APPROXIMATE SOLUTION FOR PRESSURE DROP IN MICROCHANNELS OF ARBITRARY CROSS-SECTIONS

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Flow in Microchannels

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motivations and objectives

- **Motivation and Objective**
- **Ideas and Issues**
- **Characteristic Length Scales**
- **Solution for Arbitrary Cross-Section Channels**
- **Comparisons with Experimental Data**
- **Comparisons with Numerical Data**
- **Summary and Conclusions**

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motivations and objectives

Applications:

- **Microelectronics cooling and high capacity heat exchangers**
- **Fuel cell technologies**
- **Biomedical devices**

Features:

- **High surface area to volume ratio**
- **High heat transfer coefficient (low film resistance heatsinks)**
- **Small size, compact heat exchangers**

Microchannel heatsinks

ideas and issues

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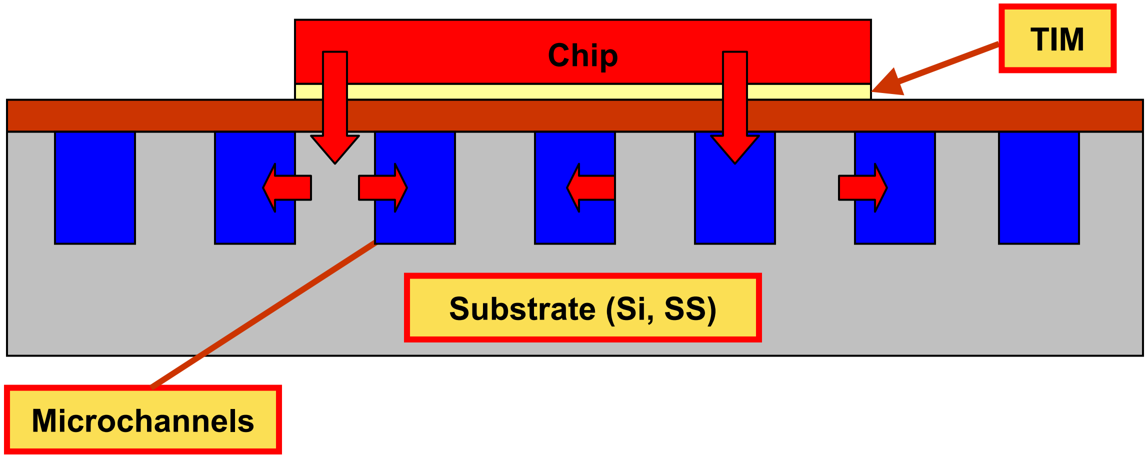
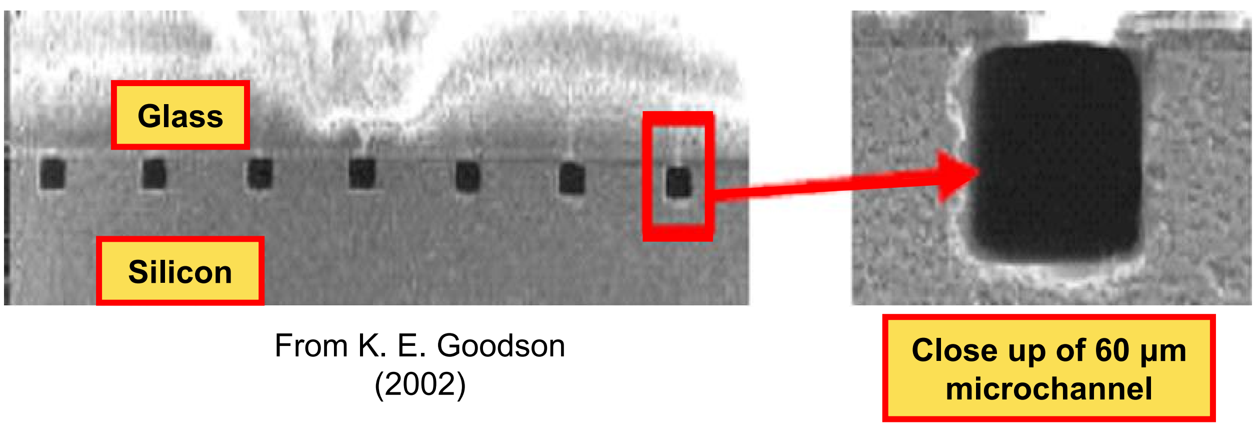
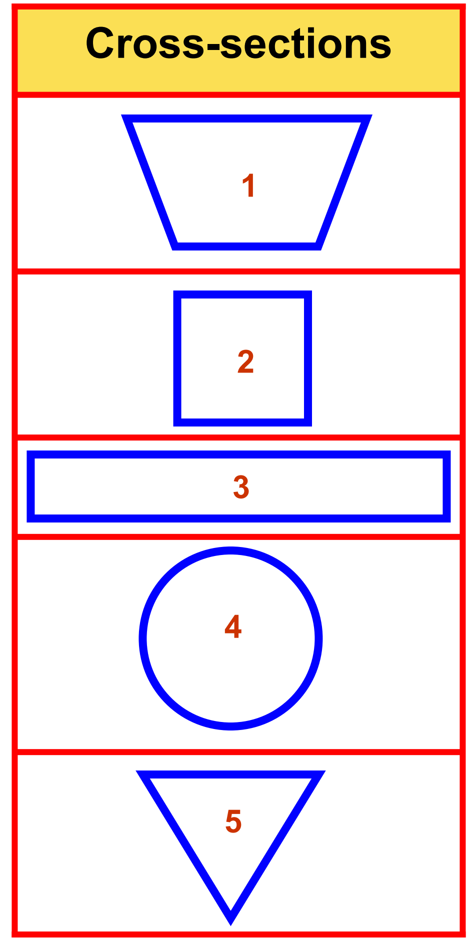




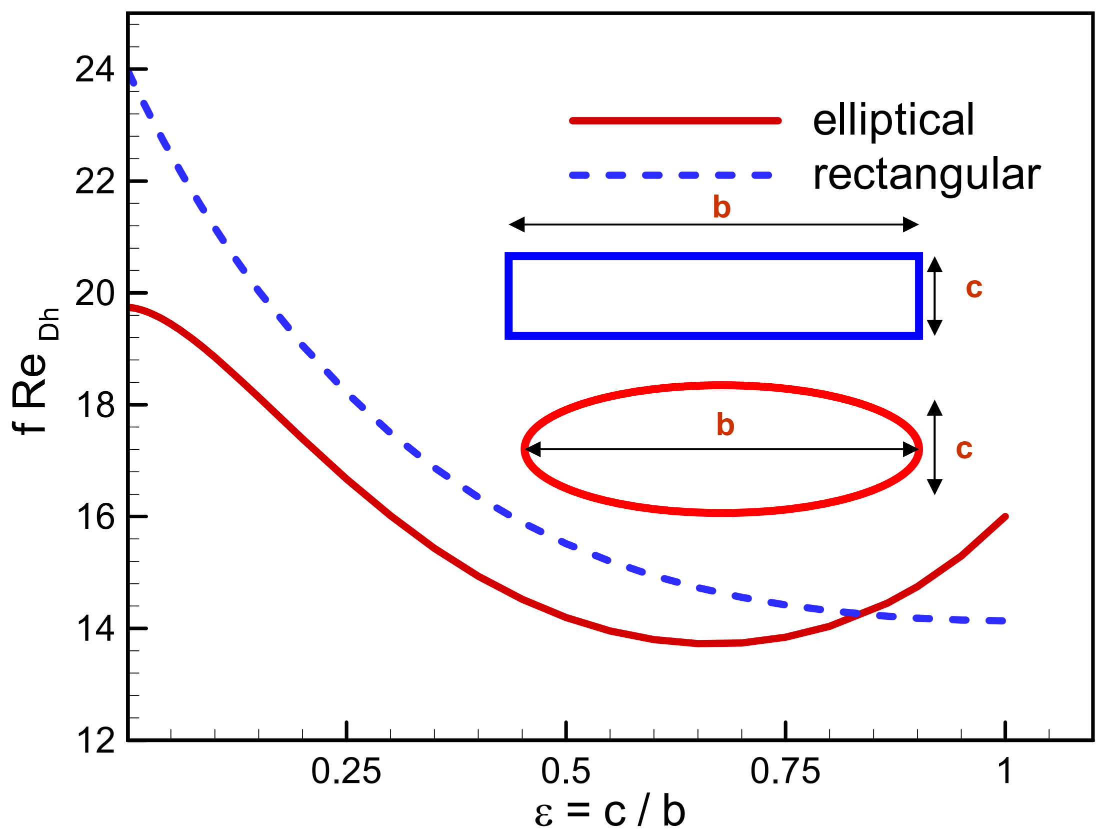
From K. E. Goodson (2002)



Characteristics Length

D_h: hydraulics diameter

Analytical solutions for elliptical and rectangular channels where D_h is used as length scale

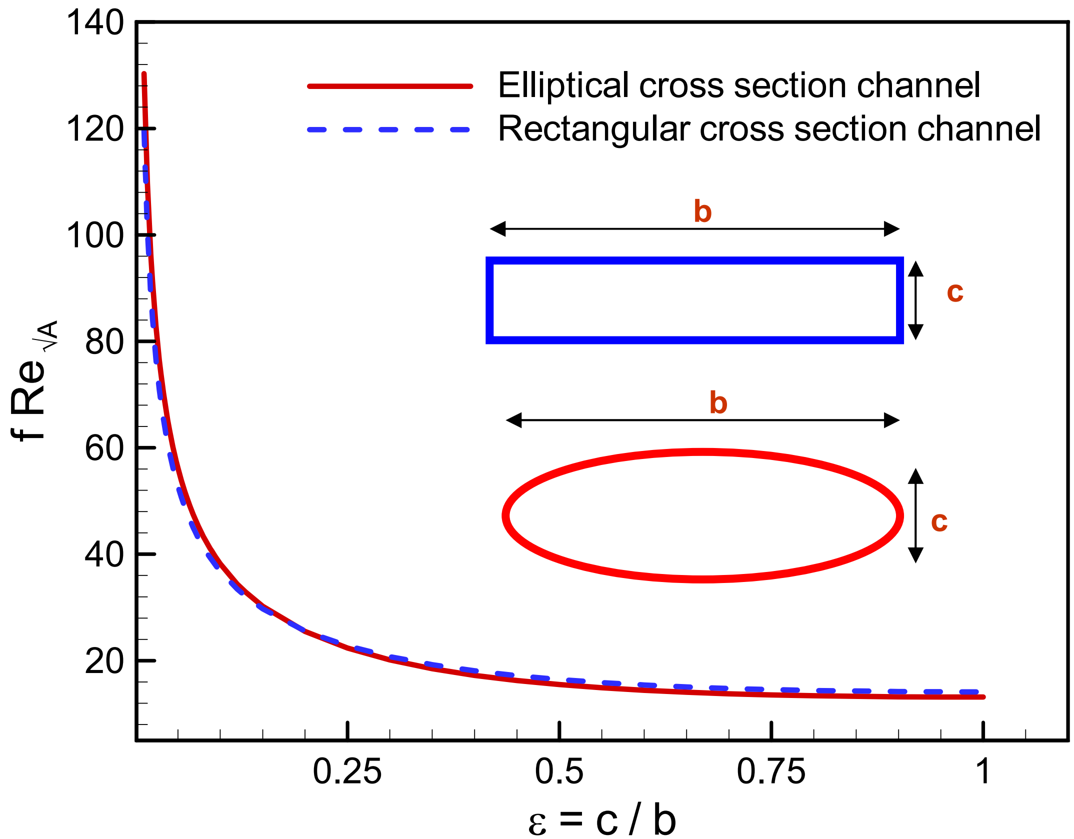


Approximate Solution for Pressure Drop in Microchannels of Arbitrary Cross-Sections
AIAA 2006-3119, June 5 -8, 2006, San Francisco, CA.

Characteristics Length

√A: a more appropriate length scale

Analytical solutions for elliptical and rectangular channels where √A is used as length scale



Microchannels: Pressure Drop

solution for arbitrary cross sections

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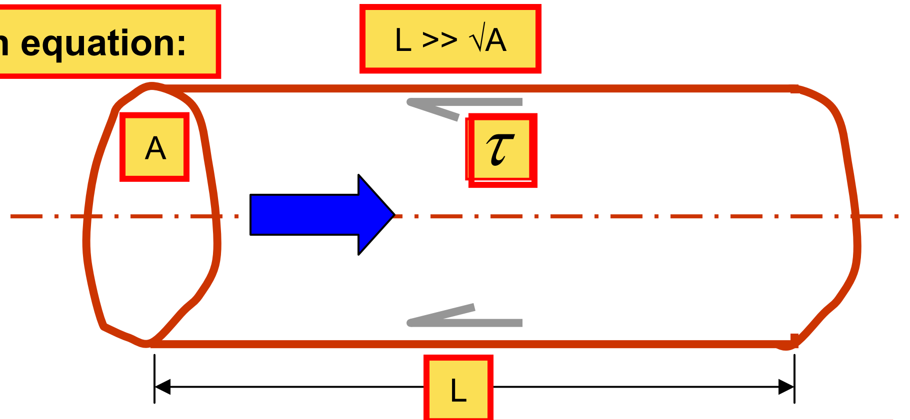
Navier-Stokes equations reduce to the Poisson equation:

$$\nabla^2 w = \frac{1}{\mu}\frac{dP}{dz} \qquad \text{with} \quad w = 0 \;\; \text{on} \;\; \Gamma$$

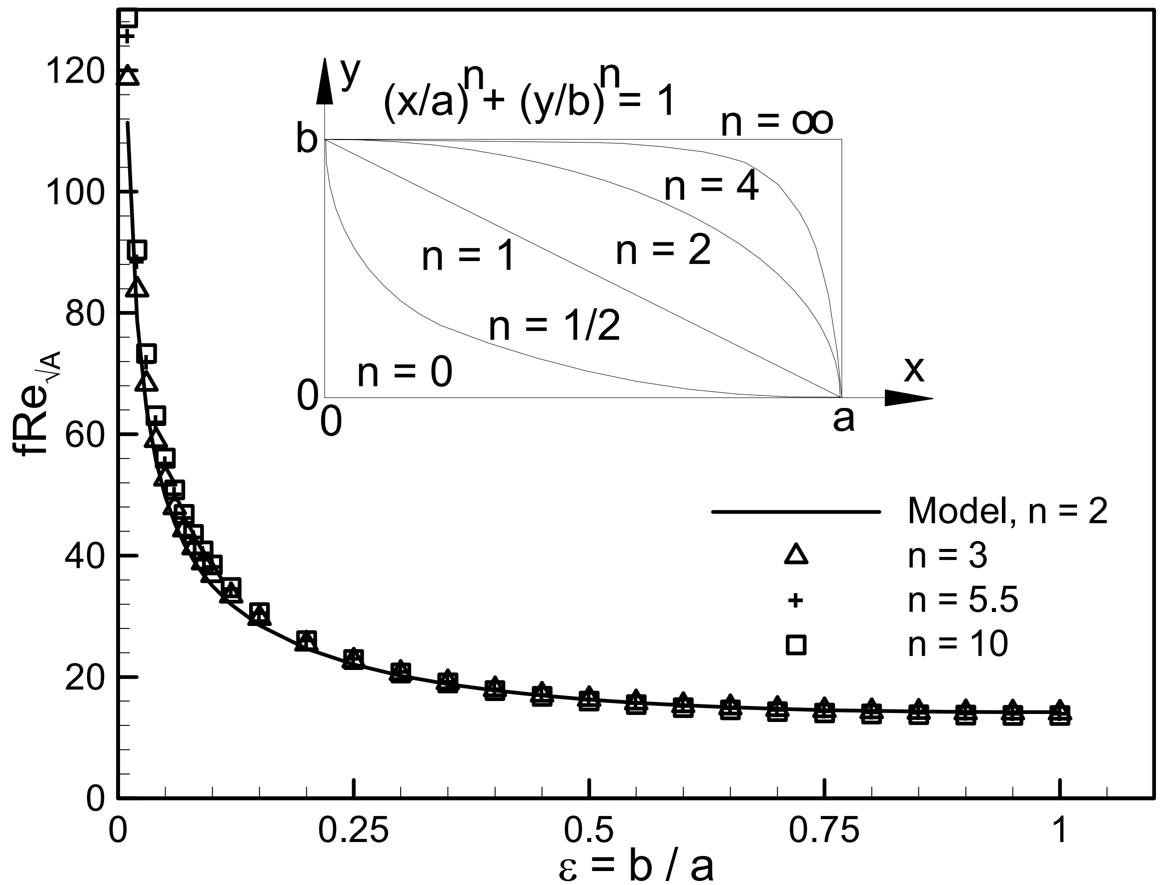


- **Torsion in beams and fully developed, laminar flow in ducts are mathematically similar**
- **Saint-Venant (1880) found that the torsional rigidity of a singly-connected arbitrary cross-section shaft can be accurately approximated by using an equivalent elliptical cross-section**
- **Solution for the elliptical duct has a unique geometrical property**

$$f\,\mathrm{Re}_{\sqrt{A}} = 32\pi^2 I_p^* \frac{\sqrt{A}}{P} \quad \text{where} \quad I_p^* = I_p / A^2 \quad \text{and} \quad I_p = \int_A \left(x^2 + y^2\right) dA$$

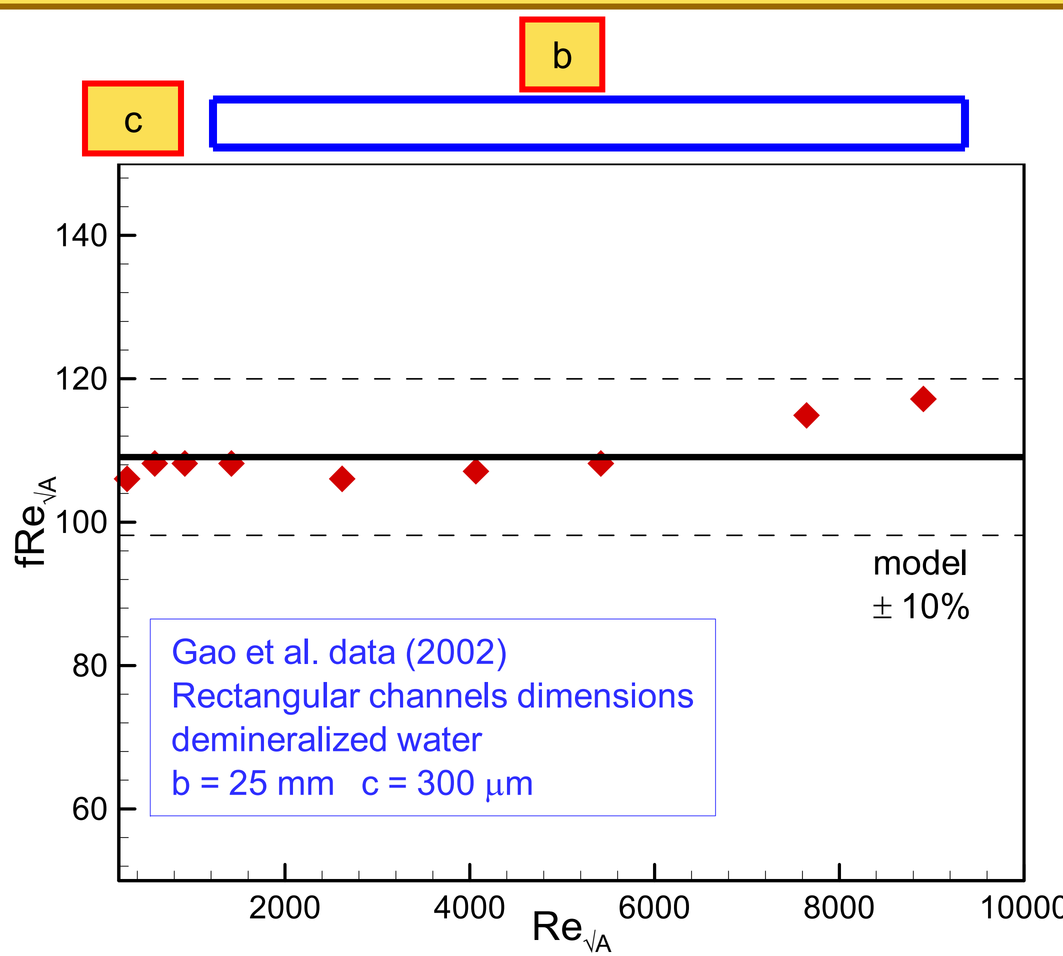
Approximate Model

hyper-ellipse channels



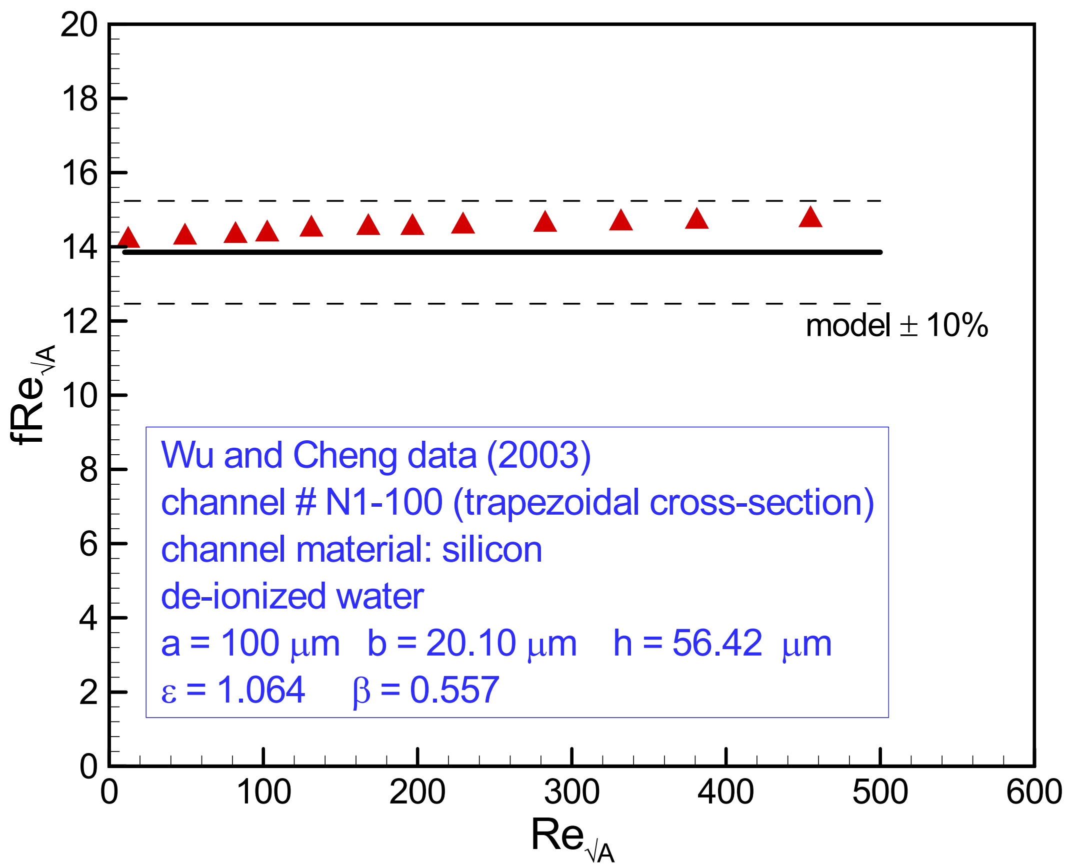
Comparison with Data

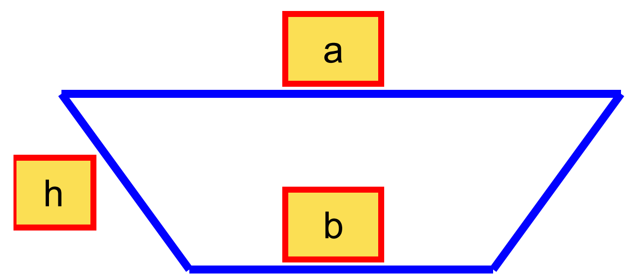
parallel plates microchannels



Comparison with Data

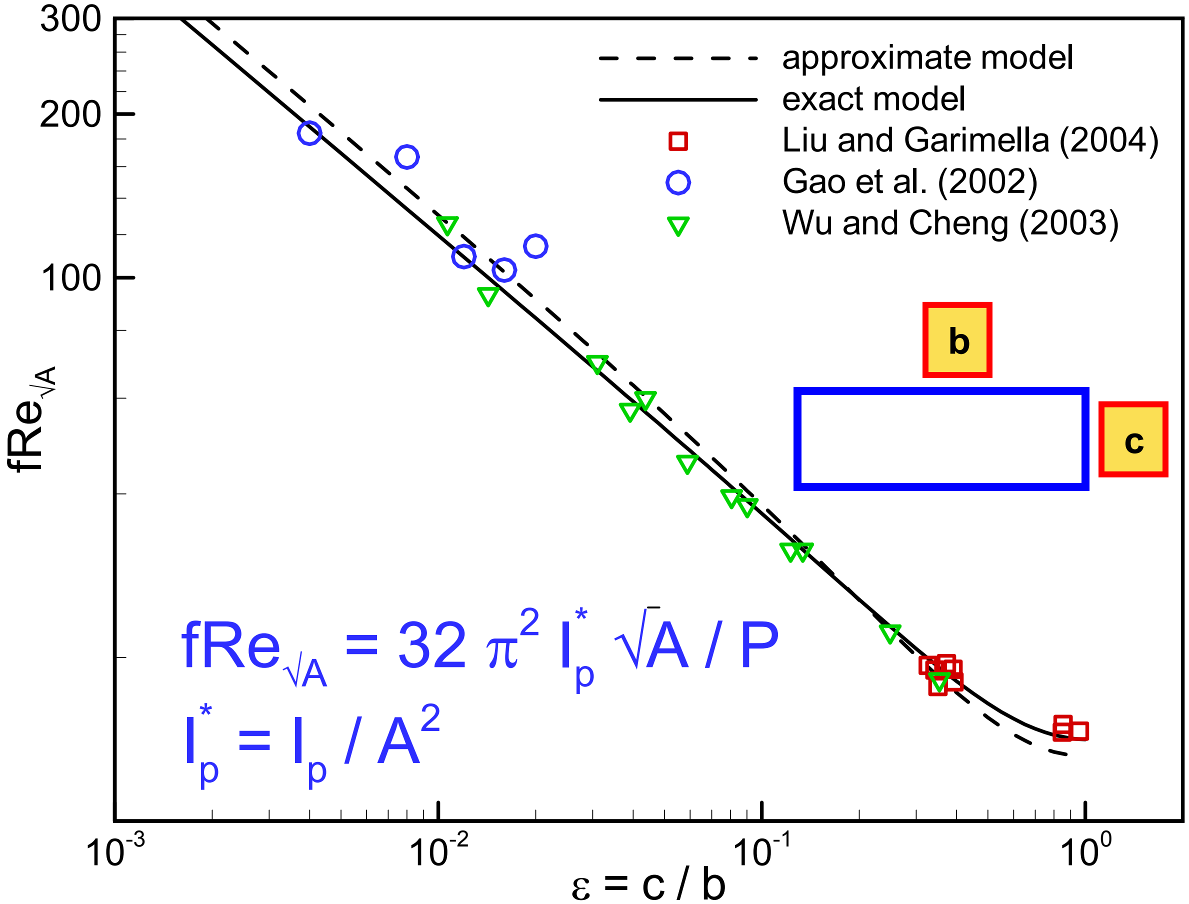
trapezoidal microchannels





Comparison with Data

rectangular microchannels



Comparison with Data

triangular and trapezoidal microchannels

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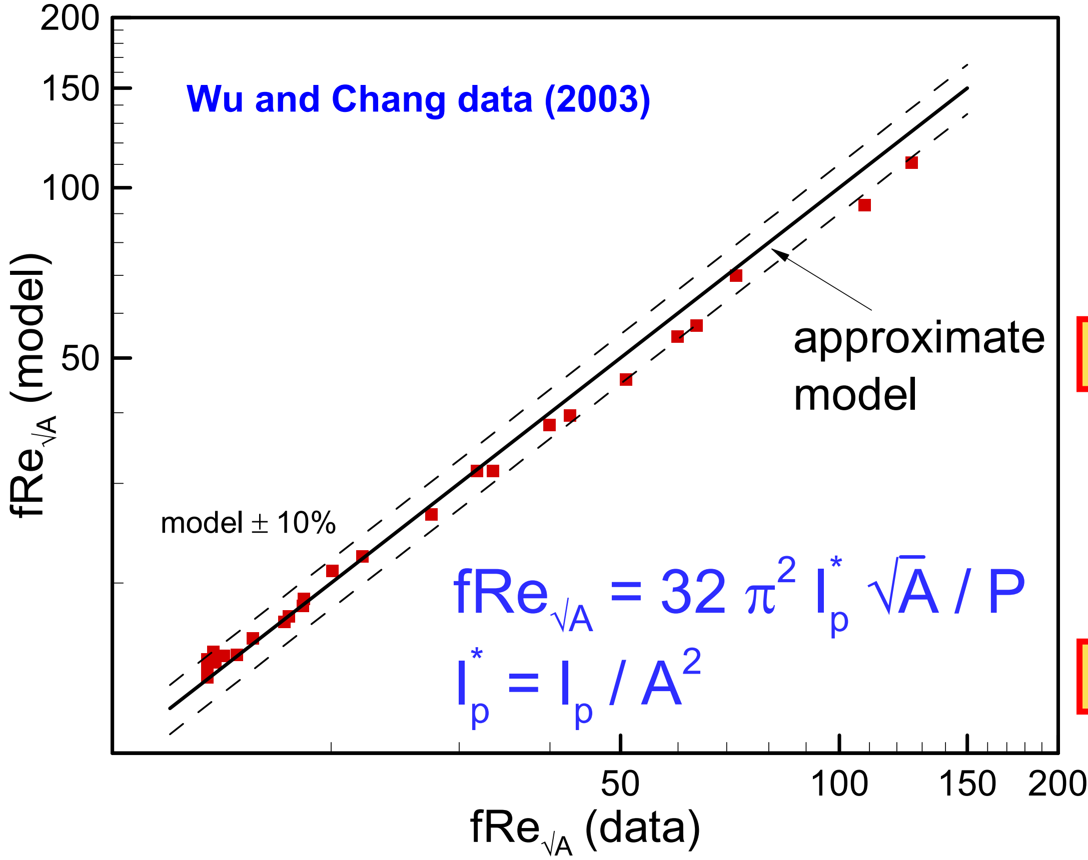
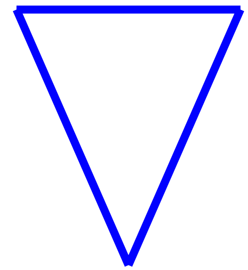


$$fRe_{\sqrt{A}} = 32\,\pi^2\, I_p^*\, \sqrt{A}\,/\,P$$

$$I_p^* = I_p\,/\,A^2$$



Isosceles triangular microchannels

Isosceles trapezoidal microchannels

Comparison with Numerical Data

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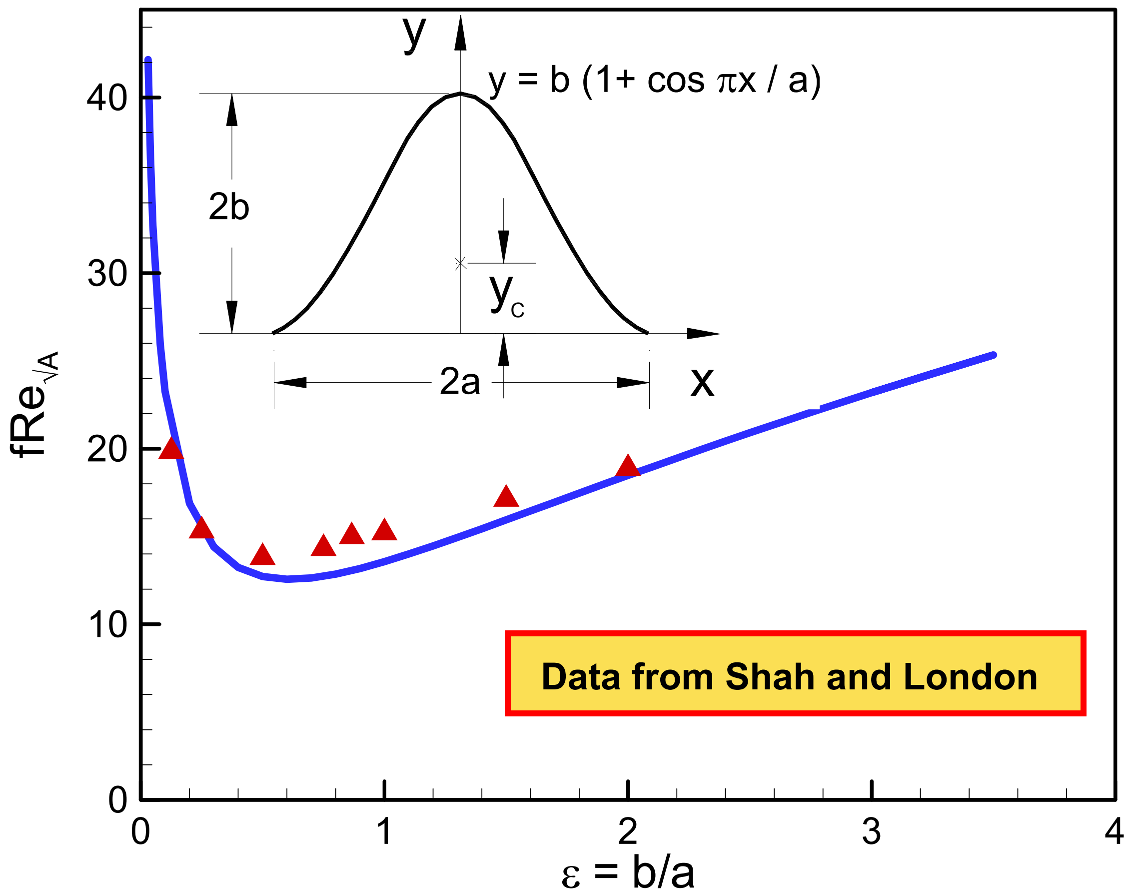
sine duct



Comparison with Numerical Data

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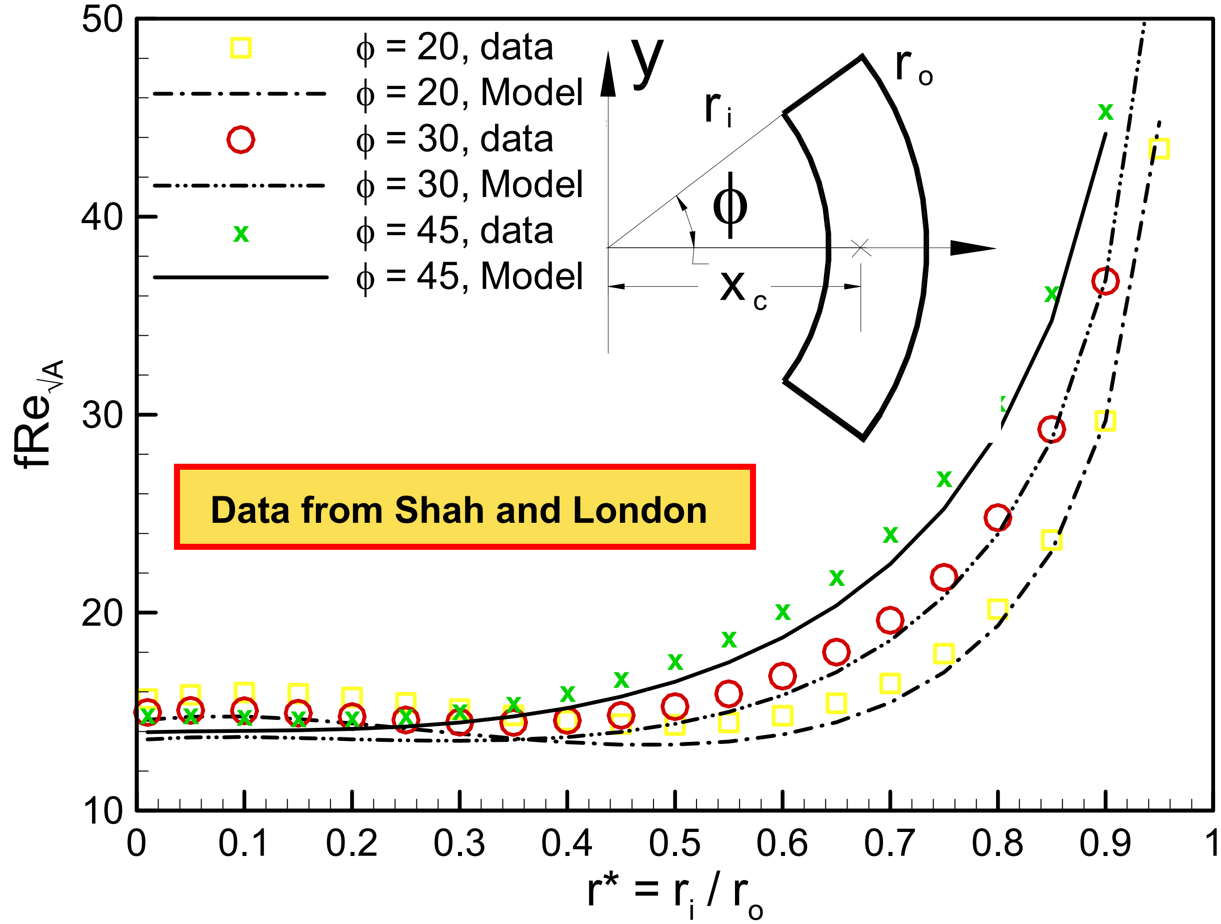
annular sector



Comparison with Numerical Data

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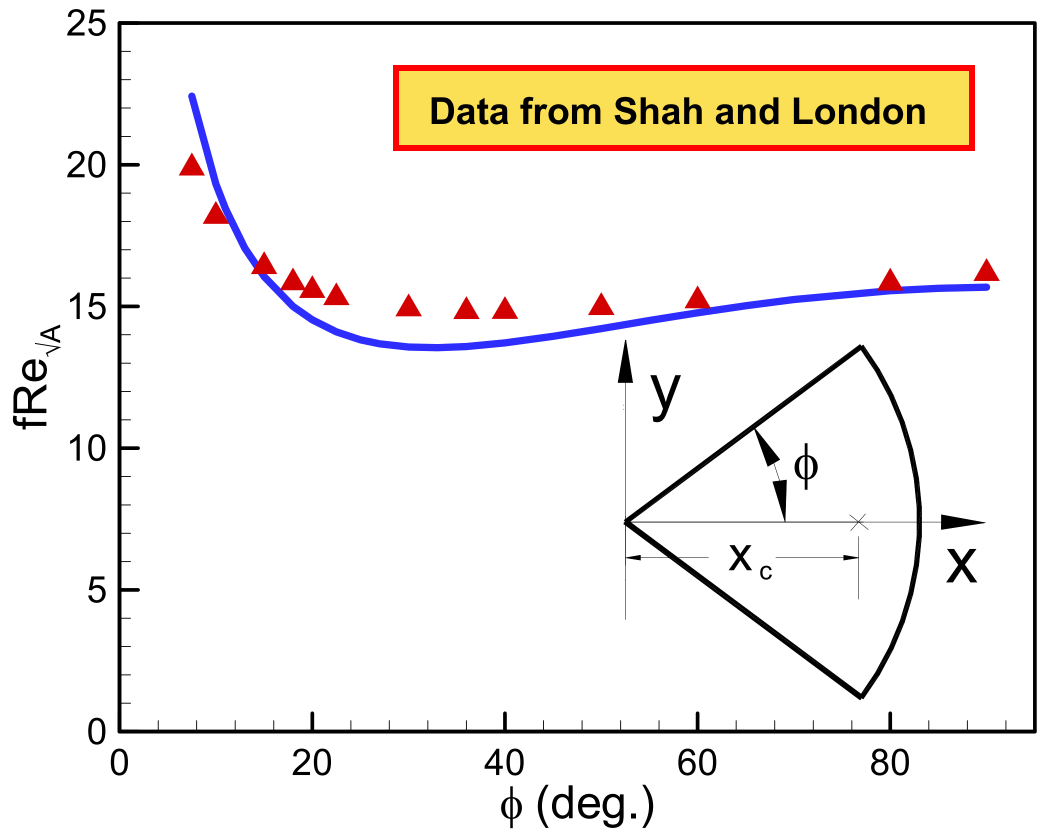
circular sector



Comparison with Numerical Data

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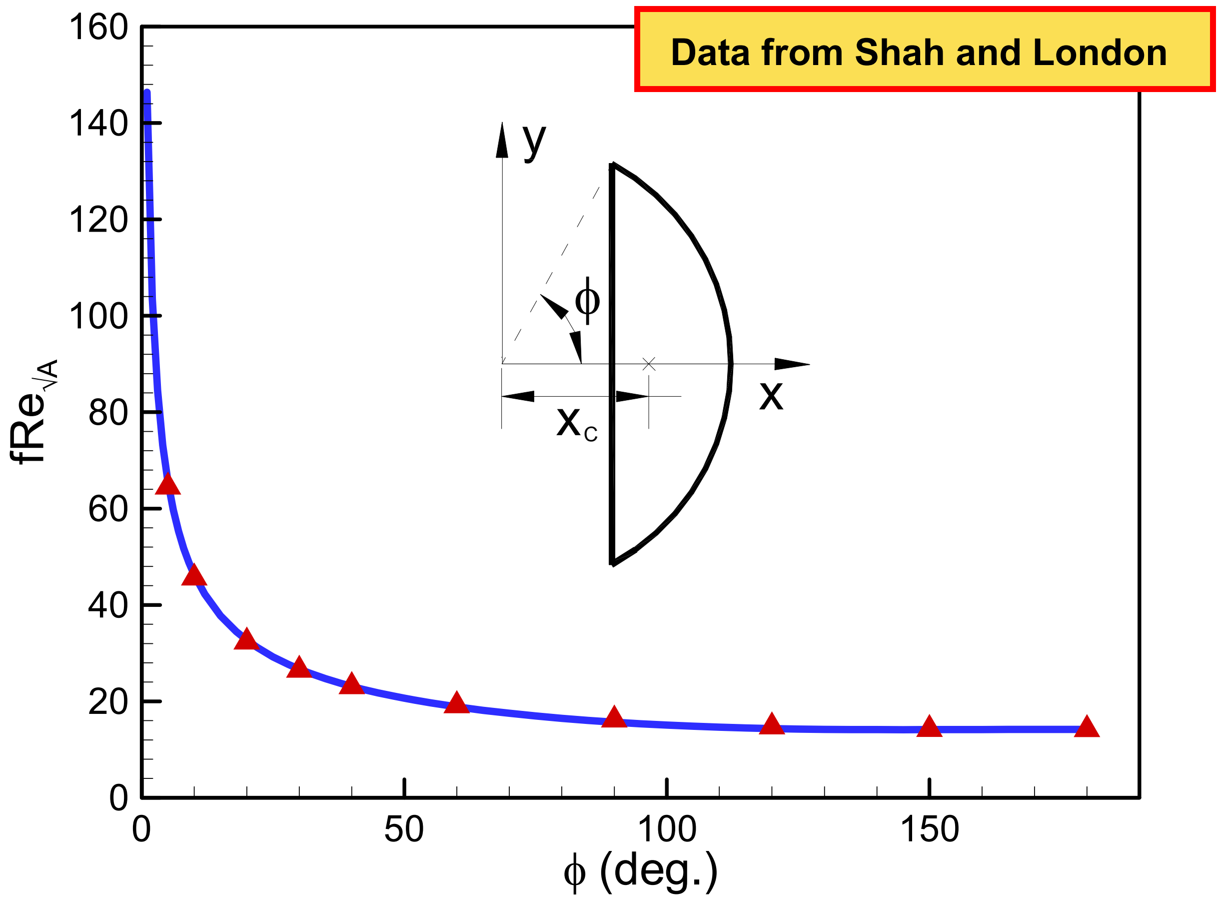
circular segment



Comparison with Numerical Data

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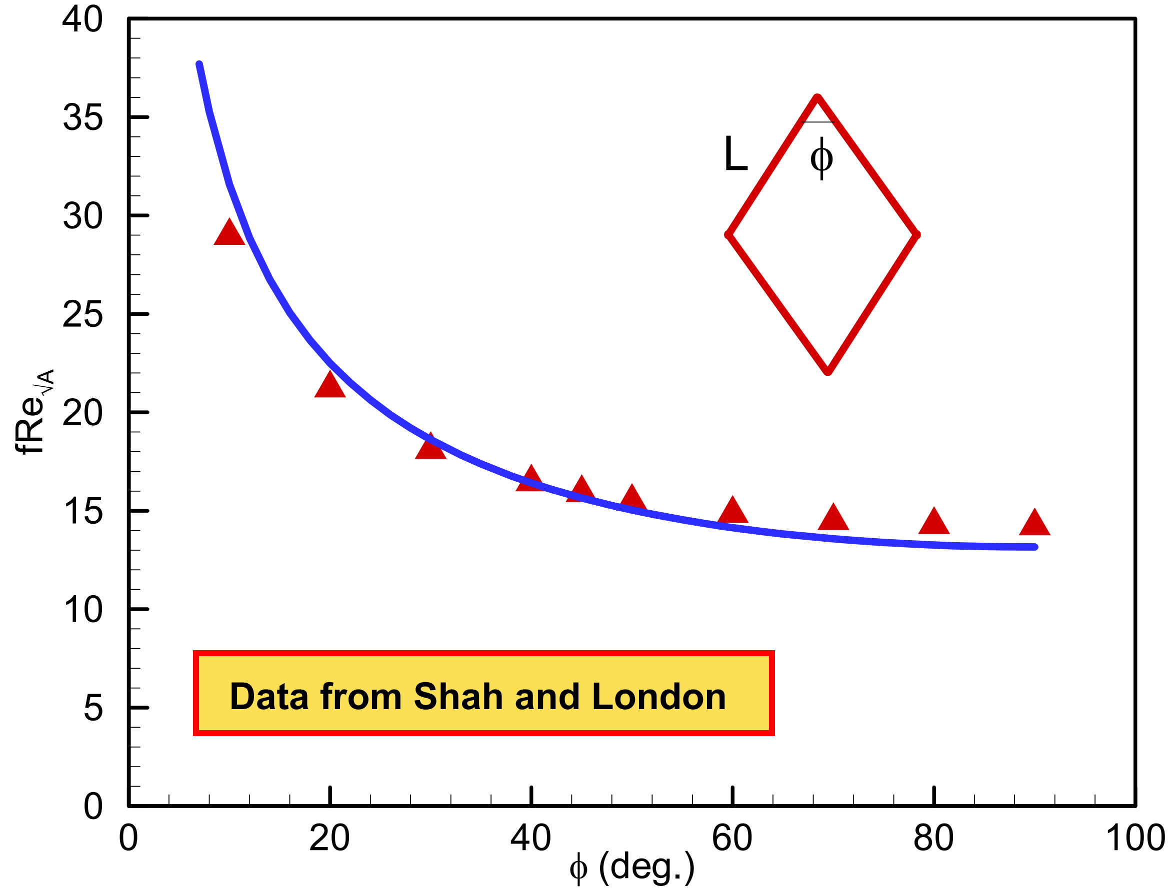
rhombic duct



Comparison with Numerical Data

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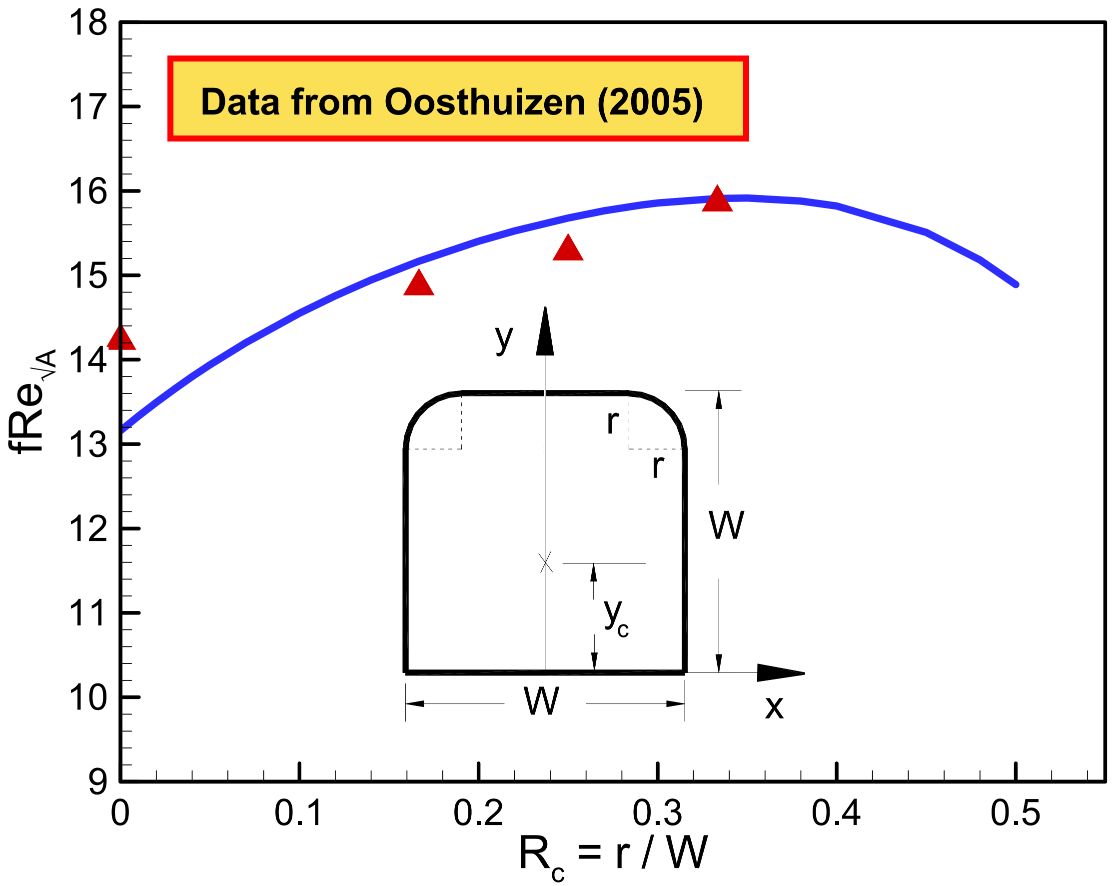
square duct with 2 adjacent round corners



Comparison with Numerical Data

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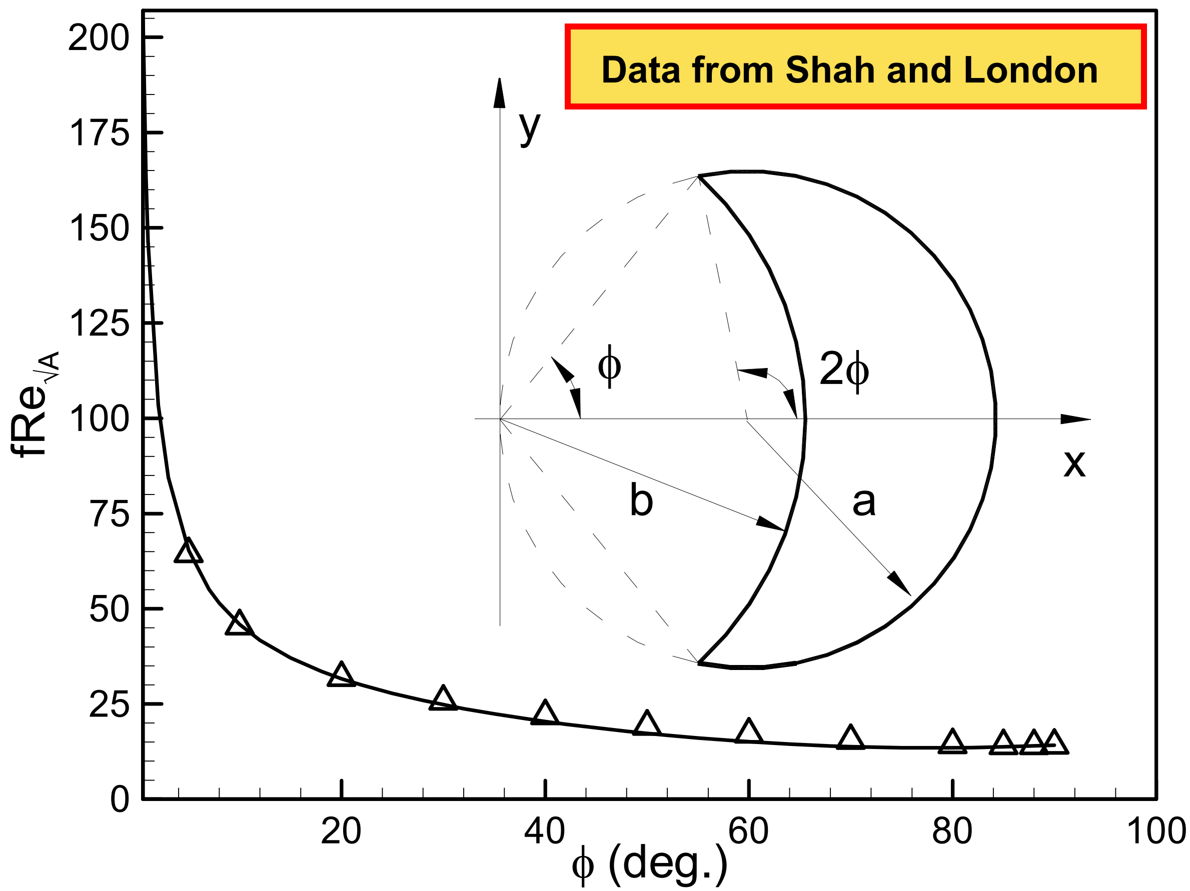
moon-shaped duct



Comparison with Numerical Data

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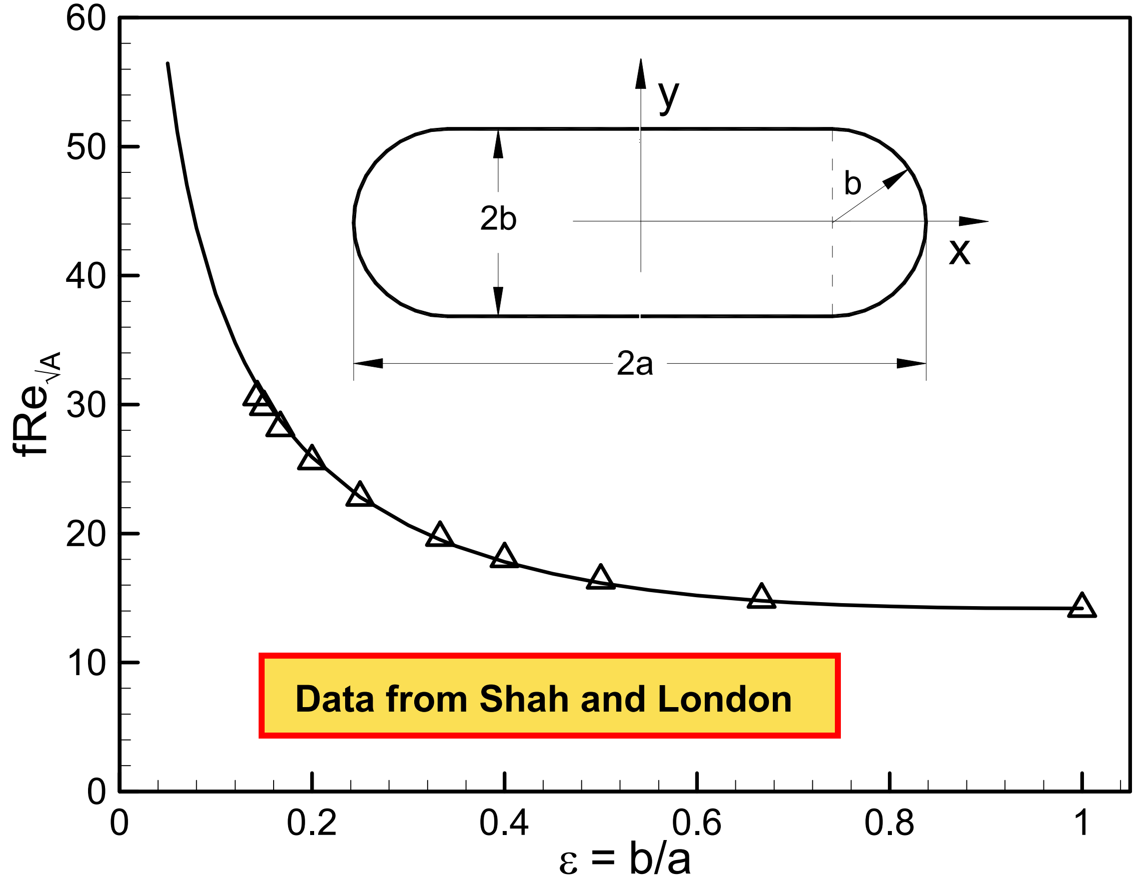
Rectangular duct with semi-circular ends



Summary and Conclusion

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A new compact analytical model is developed and validated with experimental and numerical data for a variety of microchannel cross-sections including:

- **Rectangular**
- **Trapezoidal**
- **Isosceles triangular**
- **Square**
- **Circular**
- **Other cross-sections**

Square root of area, as the characteristic length scale, is superior to the hydraulic diameter