Thermal Contact Resistance of Non-Conforming Rough Surfaces Part 1: Contact Mechanics Model

M. Bahrami
J. R. Culham
M. M. Yovanovich
G. E. Schneider

Department of Mechanical Engineering
Microelectronics Heat Transfer Laboratory
University of Waterloo
Waterloo, ON, Canada

CONTENTS

- introduction
- objectives
- literature review
- present model
- numerical approach and results
- approximate model (dimensional analysis and correlations)
- elastic compression
- summary and conclusions

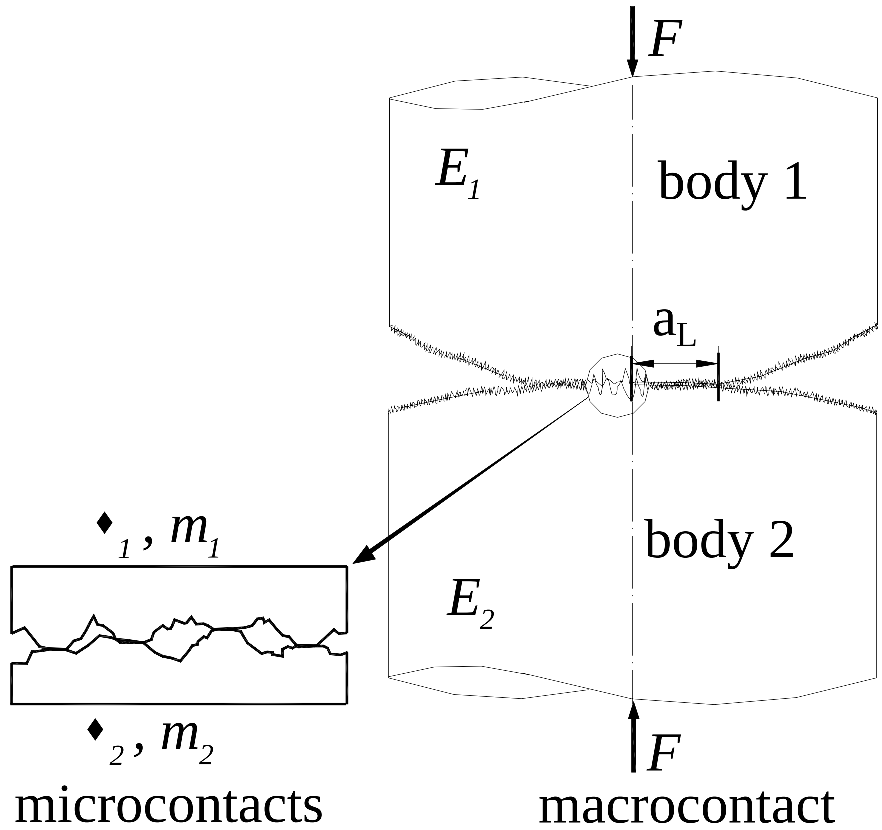
OBJECTIVES

- develop analytical model to predict contact parameters such as pressure distribution and size of the macrocontact area
- derive simple correlations for determining contact parameters used in analytical thermal contact models
- criterion to define a "*flat surface*"

INTRODUCTION

- contact of two spherical rough surfaces includes two problems:
  - microcontacts deformation or micro scale problem
  - bulk deformation or macro scale problem

- macrocontact area is the area in which the microcontacts are distributed



LITERATURE REVIEW

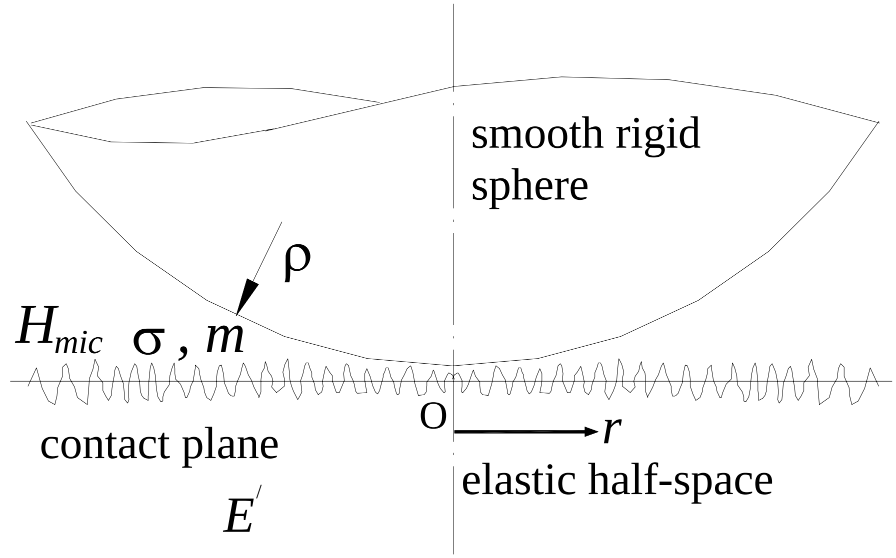
- microcontact modeling
  - Gaussian roughness
  - equivalent rough surface

$$\sqrt{\ {}_1^2 \qquad {}_2^2} \quad \text{and} \quad m \qquad \sqrt{m_1^2 \qquad m_2^2}$$

- microhardness
  - Vickers microhardness correlation, Hegazy (1985)

$$H_v \qquad c_1 \ d_v \ ^{c_2}$$

- macrocontact modeling
  - equivalent radius of curvature, Hertz (1881)



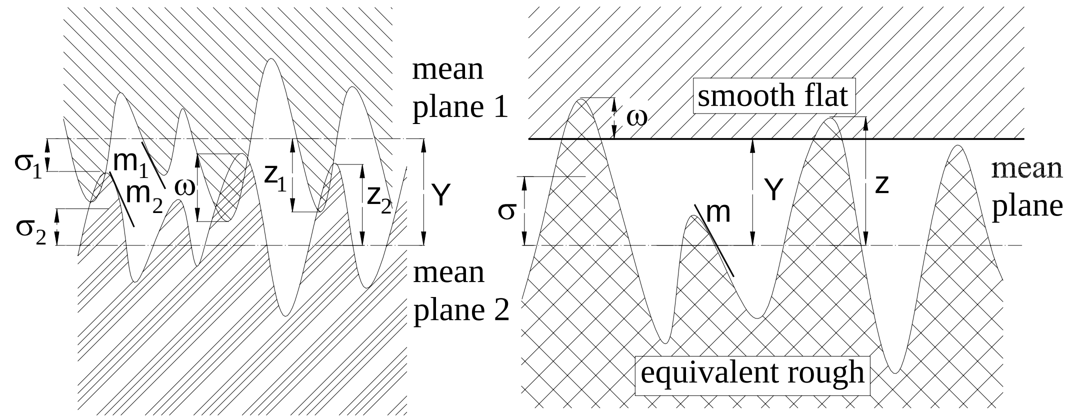
$$\frac{1}{\ } \qquad \frac{1}{\ _1} \qquad \frac{1}{\ _2}$$

$$\frac{1}{E'} \qquad \frac{1- \ _1^2}{E_1} \qquad \frac{1- \ _2^2}{E_2}$$

COOPER ET AL. MODEL

- conforming rough contacts
- Gaussian distribution for asperity
- plastically deformed

  hemispherical asperities



a) section through two contacting surfaces

b) corresponding section through equivalent rough - smooth flat

$$\frac{A_r}{A_a} \qquad \frac{1}{2}\text{erfc} \qquad\qquad Y/\sqrt{2}$$

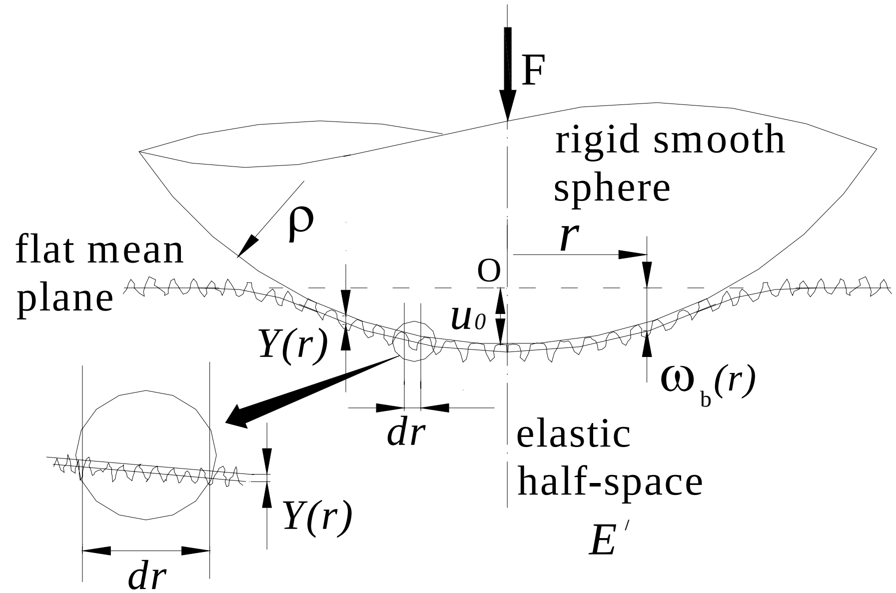
$$a_s \qquad \sqrt{\frac{8}{\ }} \quad \frac{\ }{m} \quad \exp \quad {}^{2} \quad \text{erfc}$$

$$n_s \qquad \frac{1}{16} \quad \frac{m}{\ } \quad {}^{2} \; \frac{\exp\left(-2 \;\; ^{2}\right)}{\text{erfc}} A_a$$

GREENWOOD AND TRIPP (GT) MODEL

- axisymmetric contact; elastic bulk deformation
- rough surfaces are isotropic and have Gaussian height distribution with a standard deviation ρ
- distribution of summit heights is same as surface heights standard deviation, i.e., $\rho_s = \rho$
- the deformation of each asperity is independent of its neighbors
- spherical summits all with constant radius, β; asperities deform elastically and Hertz theory applied for each individual summit.

PRESENT MODEL (ASSUMPTIONS)



- surfaces are macroscopically spherical
- microscopically, surfaces are rough with a Gaussian asperity distribution
- microcontacts deform plastically
- elastic macrocontact $\quad u \; r \qquad u_0 - r^2/2$
- first loading $\quad Y \; r \qquad {}_b \; r \; - u \; r \qquad {}_b \; r \; - u_0 \quad r^2/2$

GOVERNING RELATIONSHIPS

$$Y\ r \quad {}_b\ r\ - u\ r \quad {}_b\ r\ - u_0 \quad r^2/2$$

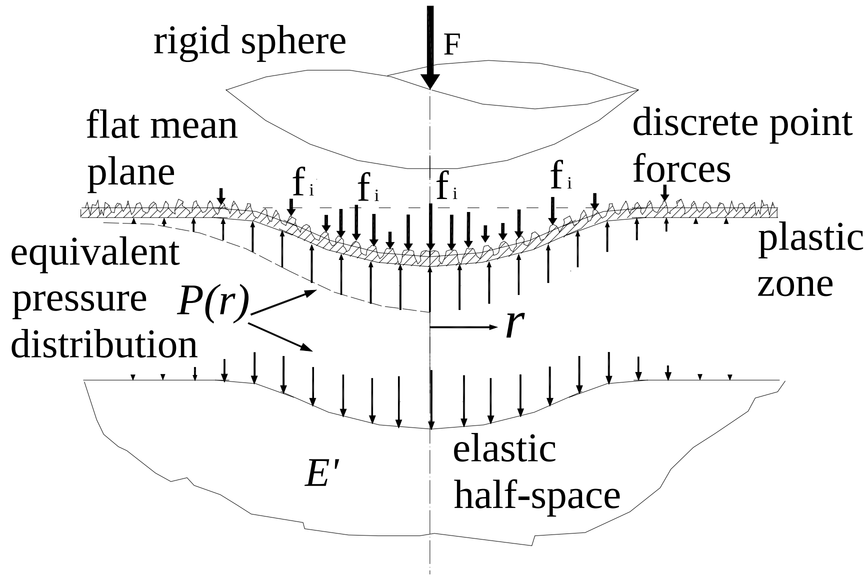
$$a_s\ r \quad \sqrt{\frac{8}{\ }}\ \frac{\ }{m}\ \exp\ {}^2\ r \quad \mathrm{erfc}\ \ r$$

$$H_{mic}\ r \quad c_1\left[\sqrt{2}\ \ a_s\ r\ \right]^{c_2}$$

$$P\ r \quad \tfrac{1}{2} H_{mic}\ r\ \mathrm{erfc}\ \ r$$

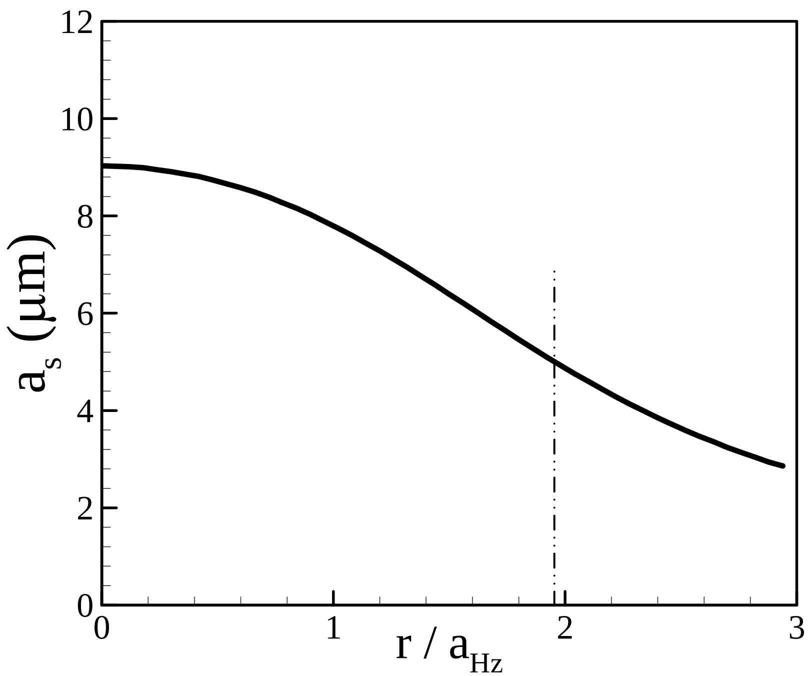
$$\omega_b(r) = \begin{cases} \dfrac{2}{E'} \displaystyle\int_0^{\infty} P(s)\, ds & r = 0 \\ \dfrac{4}{\pi E' r} \displaystyle\int_0^{r} s P(s) K\left(\dfrac{s}{r}\right) ds & r > s \\ \dfrac{4}{\pi E'} \displaystyle\int_r^{\infty} P(s) K\left(\dfrac{r}{s}\right) ds & r < s \end{cases}$$

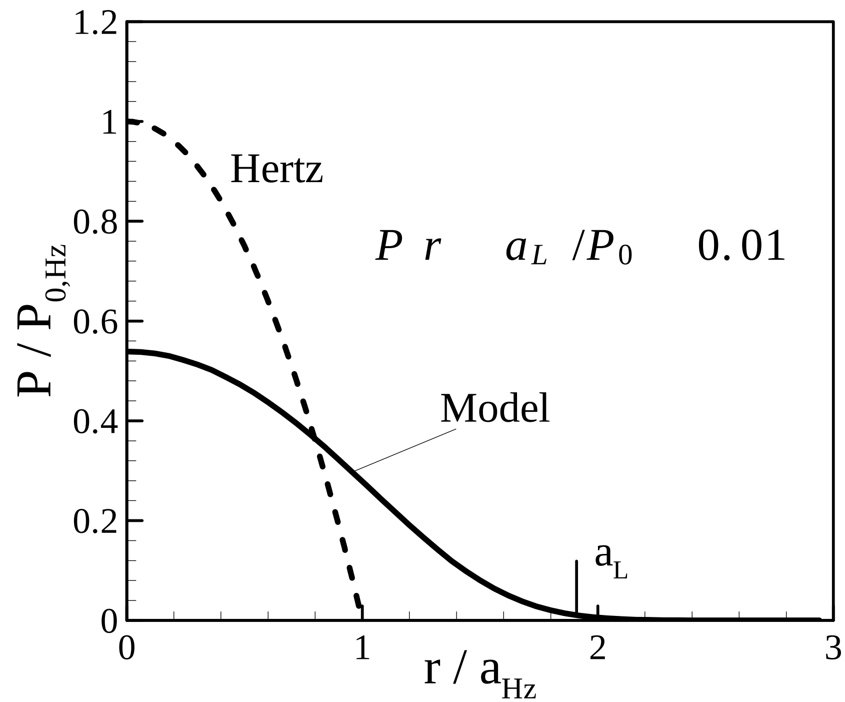
$$F \quad 2 \quad {}_0\ P\ r\ rdr$$



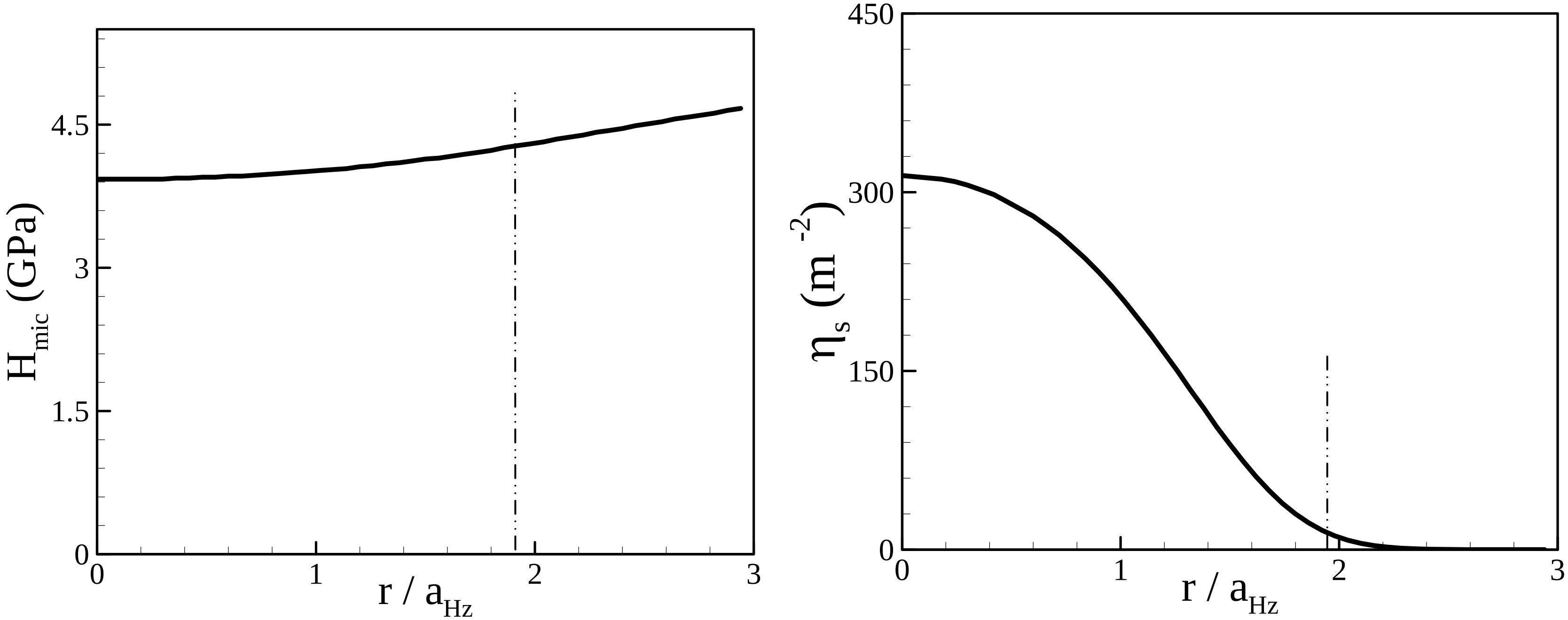
NUMERICAL RESULTS

25 mm	F 50 N
1.41 m	E' 112.1 GPa
m 0.107 –	c_1/c_2 6.27 GPa / – 0.15 –

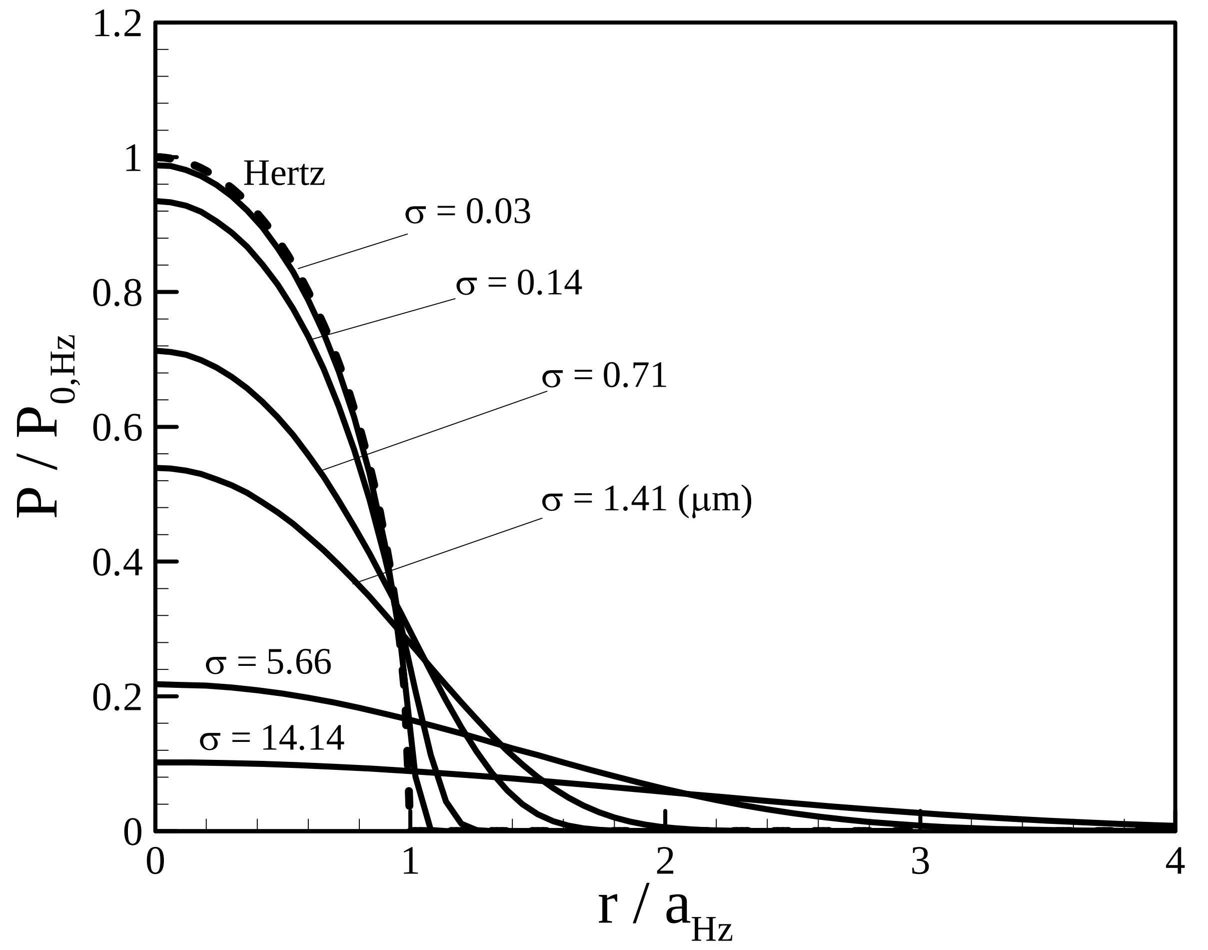




NUMERICAL RESULTS (Cont'd)



EFFECT OF ROUGHNESS



APPROXIMATE MODEL

- effective microhardness, H_{mic} = Const.

- surface slope *m* is assumed to be a function of surface roughness, Lambert (1995)

$$m \qquad 0.076 \qquad ^{0.52}$$

- maximum contact pressure is a function of

$$P_0 \qquad P_0 \quad , \quad , E', F, H_{mic}$$

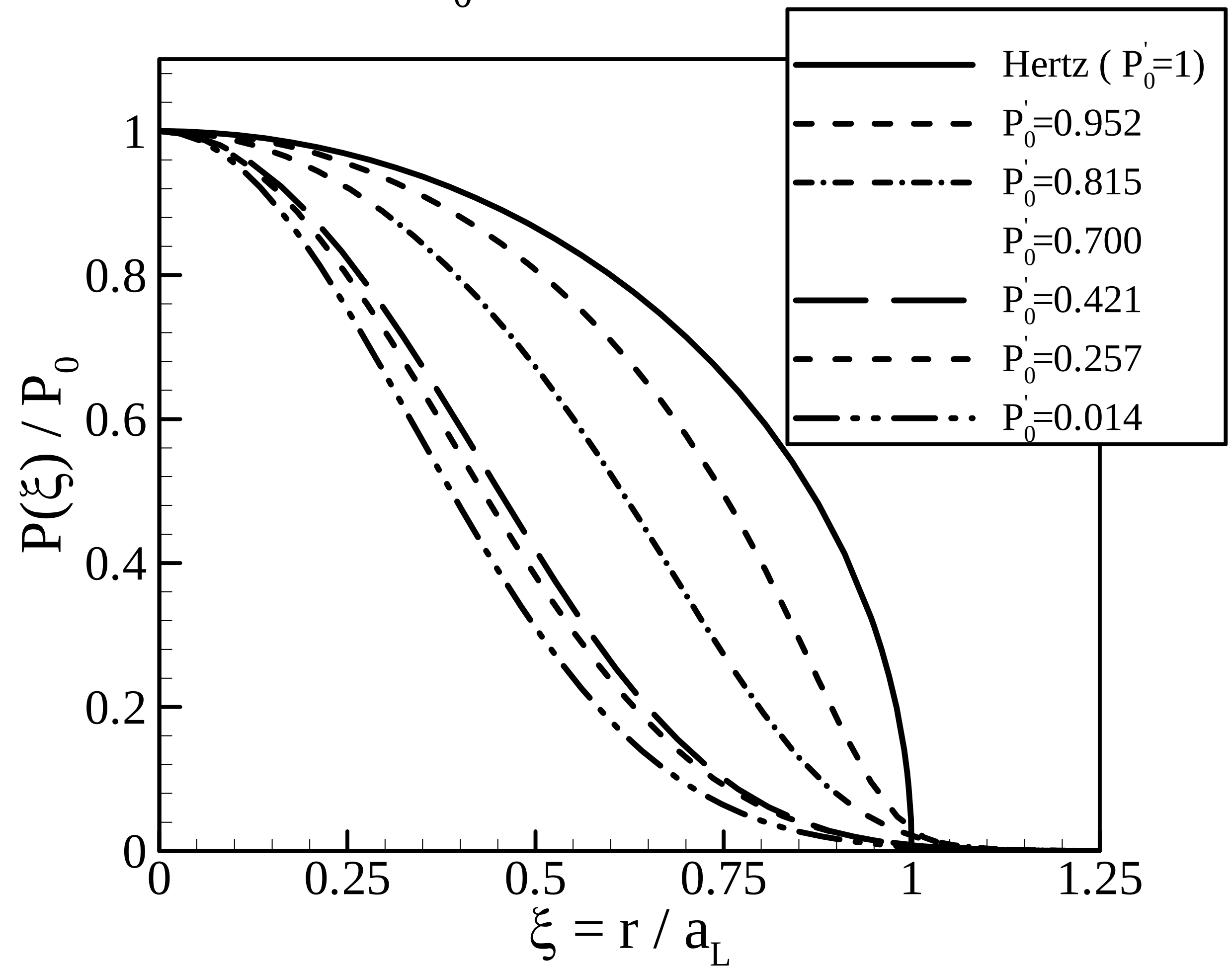
- Hertzian pressure distribution is

$$P_{Hz} \;\; r/a_{Hz} \qquad P_{0,Hz}\sqrt{1 - \;\; r/a_{Hz} \;\; ^2}$$

University of Waterloo

GENERAL PRESSURE DISTRIBUTION

$P \quad P_0 \; 1 - \quad ^2 \qquad P_0^{'} \quad P_0/P_{0,Hz}$



DIMENSIONAL ANALYSIS

Parameter	Dimension
Effective elastic modulus, E'	$ML^{-1}T^{-2}$
Force, F	MLT^{-2}
Microhardness, H_{mic}	$ML^{-1}T^{-2}$
Radius of curvature,	M
Roughness,	M
Max. contact pressure, P_0	$ML^{-1}T^{-2}$

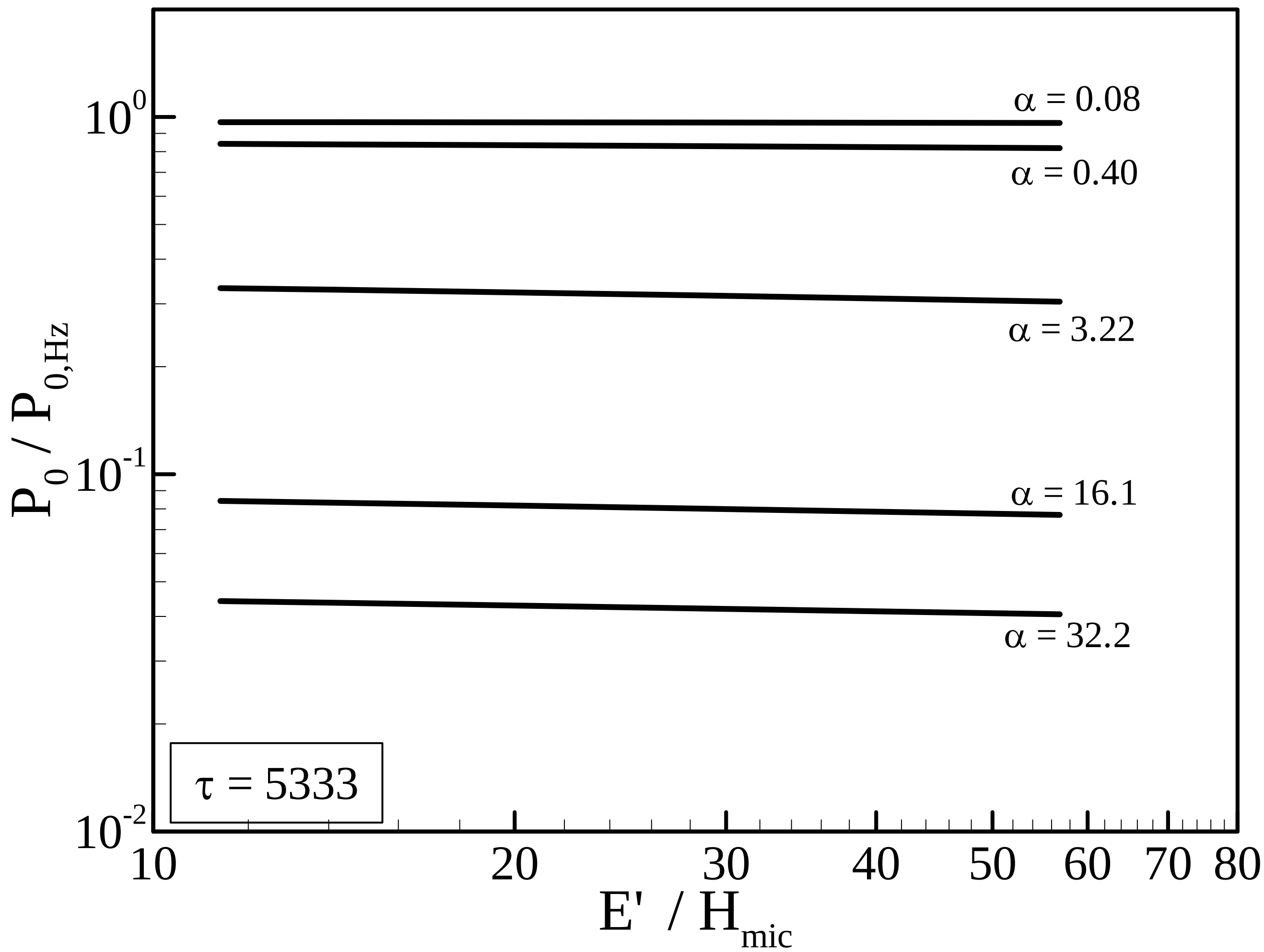
three non-dimensional parameters

$$\frac{}{_{0,Hz}} \equiv \frac{}{a_{Hz}^2} \qquad \left(\frac{16\ E'^2}{9F^2}\right)^{1/3}$$

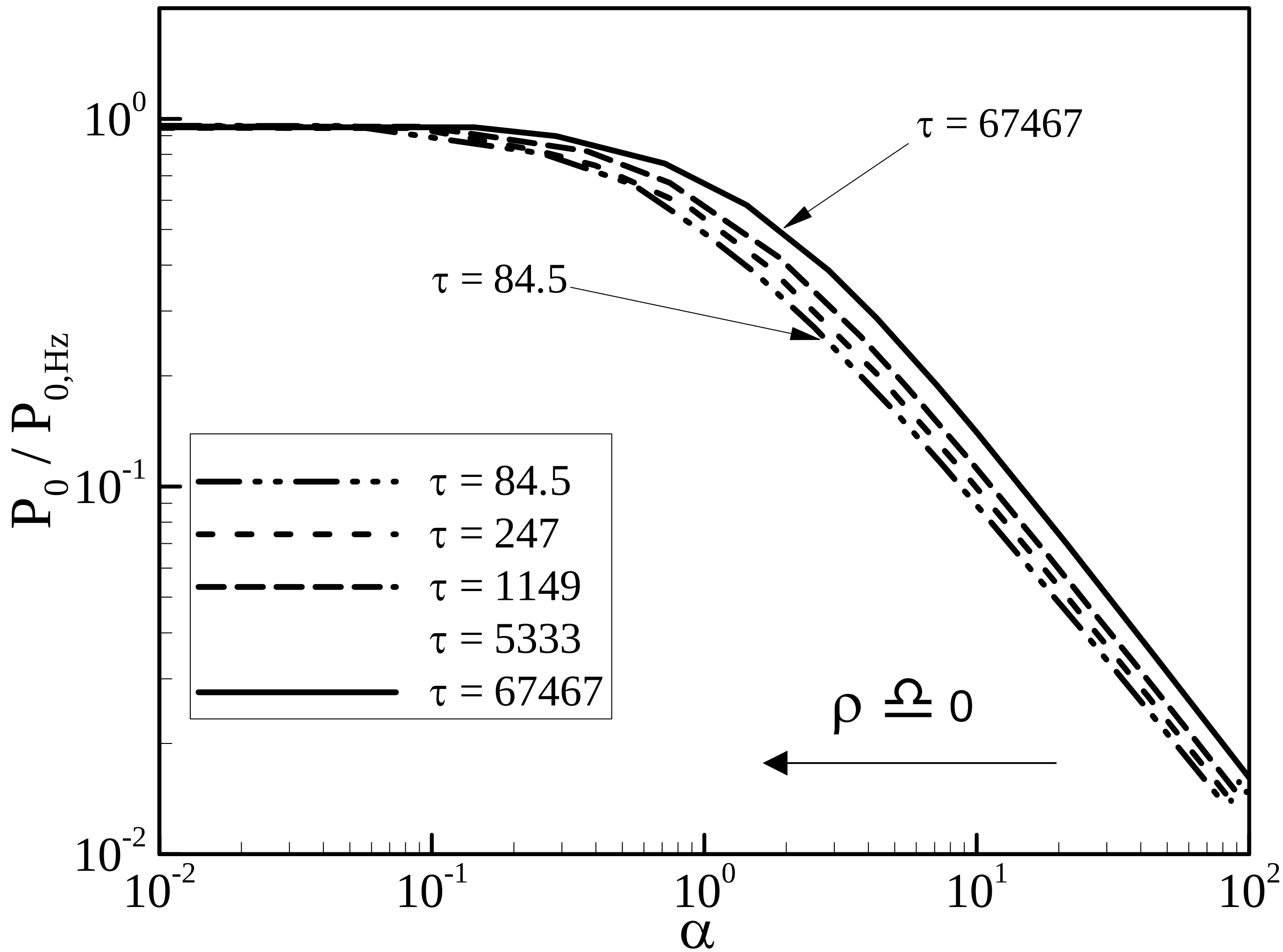
$$\frac{}{a_{Hz}} \qquad \left[\frac{4E'\quad 2}{3F}\right]^{1/3}$$

$$E \qquad \frac{E'}{H_{mic}}$$

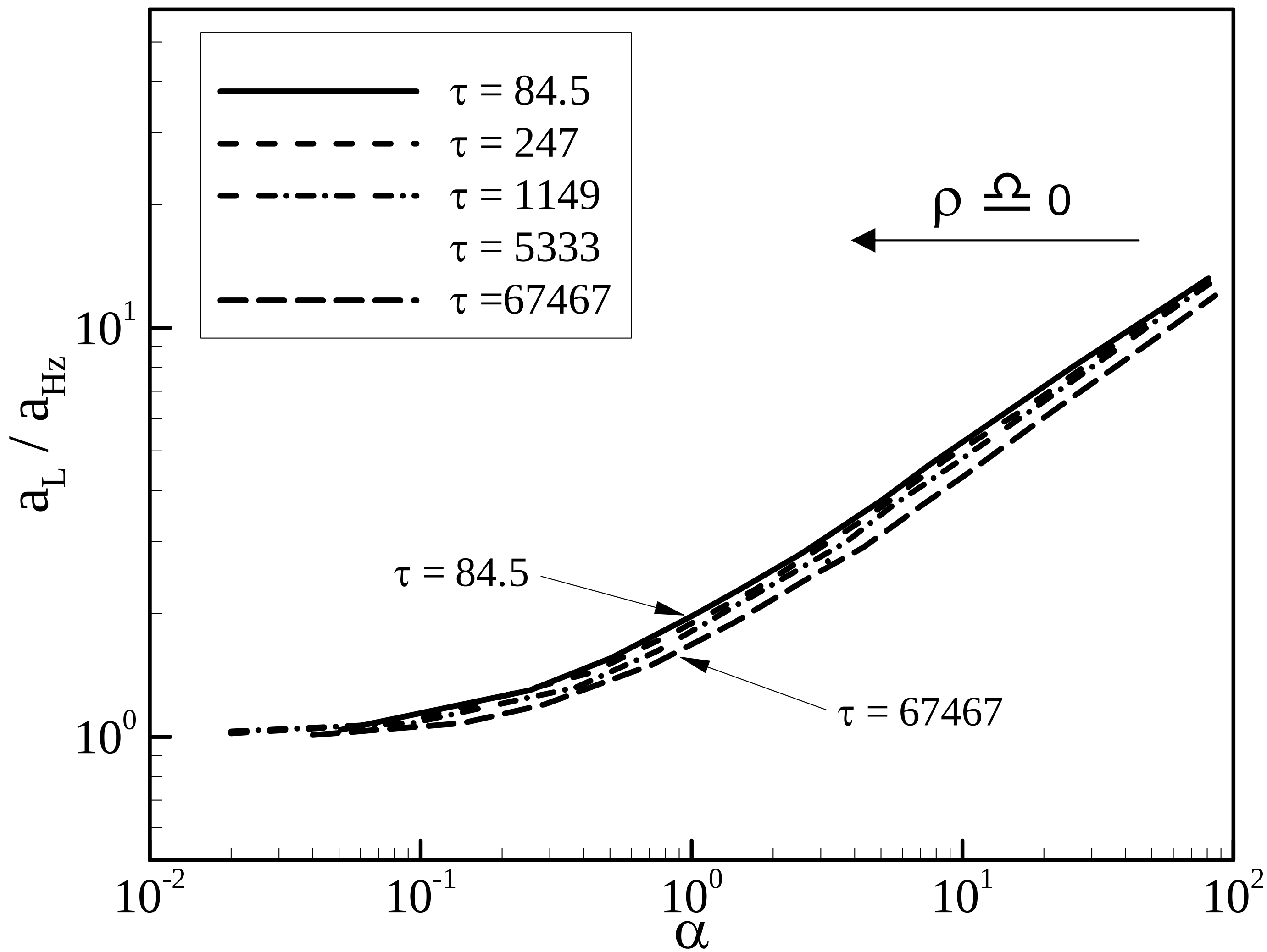
EFFECT OF MICROHARDNESS



MAXIMUM CONTACT PRESSURE



RADIUS OF MACROCONTACT



CORRELATIONS

$$P_0' \qquad \frac{P_0}{P_{0,Hz}} \qquad \frac{1}{1 \quad 1.37 \qquad {}^{-0.075}}$$

$$a_L' \qquad \frac{a_L}{a_{Hz}} \qquad 1 - 1.50 \ln P_0' - 0.14 \ln^2 P_0' - 0.11 \ln^3 P_0'$$

Or,

$$a_L' \qquad \frac{a_L}{a_{Hz}} \qquad 1.80 \frac{\sqrt{\quad 0.31 \quad {}^{0.056}}}{0.028}$$

$$P \qquad P_0 \; 1 - \qquad {}^{2}$$

using a force balance, $F \quad 2 \quad {}_{0}^{a_L} P \;\; r \;\; rdr$

$$1.5 P_0' \;\; a_L' \quad {}^{2} - 1$$

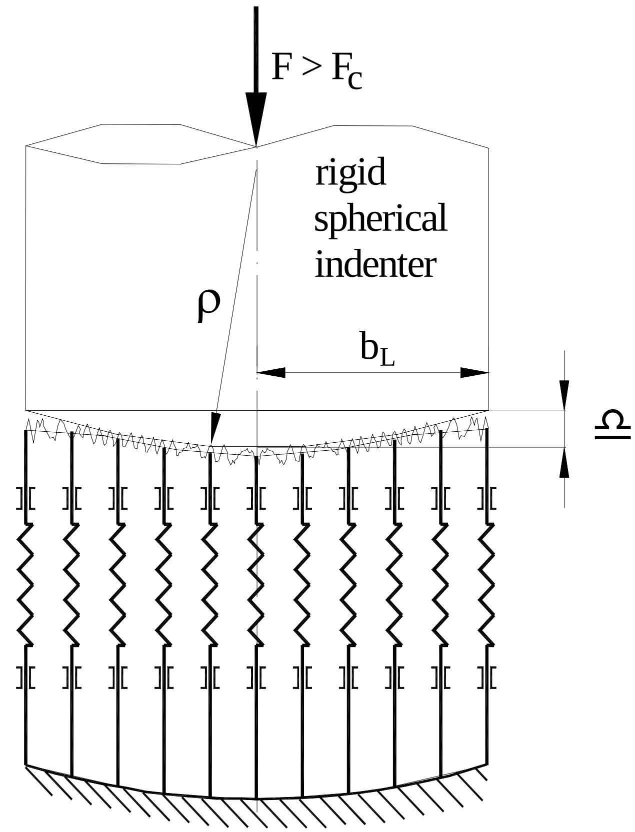
ELASTIC COMPRESSION

- critical force F_c, a_L b_L

$$F_c \quad \frac{4E'}{3} \max \ 0, \ b_L^2 - 2.25 \quad 3/2$$

- uniform increase will be added to critical pressure distribution at each point

$$P(\xi) = \begin{cases} P_0 \left[1 - \xi^2\right]^{\gamma} & F \le F_c \\ P_{0,c} \left[1 - \xi^2\right]^{\gamma_c} + \dfrac{F - F_c}{\pi b_L^2} & F \ge F_c \end{cases}$$



$$\frac{b_L^2}{2}$$

FLAT SURFACE

$$a'_L \quad 1.5\sqrt{\quad 0.45} \quad \text{and} \quad a_L \quad b_L$$

$$F_c = 0$$

if the out-of-flatness and the roughness of a surface are in the same order of magnitude, the surface is flat,

$$\text{—} \leq 1.12$$

SUMMARY AND CONCLUSIONS

- closed set of governing relationships was derived and solved numerically
- general pressure distribution was proposed that yields Hertzian pressure at limit, where roughness approaches zero
- using curve-fitting techniques, simple correlations were proposed for calculating contact parameters, as functions of governing non-dimensional parameters
- criterion was derived to identify flat surface

PRESENT MODEL VS GT MODEL

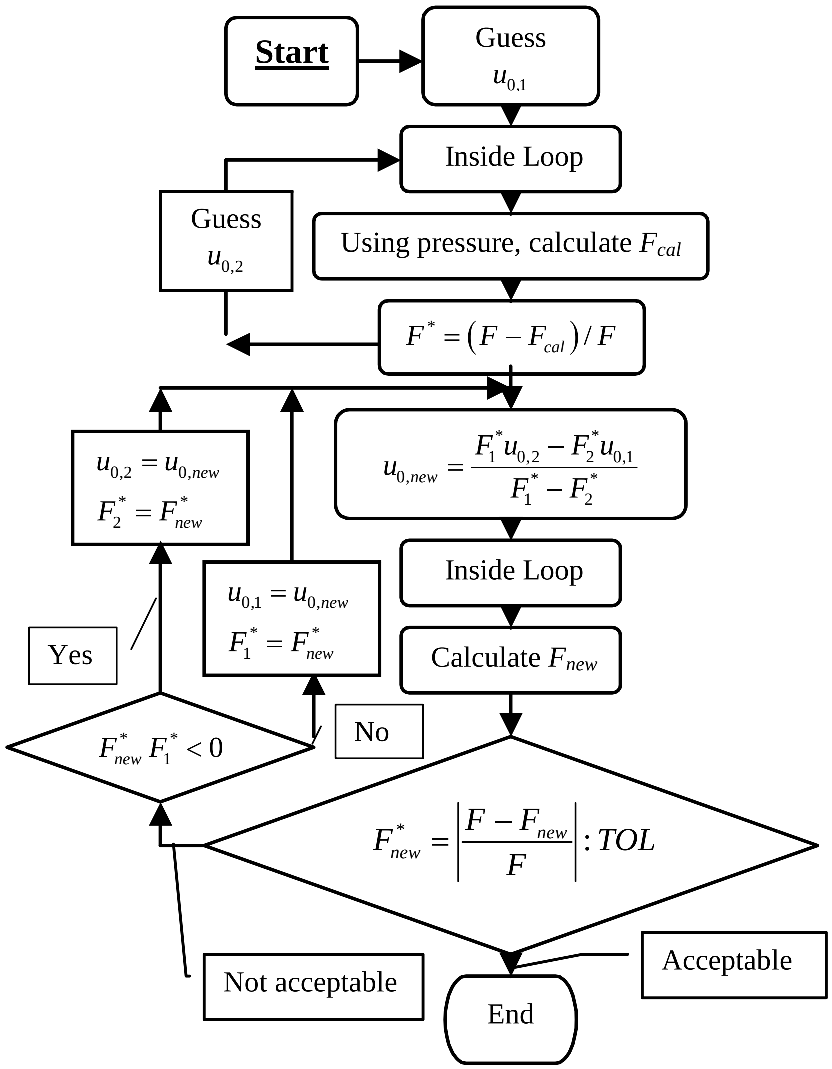
PRESENT MODEL	GT MODEL
elastic bulk **plastic** microcontacts	**elastic** bulk **plastic** microcontacts
requires **2 input** surface parameters ρ, m	requires **3 input** surface parameters ρ, β, γ_s
input parameters can be measured directly and are **not sensitive** to measurements	β, γ_s must be calculated through statistical relationships and are **sensitive** to measurements
simple correlations	requires **computer programming** and numerically intensive solutions

ACKNOWLEDGEMENTS

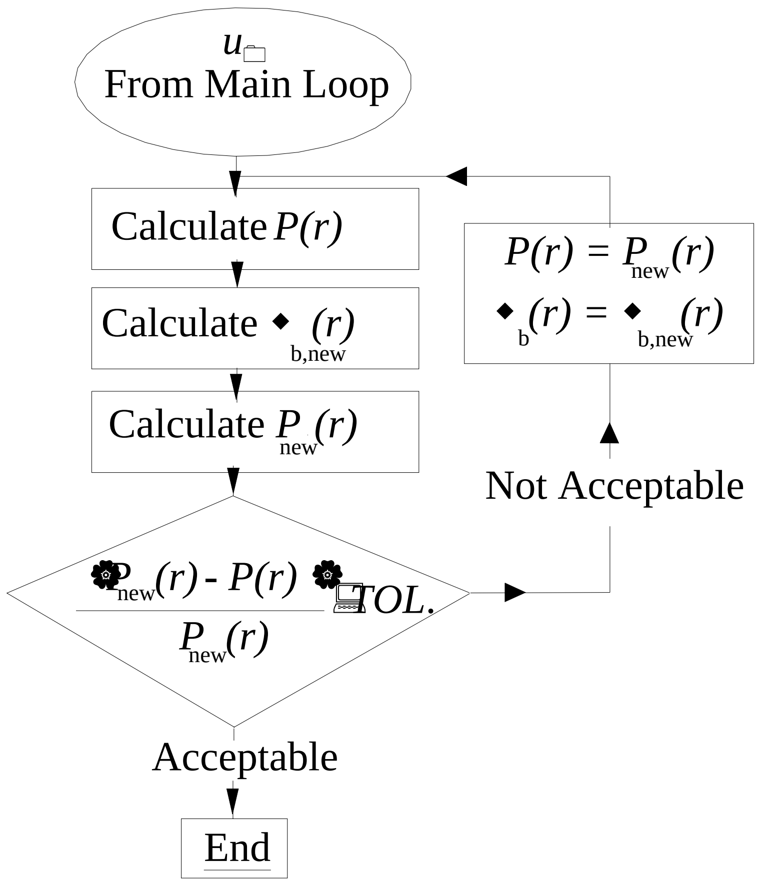
• Natural Sciences and Engineering Research Council of Canada (NSERC)

• The Center for Microelectronics Assembly and Packaging (CMAP)

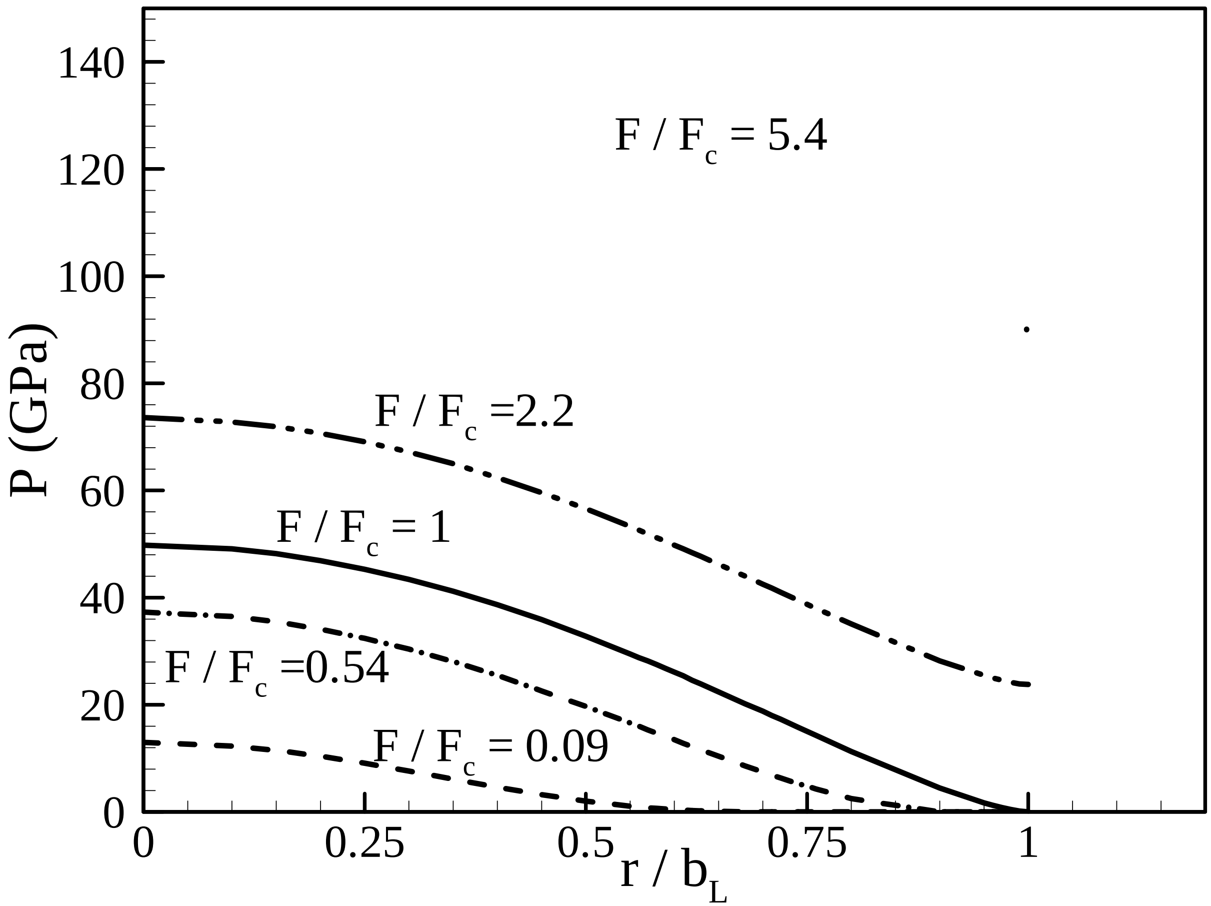
NUMERICAL ALGORITHM



THE INNER LOOP ALGORITHM



CONTACT PRESSURE DISTRIBUTION



10 m

E' 112 GPa

2 m

b_L 12 mm

GT MODEL SHORTCOMINGS

- A constant summit radius β is unrealistic
- Two of its input parameters, i.e., radius of summits β and density of summits γ_s cannot be measured directly and must be estimated through statistical calculations. These parameters are sensitive to the surface measurements
- Applying the model is complex and requires computer programming and numerically intensive solutions
- All asperities are assumed to deform elastically.