APPROXIMATE SOLUTION FOR PRESSURE DROP IN MICROCHANNELS OF ARBITRARY CROSS-SECTIONS

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Flow in Microchannels
motivations and objectives

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• Characteristic Length Scales
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Flow in Microchannels

motivations and objectives

Applications:

- Microelectronics cooling and high capacity heat exchangers
- Fuel cell technologies
- Biomedical devices

Features:

- High surface area to volume ratio
- High heat transfer coefficient (low film resistance heatsinks)
- Small size, compact heat exchangers
Microchannel heatsinks

ideas and issues

From K. E. Goodson (2002)

Close up of 60 µm microchannel
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Characteristics Length

$D_h$: hydraulics diameter

Analytical solutions for elliptical and rectangular channels where $D_h$ is used as length scale
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Navier-Stokes equations reduce to the Poisson equation:

\[ \nabla^2 w = \frac{1}{\mu} \frac{dP}{dz} \quad \text{with} \quad w = 0 \quad \text{on} \quad \Gamma \]

- Torsion in beams and fully developed, laminar flow in ducts are mathematically similar

- Saint-Venant (1880) found that the torsional rigidity of a singly-connected arbitrary cross-section shaft can be accurately approximated by using an equivalent elliptical cross-section

- Solution for the elliptical duct has a unique geometrical property

\[ f \frac{\text{Re}}{\sqrt{A}} = 32\pi^2 I_p^* \frac{\sqrt{A}}{P} \quad \text{where} \quad I_p^* = I_p / A^2 \quad \text{and} \quad I_p = \int_A \left( x^2 + y^2 \right) dA \]
Approximate Model

hyper-ellipse channels

\[ \left( \frac{x}{a} \right)^n + \left( \frac{y}{b} \right)^n = 1 \]

\[ \varepsilon = \frac{b}{a} \]

Model, \( n = 2 \)

- \( n = 3 \)
- \( n = 5.5 \)
- \( n = 10 \)
Comparison with Data

parallel plates microchannels

Gao et al. data (2002)
Rectangular channels dimensions
demineralized water
\( b = 25 \text{ mm} \quad c = 300 \mu \text{m} \)
Comparison with Data

trapezoidal microchannels

channel # N1-100 (trapezoidal cross-section)
channel material: silicon
de-ionized water

$a = 100 \, \mu m$  
$b = 20.10 \, \mu m$  
$h = 56.42 \, \mu m$

$\varepsilon = 1.064$  
$\beta = 0.557$

$f_{Re_{\sqrt{A}}} = \mu Re_{\sqrt{A}} \sqrt{A}$

model ± 10%
Comparison with Data

rectangular microchannels

\[
\frac{f \cdot \text{Re}_{\sqrt{A}}}{A} = 32 \pi^2 I_p^* \sqrt{A/P}
\]

\[
I_p^* = I_p / A^2
\]

\[\epsilon = c / b\]
Comparison with Data

triangular and trapezoidal microchannels

\[ f \text{Re}_{\sqrt{A}} = 32 \pi^2 I_p^* \sqrt{A/P} \]

\[ I_p^* = I_p / A^2 \]

Wu and Chang data (2003)

Isosceles triangular microchannels

Isosceles trapezoidal microchannels
Comparison with Numerical Data

sine duct

\[ y = b (1 + \cos \pi x / a) \]

Data from Shah and London
Comparison with Numerical Data

Data from Shah and London
Comparison with Numerical Data

circular sector

Data from Shah and London

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Comparison with Numerical Data

circular segment

Data from Shah and London
Comparison with Numerical Data

rhombic duct

Data from Shah and London
Comparison with Numerical Data
square duct with 2 adjacent round corners

Data from Oosthuizen (2005)
Comparison with Numerical Data

moon-shaped duct

Data from Shah and London
Comparison with Numerical Data

Rectangular duct with semi-circular ends

Data from Shah and London
A new compact analytical model is developed and validated with experimental and numerical data for a variety of microchannel cross-sections including:

- Rectangular
- Trapezoidal
- Isosceles triangular
- Square
- Circular
- Other cross-sections

Square root of area, as the characteristic length scale, is superior to the hydraulic diameter