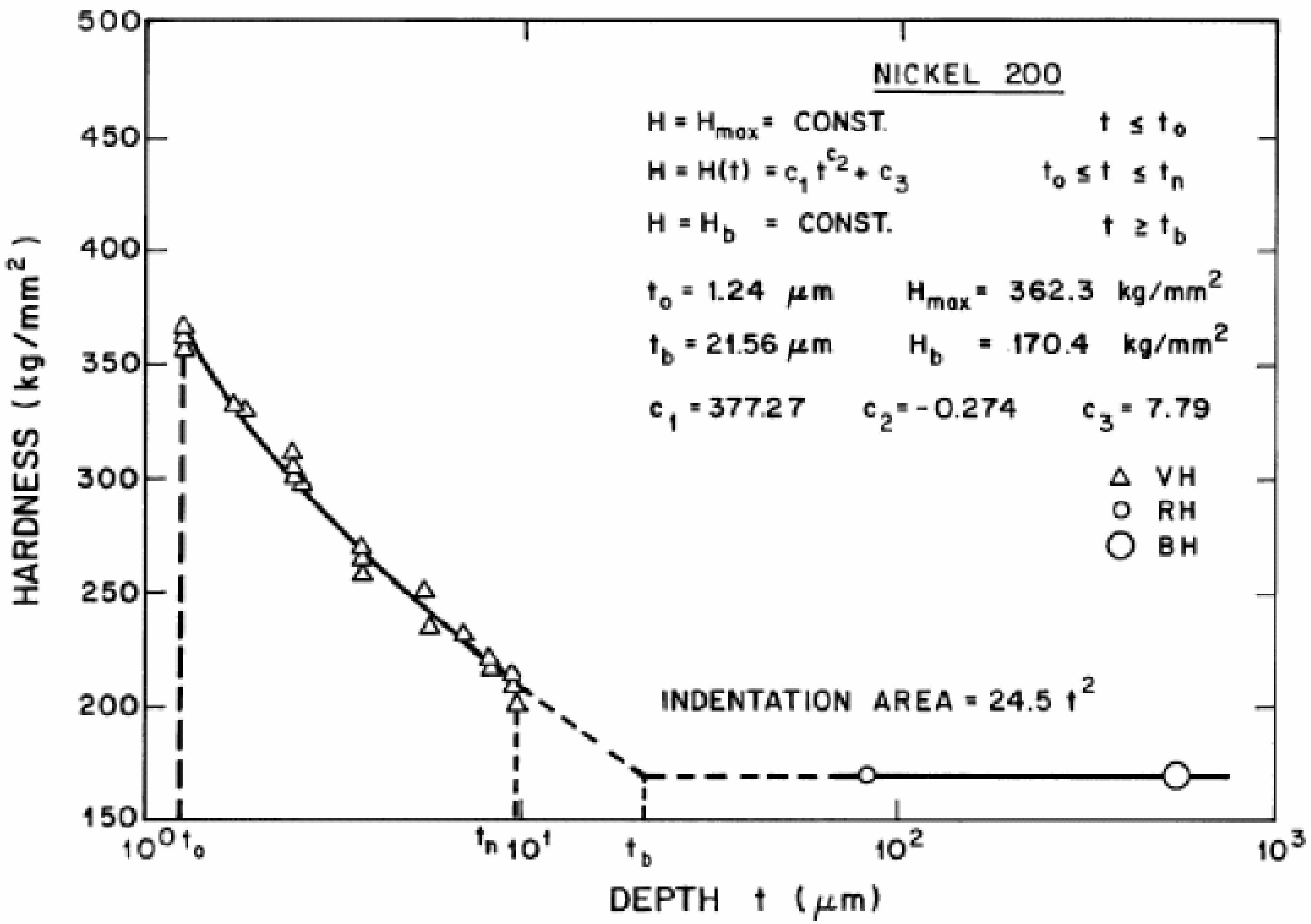
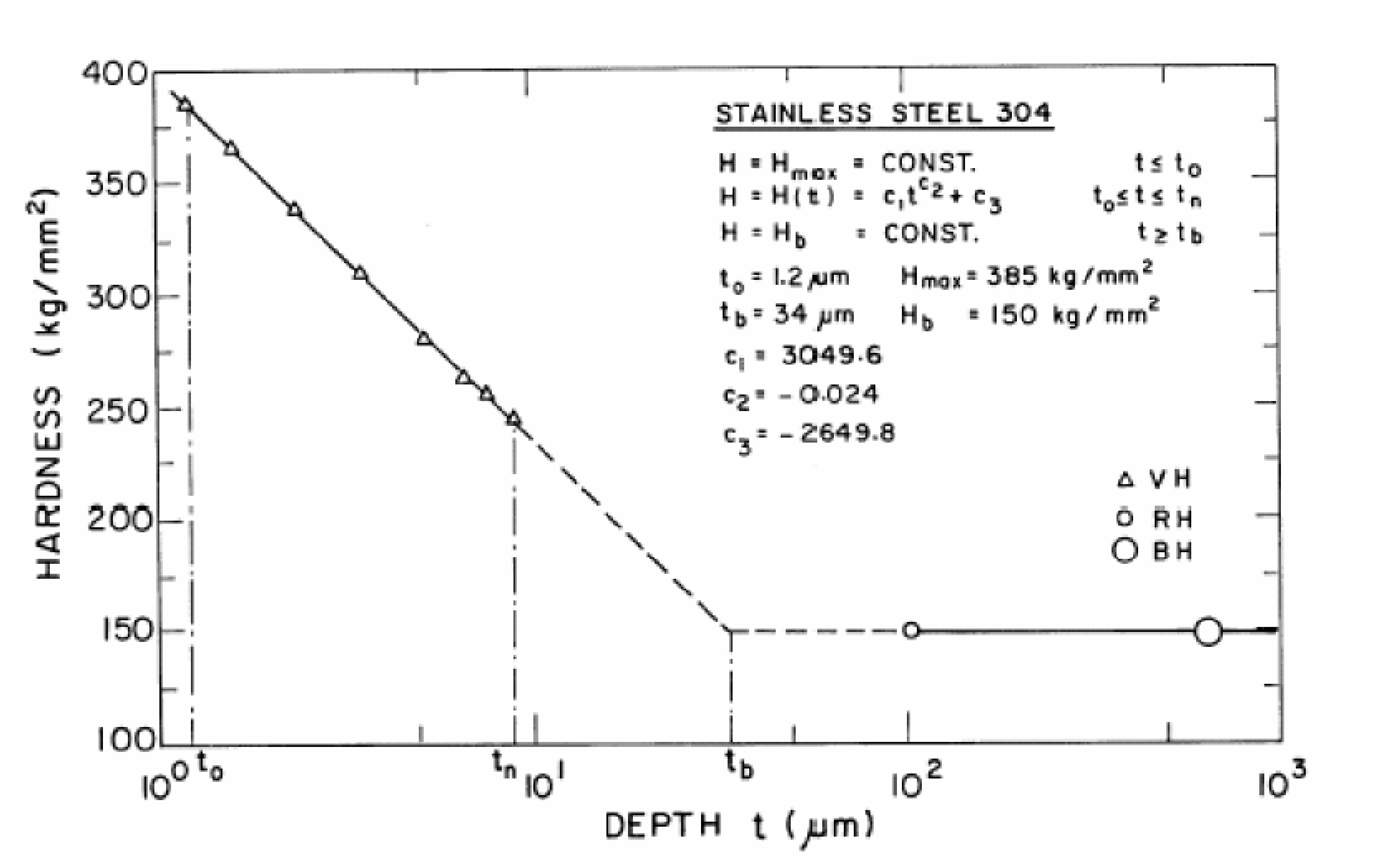
Micro and Macrohardness Test Results



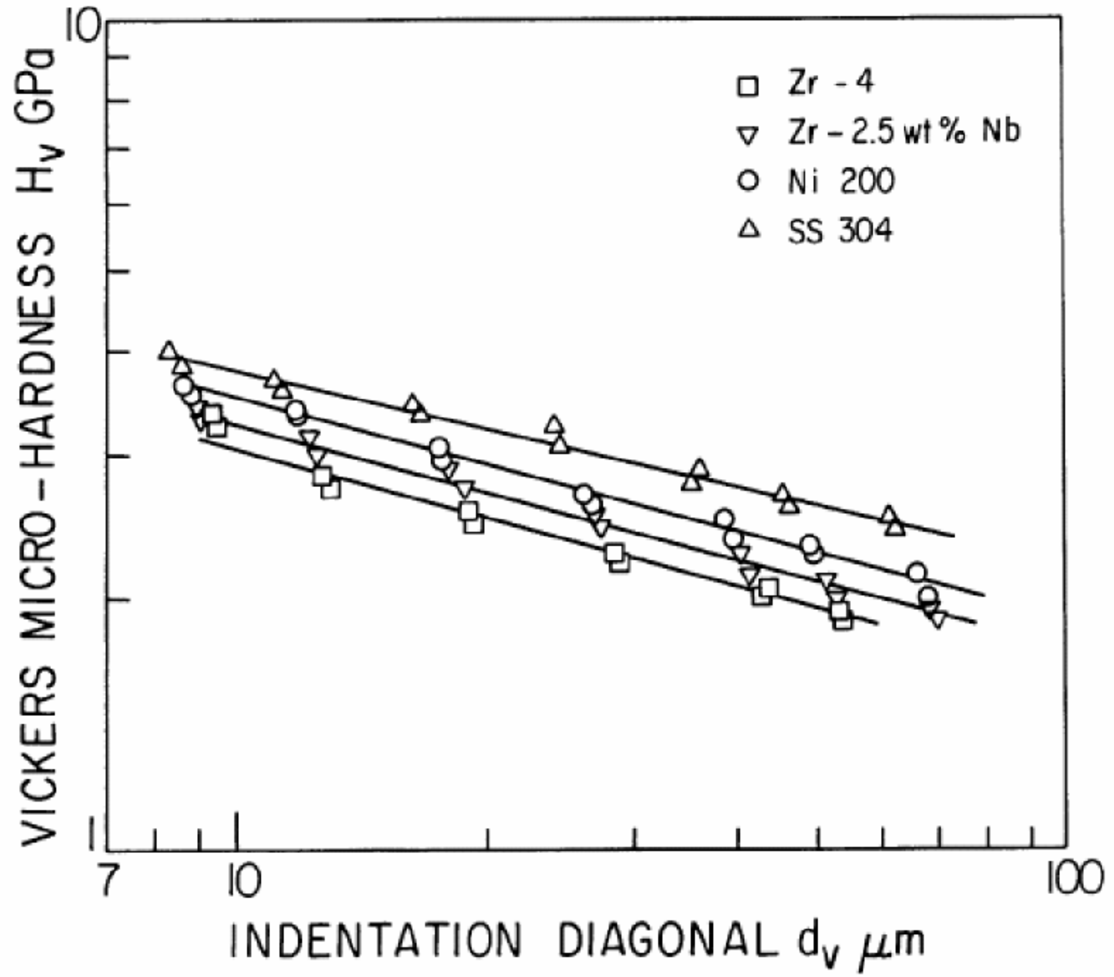
Micro and Macrohardness Test Results



Vickers Microhardness Data and Correlation

$$H_V = c_1 \left(\frac{d_V}{d_0} \right)^{c_2}$$

$$\text{with} \quad d_0 = 1\,\mu\text{m}$$



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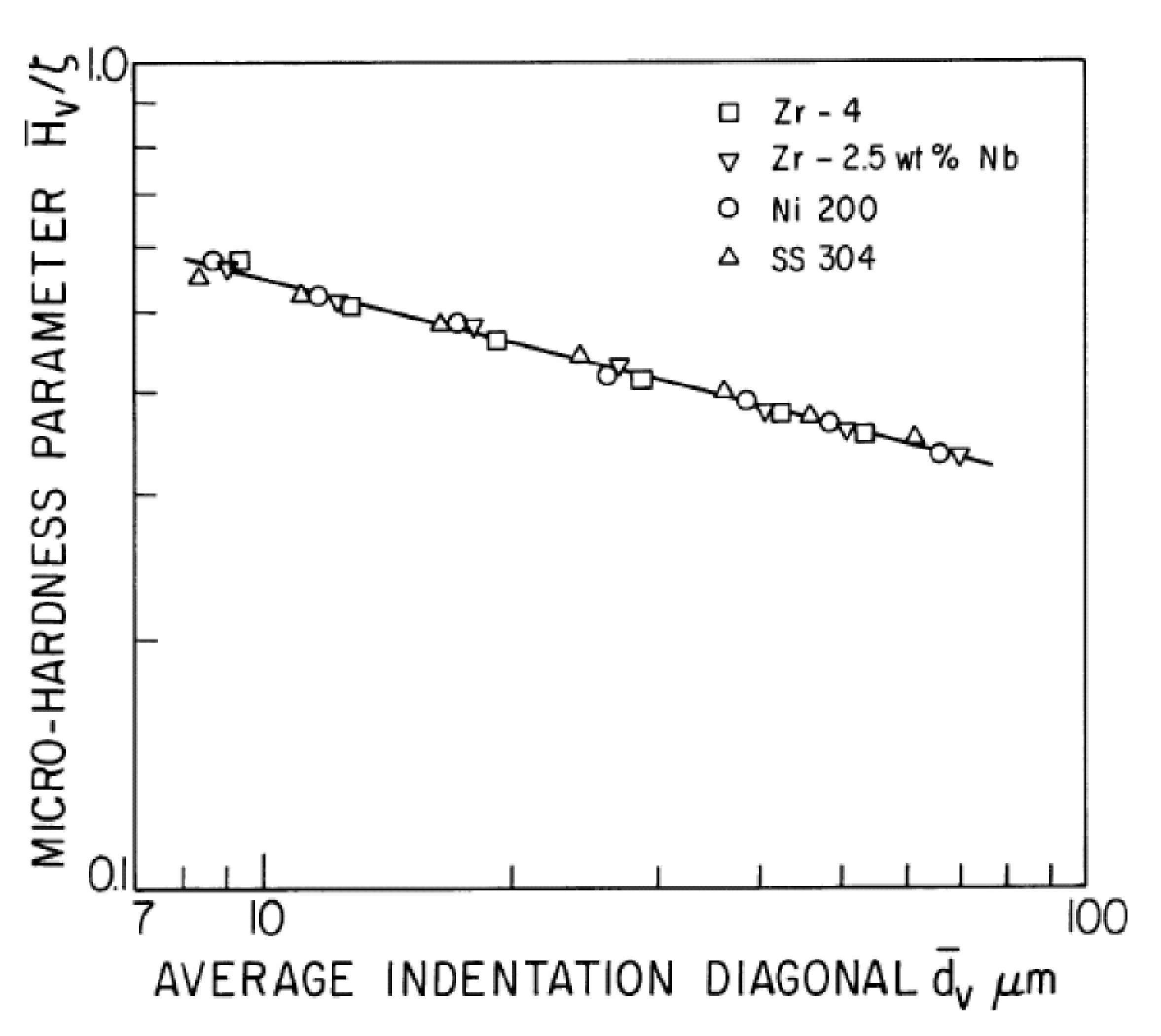

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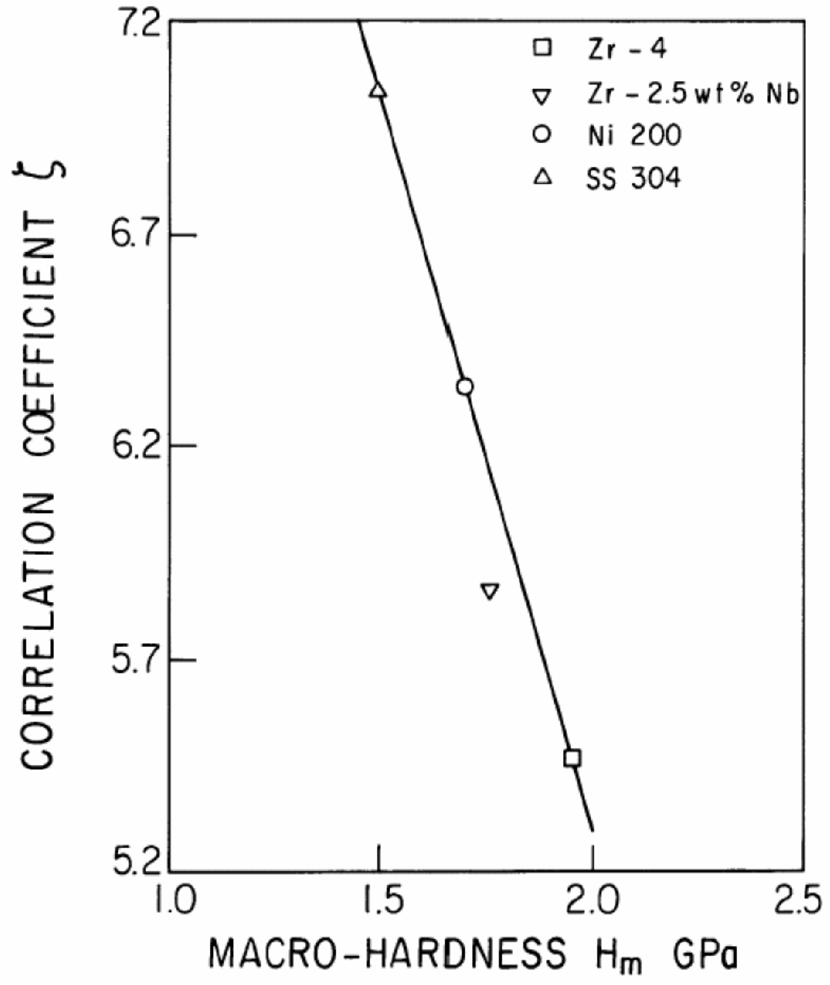

Vickers Microhardness Correlations

$$H_V = c_1 \left(\frac{d_V}{d_0} \right)^{c_2} \qquad \text{with} \quad d_0 = 1\,\mu\text{m}$$

Metal	H_m(GPa)	c_1(GPa)	c_2	Max % Diff.	RMS % Diff.
Ni 200	1.668	6.304	−0.264	4.8	1.8
SS 304	1.472	6.271	−0.229	4.2	1.4
Zr-4	1.913	5.677	−0.278	3.4	1.7
Zr-Nb	1.727	5.884	−0.267	10.2	2.7

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Vickers Microhardness Correlations

Metal	H_m(GPa)	ξ(GPa)	η	Max % Diff.	RMS % Diff.
Ni 200	1.668	6.217	−0.260	5.2	1.8
SS 304	1.472	6.906	−0.260	5.9	2.4
Zr-4	1.913	5.367	−0.260	3.9	1.8
Zr-Nb	1.727	5.750	−0.260	9.7	2.7

Vickers Microhardness Correlations

$$H_V = (12.04 - 3.49\, H_m) \left(\frac{d_V}{d_0} \right)^{-0.260} \qquad \text{GPa}$$

Metal	H_m(GPa)	ξ(GPa)	Corr. Eq.	% Diff.
Ni 200	1.668	6.217	6.219	0.03
SS 304	1.472	6.906	6.903	−0.04
Zr-4	1.913	5.367	5.364	−0.06
Zr-Nb	1.727	5.750	6.013	4.60

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Iterative Model for Contact Microhardness

Given parameters: $(\sigma/m, c_1, c_2, P, H_b)$

$$
\begin{aligned}
(i) \quad & \lambda &=& \quad \sqrt{2}\,\mathrm{erfc}^{-1}\left(\frac{2P}{H_c}\right) \\
(ii) \quad & a &=& \quad \left(\frac{\sigma}{m}\right)\sqrt{\frac{8}{\pi}}\exp\left(\frac{\lambda^2}{2}\right)\mathrm{erfc}\left(\frac{\lambda}{\sqrt{2}}\right) \\
(iii) \quad & d_V &=& \quad \sqrt{2\pi}\,a \\
(iv) \quad & H_c &=& \quad c_1\left(\frac{d_V}{d_0}\right)^{c_2}
\end{aligned}
$$

2 to 3 iterations are required for convergence.

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Direct Approximate Method (DAM)

$$A_V = d_V^2/2 = 49t^2/2$$

$$A_c = \pi a^2$$

$$t = \left(\frac{\pi a^2}{24.5}\right)^{1/2} = 0.358\,a$$

$$H_c = c_1\,(0.358\,a)^{c_2} + c_3$$

$$a = 0.99\left(\frac{\sigma}{m}\right)\left[-\ln\left(3.132\,\frac{P}{H}\right)\right]^{-0.547}$$

Direct Approximate Method (DAM)

$$H = 501.3\left(\frac{m}{\sigma}\right)^{0.274}\left[-\ln\left(3.132\,\frac{P}{H}\right)\right]^{0.150}$$

$$H_c = 501.3\left(\frac{m}{\sigma}\right)^{0.274}\left[-\ln\left(3.132\,\frac{P}{H_b}\right)\right]^{0.150}$$

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Temperature Effects on Yield Strength and Vickers Hardness

$T(°\mathrm{C})$	$S_y(\mathrm{MPa})$	$S_y(T)/S_y(25)$	$H_V(\mathrm{MPa})$	$H_V(T)/H_V(25)$
25	274	1.00	1570	1.00
200	223	0.81	1180	0.75
400	198	0.72	1090	0.69
600	157	0.57	863	0.55
800	78.9	0.29	392	0.25

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Temperature Effects on Yield Strength and Brinell Hardness (Ni 200)

T(°C)	d(mm)	H_B(MPa)	$H_B(T)/H_B(23.1)$
23.1	3.51	1472	1.00
58.5	3.62	1381	0.94
95.1	3.70	1320	0.90
121.8	3.80	1249	0.85
159.0	3.91	1177	0.80
209.3	4.17	1028	0.70

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Temperature Effects on Yield Strength and Brinell Hardness (SS 304)

$T(°C)$	$d(mm)$	$H_B(MPa)$	$H_B(T)/H_B(23.8)$
23.8	3.68	1335	1.00
55.6	3.79	1256	0.94
89.9	3.89	1189	0.89
123.0	4.00	1122	0.84
152.4	4.07	1082	0.81
186.1	4.15	1039	0.78

Temperature Effects on Brinell Hardness (Al 6061 T-6)

$T(°C)$	d(mm)	H_B(MPa)	$H_B(T)/H_B(22.3)$
22.3	2.59	915	1.00
48.9	2.61	901	0.98
71.1	2.64	880	0.96
102.3	2.68	854	0.93
124.3	2.72	828	0.91
140.8	2.77	798	0.87
164.3	2.81	775	0.85
195.6	2.86	748	0.82

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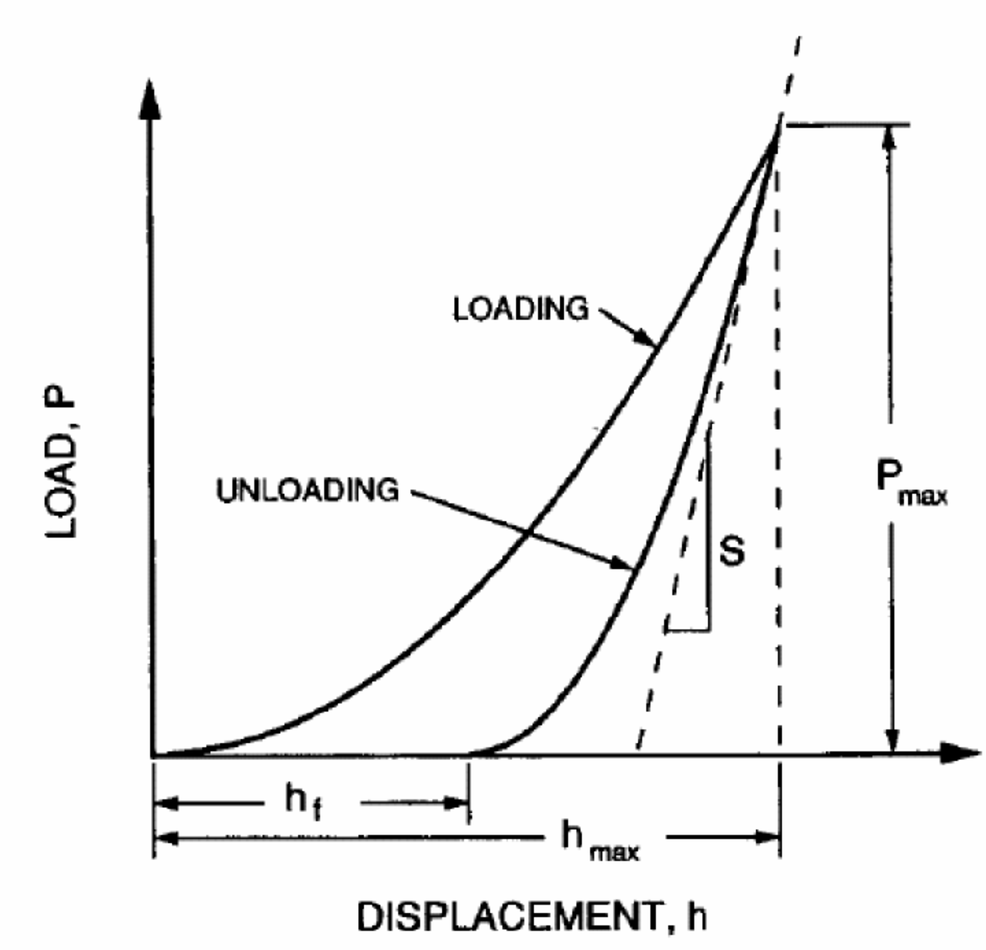
Temperature Effects on Vickers Microhardness Coefficient

$$C_T = -1.20 \times 10^{-3}\,\mathrm{C}^{-1}$$

$$\frac{c_1(T)}{c_1(T_{rm})} = \exp\left[C_T\left(T - T_{rm}\right)\right] \qquad 25 \leq T \leq T_{max}$$

Metal	$c_1(T_{rm})$(MPa)	$10^3 C_T(\mathrm{C}^{-1})$	T_{max}(°C)
Ni 200	6636	–1.372	186
SS 304	7339	–1.675	190
Al 6061 T-6	1123	–1.190	183

Nanoindentation Loading and Unloading Tests



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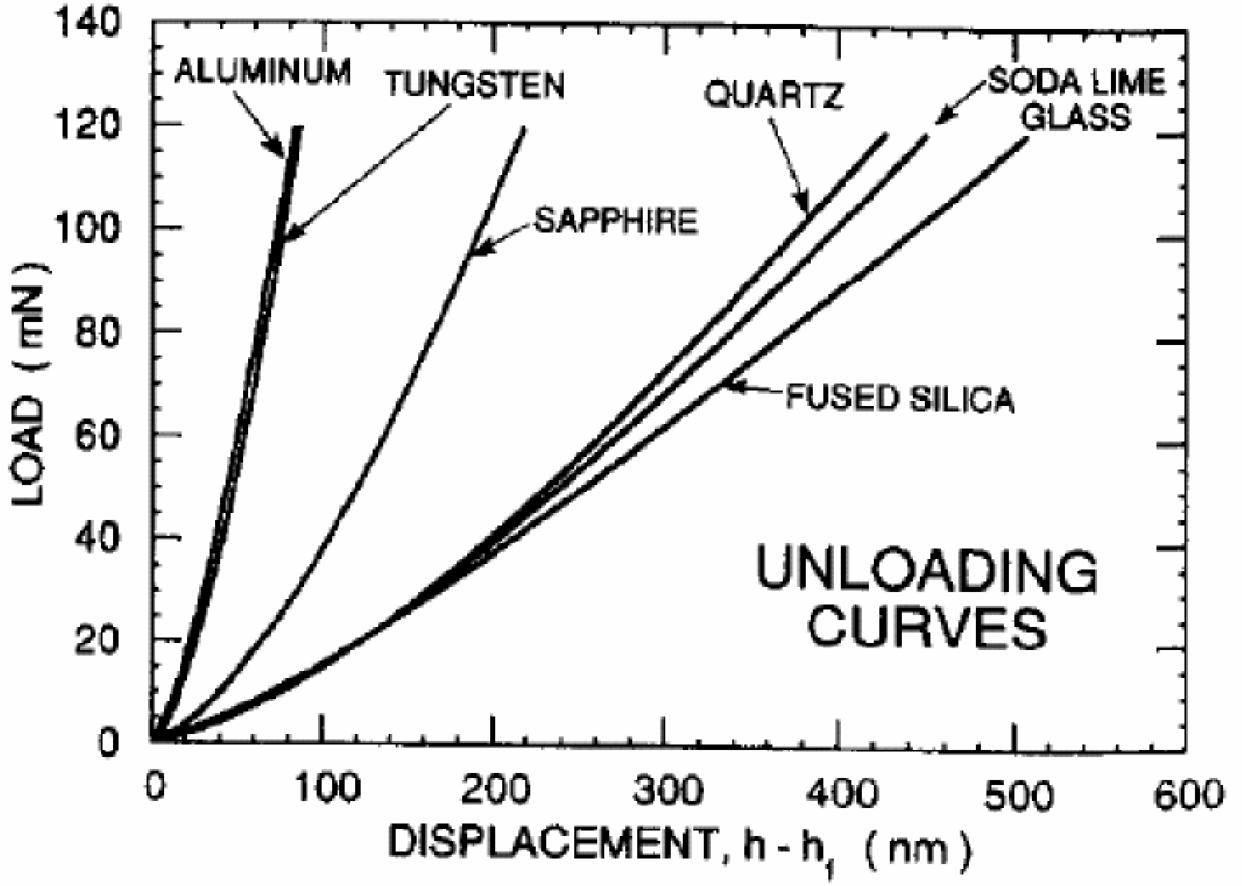
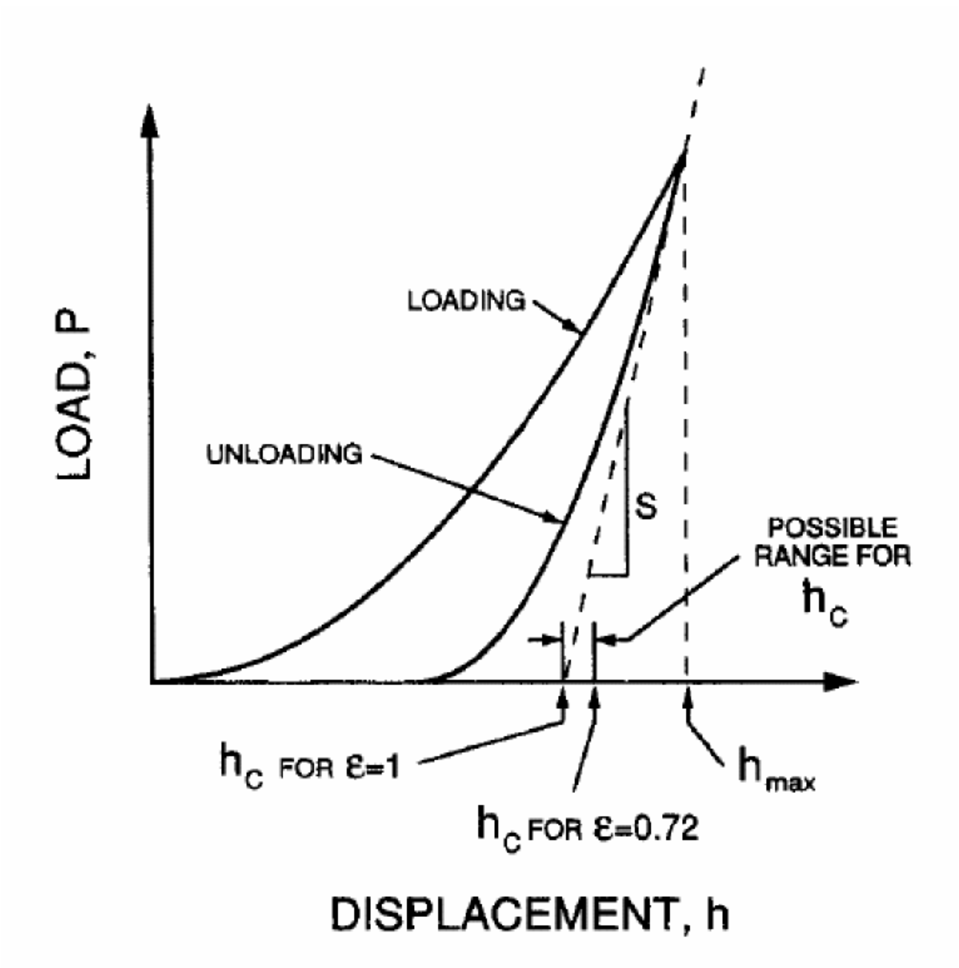
Nanoindentation Loading and Unloading Tests



Nanoindentation Loading and Unloading Tests



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Nanohardness and Elastic Modulus

$$H = \frac{P}{A}$$

$$E_r = \frac{\sqrt{\pi}}{2\beta} \frac{S}{\sqrt{A}}$$

$$\frac{1}{E_r} = \frac{1 - \nu^2}{E} + \frac{1 - \nu_i^2}{E_i}$$

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Curve Fit of Unloading Curve and Contact Stiffness

$$P = B(h - h_f)^m$$

$$S = \left(\frac{dP}{dh}\right)_{h=h_{max}} = Bm(h_{max} - h_f)^{m-1}$$

$$h_c = h_{max} - \epsilon \frac{P_{max}}{S}$$

$$A = 24.5h_c^2 + \sum_{i=1}^{n} C_i \, h_c^{1/2^i}$$

Nanoindentation Elastic Modulus and Microhardness of Tungsten

Load(mN)	Hardness(GPa)	Load(mN)	Modulus(GPa)
0.512	5.86	0.499	372.0
1.49	5.67	1.48	495.0
4.53	5.21	4.49	427.0
13.2	4.55	13.6	403.0
40.9	3.88	41.1	401.0
119	3.75	120	400.0

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Nanoindentation Elastic Modulus and Microhardness of Aluminum

F(mN)	H(GPa)	ratio	F(mN)	E(GPa)	ratio
0.504	0.271	0.886	0.504	77.7	0.970
1.50	0.306	1.000	1.53	80.1	1.000
4.53	0.295	0.964	4.62	74.7	0.933
13.5	0.258	0.843	14.0	72.1	0.900
41.0	0.231	0.755	40.7	71.9	0.898
119	0.202	0.661	118	70.9	0.885

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Summary and Conclusions

- Brief review of macro, micro, and nanohardness was presented.

- Methods to calculate contact microhardness from indentation data are given.

- Vickers microhardness correlation coefficients are related to Brinell hardness.

Summary and Conclusions (Contd.)

• Effect of temperature level on yield strength, Brinell and Vickers hardness is significant.

• Review of nanohardness indentation tests and unloading force-displacement data reduction for elastic modulus and nanohardness was given.

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