Thermal Contact Resistance of Non-Conforming Rough Surfaces
Part 2: Thermal Model

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CONTENTS

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• present model
• numerical approach and results
• parametric study
• alternative approach (correlations)
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INTRODUCTION

• conduction (microcontacts)
• conduction (interstitial fluid)
• radiation across the gap

• two sets of resistances in series represent TCR in a vacuum

• many researchers assumed

\[ R_j : R_{mic} + R_{mac} \]

• Bahrami et al. (2003) reviewed existing non-conforming rough models
Thermal resistance is defined as,

\[
R : = \frac{T}{Q}
\]

\[
R_j : R_{L,1} + R_{s,1} + R_{s,2} + R_{L,2}
\]

\[
\left( \frac{1}{R_s} \right)_{1,2} : \left( \begin{array}{c}
\frac{1}{R_{s,1}} \\
\vdots \\
\frac{1}{R_{s,2}} \\
\end{array} \right)_{1,2}
\]
TCR PROBLEM STATEMENT

Geometrical Analysis

Macro-Geometry

Micro-Geometry

Mechanical Analysis

Macro-Contact (Bulk Deformations)

Coupled

Micro-Contacts (Asperity Deformations)

Mechanical Analysis

Macro-Contact (Bulk Deformations)

Micro-Contacts (Asperity Deformations)

Thermal Analysis

Macro-Constriction Resistance

Micro-Constriction Resistance

Superposition

Thermal Joint Resistance
OBJECTIVES

• develop analytical TCR model for entire range of contacts:
  – conforming rough
  – elastoconstriction
  – transition region

• study effects of input parameters on TCR

• derive simple correlations for determining TCR
GEOMETRICAL MODELING

a) contact of non-conforming rough surfaces

b) contact of two rough spherical segments

c) rough sphere-flat contact, effective radius of curvature

d) equivalent sphere-flat contact, effective radius and roughness

\[
\begin{bmatrix}
\frac{b_L^2}{2L}
\end{bmatrix}
\]
FLUX TUBE SOLUTION

• inside macrocontact area a number of parallel cylindrical heat channels exist

• basic element for macro and micro thermal analysis

• Cooper et al. (1969)

\[ R_{\text{flux tube 1}} + R_{\text{flux tube 2}} : \frac{b \sqrt{L b}}{2k_s a} : \frac{\sqrt{L b^{1.5}}}{2k_s a} \]

\[ L : \frac{a}{b} \quad k_s : \frac{2k_1 k_2}{k_1 + k_2} \]
PRESENT MODEL (ASSUMPTIONS)

- solids are isotropic
- radiation heat transfer is negligible
- microcontacts are circular and steady-state heat transfer
- isothermal microcontacts
- microcontacts are flat
- surfaces are clean

\[ R_j : R_L + R_s \]

\[ R_L : \frac{\dot{Q} a_L}{b_L} \frac{b^{3/2}}{2k_s a_L} \]
MICROCONTACTS RESISTANCE

\[ R_s \hat{Y}_r \hat{p} : \frac{b \hat{Y}_r \hat{p}}{2k_s a_s \hat{y}_r \hat{p}} \]

\[ L \hat{Y}_r \hat{p} : \sqrt{\frac{A_r \hat{Y}_r \hat{p}}{A_a \hat{y}_r \hat{p}}} : \sqrt{\frac{1}{2} \text{erfc} \ R \hat{Y}_r \hat{p}} \]

\[ n_s : \frac{1}{16} \hat{Y}_m \hat{p}^2 \frac{\exp \left[ -2R \hat{Y}_r \hat{p}^2 \right]}{\text{erfc} \ R \hat{y}_r \hat{p}} A_a \]

\[ \frac{dR_s \hat{Y}_r \hat{p}}{n_s \hat{y}_r \hat{p}} \]

\[ R_s : \frac{1}{1/dR_s \hat{Y}_r \hat{p}} \]

Thermal Contact Resistance of Non-Conforming Rough Surfaces, Part II
NUMERICAL RESULTS

\[
R^*_s = 2b_L k_s R_s
\]

\[
\epsilon = a_s / b_s
\]

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Thermal Contact Resistance of Non-Conforming Rough Surfaces, Part II
EFFECT OF ROUGHNESS

\[ R^* = 2\beta L k_s R \]

\[ R_{Hz}^* \]

\[ R_{j min}^* \]

\[ R_L^* \]

\[ R_s^* \]
EFFECT OF LOAD

\[ R^* = 2 b_L k_s R \]

\[ F \text{ (N)} \]

\[ 10^0 \] \[ 10^1 \] \[ 10^2 \] \[ 10^3 \] \[ 10^4 \]

\[ 10^0 \] \[ 10^1 \] \[ 10^2 \] \[ 10^3 \] \[ 10^4 \]
EFFECT OF RADIUS OF CURVATURE

\[ R^* = 2 b L k_s R \]

\[ \rho (m) \]

\[ R_{Hz}^* \]
\[ R_j^* \]
\[ R_s^* \]
\[ R_L^* \]
ALTERNATIVE APPROACH

- conforming rough conductance, Yovanovich (1982)

\[ h_s : 1.25 k_s \dot{Y}^m \frac{P}{H_{mic}}^{0.95} \]

- heat transfer in non-conforming rough contact:

\[ Q : \pi \frac{h_s \dot{Y}_r \delta}{T_s dA_a} \]

- using pressure distribution, Part I and

\[ R : \frac{1}{\dot{Y}_h A_a} \]

\[ R_s : \frac{\int \dot{Y}_l + s HP}{1.25Z m k_s a_L^2} \left( \frac{H^r}{P_0} \right)^s \]

Thermal Contact Resistance of Non-Conforming Rough Surfaces, Part II
PROCEDURE FOR UTILIZING THE MODEL

If an estimate of microhardness is available, set $c_1 = H_{micro}$, $c_2 = 0$.

If $m$ may be estimated from:

$m = 0.076 \sigma^{0.52}$

$\sigma [\mu m]$

If an estimate of microhardness is available, set $c_1 = H_{micro}$, $c_2 = 0$.

$m = 0.076 \sigma^{0.52}$

$\sigma [\mu m]$

Start

Input

$F, \rho, \sigma, m, k_s$

$E', b_l, c_1, c_2$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$

$m = \sqrt{m_1^2 + m_2^2}$

$\rho = \left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right]^{-1}$

$k_s = 2k_1k_2/(k_1 + k_2)$

$E' = \left[\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right]^{-1}$

$F_c = \frac{4E'}{3\rho} \left(\max\{0, (b_i^2 - 2.25 \sigma \rho)\}\right)^{1/2}$

$s = 0.95 \left(1 + 0.071c_2\right)$

$H' = c_1 \left(1.62 \sigma^*/m\right)^{1/2}$

$\sigma^* = \frac{\sigma}{\sigma_0 [\mu m]}$

$\alpha = \frac{\sigma}{\rho/a_{hc}^2}$

$\tau = \frac{\rho}{a_{hc}}$

$a_{hc} = (0.75F \rho/E')^{1/3}$

$P_{0,hc} = 1.5F/\pi a_{hc}^2$

$P_0 = \frac{P_0}{P_{0,hc}} = \frac{1}{1 + 1.37 \tau^{-0.075} \alpha}$

$\dot{a}_L = \frac{a_L}{a_{hc}} = 1 - 1.5 \ln P_0$

$-0.14 \ln^2 P_0 - 0.11 \ln^3 P_0$

$\gamma = 1.5P_0 (a_L)^2 - 1$

$R^*_s = \left(\pi H' b_l^2 / F\right)^s$ $

a_L = b_l$

Flat surface

$R^*_s = \left(\frac{1 + s \gamma}{a_L \left(\frac{P_0}{H'}\right)^s}\right)^s$, where

$R^*_s = 1.25\pi k_s b_l^2 (m/\sigma) R_s$

$R_L = \frac{(1 - a_L / b_l)^{1/2}}{2k_s a_L}$

$R_f = R_L + R_s$

End

$P_{0,hc} = 1.5F/\pi a_{hc}^2$

$\gamma = 1.5P_0 (a_L)^2 - 1$
**EXPERIMENTAL DATA**

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<td>M</td>
<td>Milanez et al.</td>
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### Parameter Values

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<td>$7.15 \leq b_L \leq 14.28$ mm</td>
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<td>$25.64 \leq E' \leq 114.0$ GPa</td>
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<td>$7.72 \leq F \leq 16763.9$ N</td>
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<td>$16.6 \leq k_s \leq 227.2$ W/mK</td>
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<td>$0.013 \leq \eta \leq 0.12$</td>
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COMPARISON WITH DATA

\[ E : \frac{\dot{\gamma}_f/m \cdot \gamma_f + H_p}{1.25Z_bLb^2} \left( \frac{H_f}{P_0} \right)^s + \frac{\dot{\gamma}_f B}{2B} \]

\[ R_j^D : k_s b_L R_j \]

\[ B : \frac{a_L}{b_L} : 1.80 \left( \frac{a_{Hz}}{b_L} \right) \sqrt{F + 0.31} \sim 0.056 \sim 0.028 \]

Increasing load
SUMMARY AND CONCLUSIONS

- superposition of macro and micro thermal resistance

- effects of major parameters, i.e., roughness, load, and radius of curvature on TCR were investigated

- for non-conforming rough contact, there is a value of surface roughness that minimizes TCR

- at relatively large loads effect of roughness on TCR becomes negligible

- simple correlations were derived that cover entire range of TCR
SUMMARY AND CONCLUSIONS 2

• model was compared with more than 700 experimental data points, collected by many researchers

• comparison includes all three regions of TCR and wide range of mechanical, thermal, and surfaces characteristics

• data include contact between dissimilar metals such as Ni200-Ag and SS-CS

• RMS relative difference between the model and the data was estimated to be approximately 11.4%
ACKNOWLEDGEMENTS

• Natural Sciences and Engineering Research Council of Canada (NSERC)

• The Center for Microelectronics Assembly and Packaging (CMAP)
### CONFORMING ROUGH DATA

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### ROUGH SPHERE-FLAT DATA

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<td>MM,T2</td>
<td>113.7</td>
<td>1.75/.07</td>
<td>2.44</td>
<td>4.0/0</td>
<td>22</td>
</tr>
<tr>
<td>MR,T1</td>
<td>107.1</td>
<td>4.83/-</td>
<td>21.2</td>
<td>4.2/0</td>
<td>19.9</td>
</tr>
<tr>
<td>MR,T2</td>
<td>107.1</td>
<td>3.87/-</td>
<td>39.7</td>
<td>4.2/0</td>
<td>19.9</td>
</tr>
</tbody>
</table>
CORRELATIONS

\[ F_c : \frac{4E'}{3} \lambda_{\max}^2 \frac{b_L}{\gamma} 2.25 \] \( \hat{R}_s^{3/2} \)

\[ R_s^* = \begin{cases} 
\left( \frac{\pi H' b_L^2}{F} \right)^s 
\left( \frac{b_L}{a_L} \right)^2 \left( \frac{H'}{P_0} \right)^s (1 + s\gamma) 
\left( \frac{H'}{P_0,c} \right)^s (1 + s\gamma_c) + \left( \frac{\pi H' b_L^2}{F - F_c} \right)^s & F \leq F_c \\
& F \geq F_c 
\end{cases} \]

\[ R_s^{\text{D}} : 1.25 \lambda b_L^2 k_s \gamma m/\text{Br} R_s \]

\[ s : 0.95/\gamma_1 + 0.071 c_2 \beta \]

\[ H_r : c_1 \gamma_1, 62 \] \( r/m \beta^2 \)

\[ P_r^0 : \frac{P_0}{P_{0,H\gamma}} : \frac{1}{1 + 1.37F \beta^{0.075}} \]

\[ a_r^L : \frac{a_L}{a_{H\gamma}} : 1.80 \sqrt{F + 0.31 \beta^{0.056}} \beta^{0.028} \]