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Fluid Flow and Heat Transfer From Elliptical Cylinders: Analytical Approach

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OUTLINE

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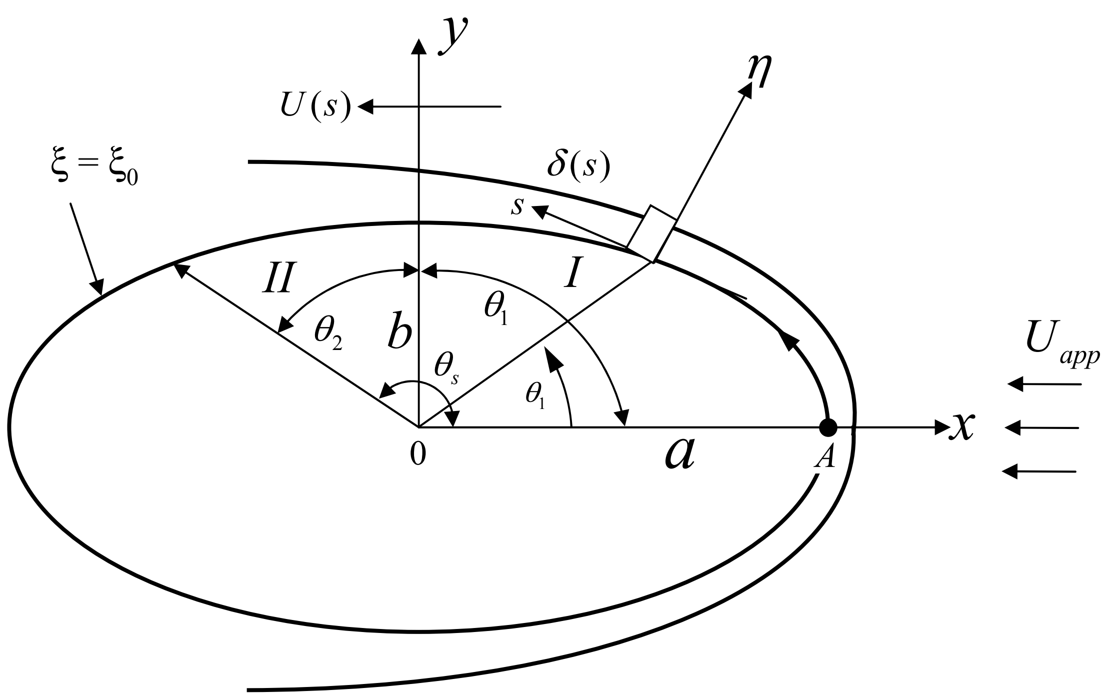
Introduction – Geometry



$$\epsilon = b/a$$

$$e = \sqrt{1-\epsilon^2}$$

$$\mathcal{L} = 4a\,E(e)/\pi$$

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Literature Review

- No analytical work available
- Experimental
- Numerical

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Objectives

Develop models for:

- Fluid flow (total drag coefficient)
- Heat transfer (local and average heat transfer coefficients under UWT and UWF)

Assumptions

- ✓ Flow normal to cylinder axis
- ✓ Steady, laminar and 2-D flow
- ✓ Incompressible fluid with constant properties
- ✓ Thin hydrodynamic boundary layer
- ✓ No energy dissipation in thermal boundary layer
- ✓ No slip at cylinder wall
- ✓ No mass flow through cylinder wall
- ✓ Inviscid flow outside hydrodynamic boundary layer

Governing Equations

Continuity:

$$\frac{\partial u}{\partial s} + \frac{\partial v}{\partial \eta} = 0$$

Momentum:

$$u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial \eta} = -\frac{1}{\rho}\frac{dP}{ds} + \nu\frac{\partial^2 u}{\partial \eta^2}$$

Energy:

$$u\frac{\partial T}{\partial s} + v\frac{\partial T}{\partial \eta} = \alpha\frac{\partial^2 T}{\partial \eta^2}$$

Velocity and Temperature Profiles

Velocity Profile :

$$\frac{u}{U(s)} = (2\eta - 2\eta^3 + \eta^4) + \frac{\lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4)$$

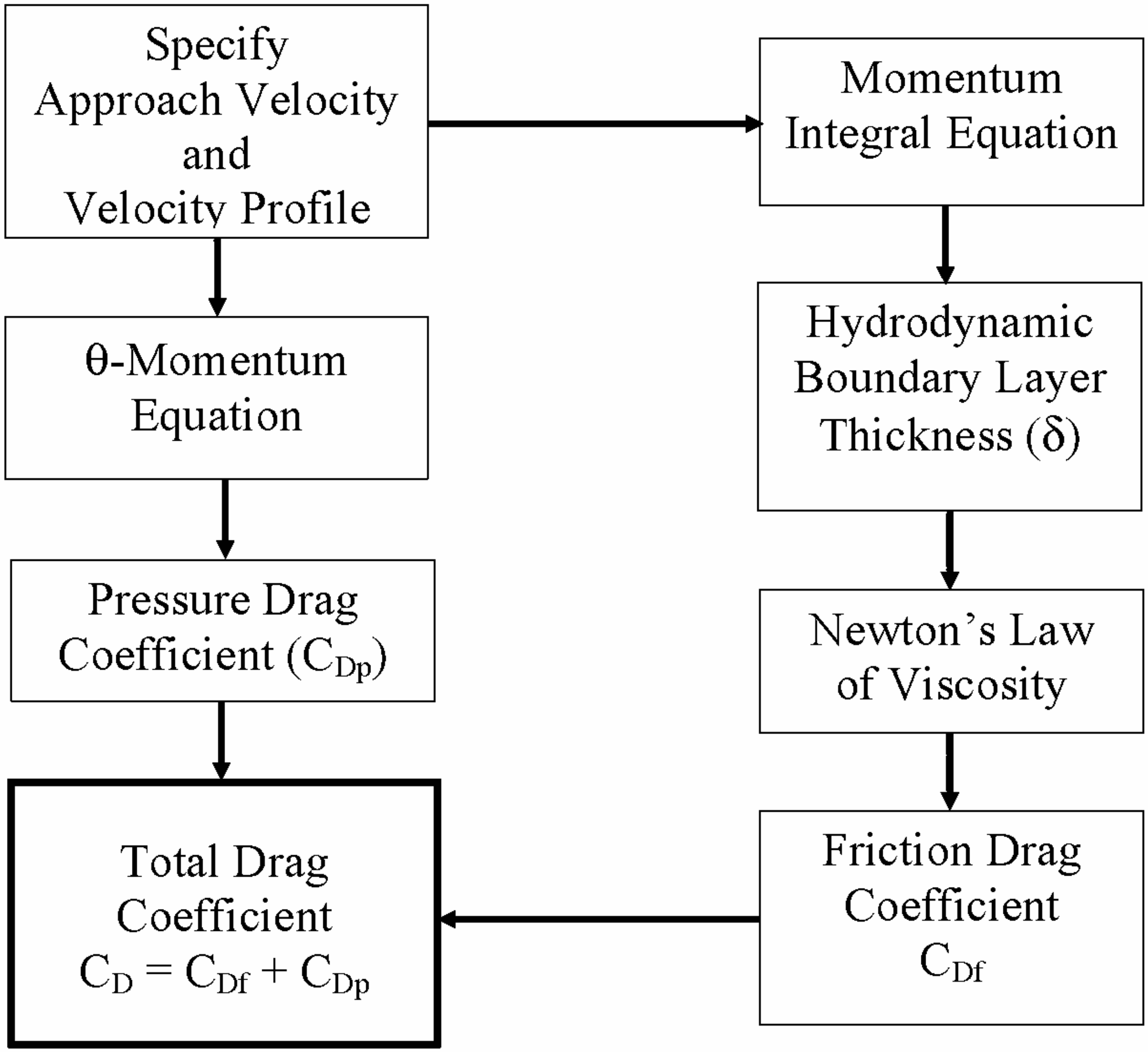
where $0 \leq \eta \leq 1$ **and** $\lambda = \frac{\delta^2}{\nu}\frac{dU(s)}{ds}$

Temperature Profiles:

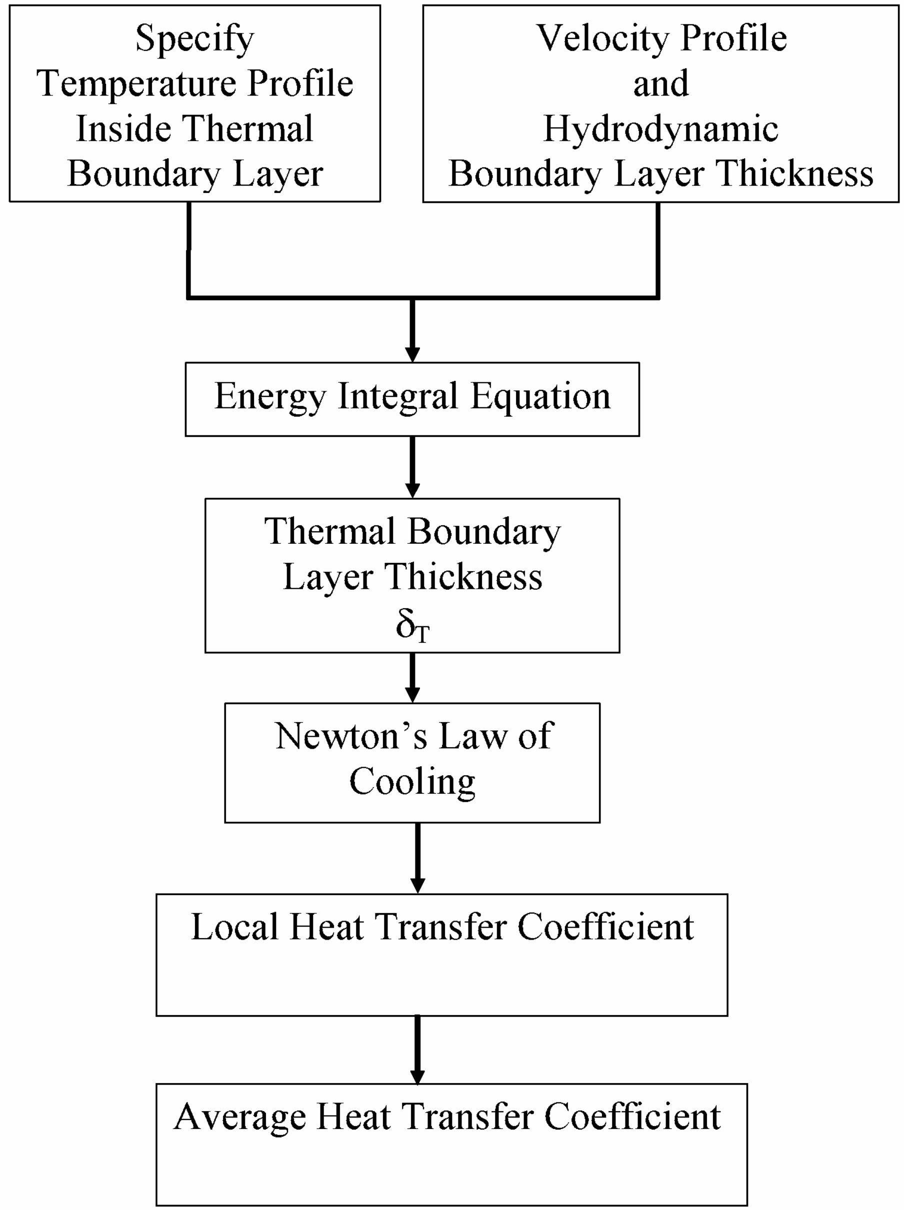
For UWT: $$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{3}{2}\eta_T + \frac{1}{2}\eta_T^3$$

For UWF: $$T - T_\infty = \frac{2q\delta_T}{3k_f}\left(1 - \frac{3}{2}\eta_T + \frac{1}{2}\eta_T^3\right)$$

Fluid Flow Model



Heat Transfer Model



Analytical Results (Fluid Flow)

Total Drag Coefficient:

$$C_D = \frac{1.353 + 4.43\epsilon^{1.35}}{\sqrt{Re_{\mathcal{L}}}} + \left(1.1526 + \frac{1.26}{Re_{\mathcal{L}}}\right)\epsilon^{0.95}$$

when $\epsilon \rightarrow 1$

$$C_D = \frac{5.786}{\sqrt{Re_D}} + 1.152 + \frac{1.260}{Re_D}$$

when $\epsilon \rightarrow 0$

$$C_D = \frac{1.353}{\sqrt{Re_L}}$$

Analytical Results (Heat Transfer)

Heat Transfer Parameter:

$$\frac{Nu_{\mathcal{L}}}{Re_{\mathcal{L}}^{1/2} Pr^{1/3}} = \begin{cases} 0.75 - 0.16 \exp\left(\dfrac{-0.018}{\epsilon^{3.1}}\right) & \text{UWT} \\ 0.91 - 0.31 \exp\left(\dfrac{-0.09}{\epsilon^{1.79}}\right) & \text{UWF} \end{cases}$$

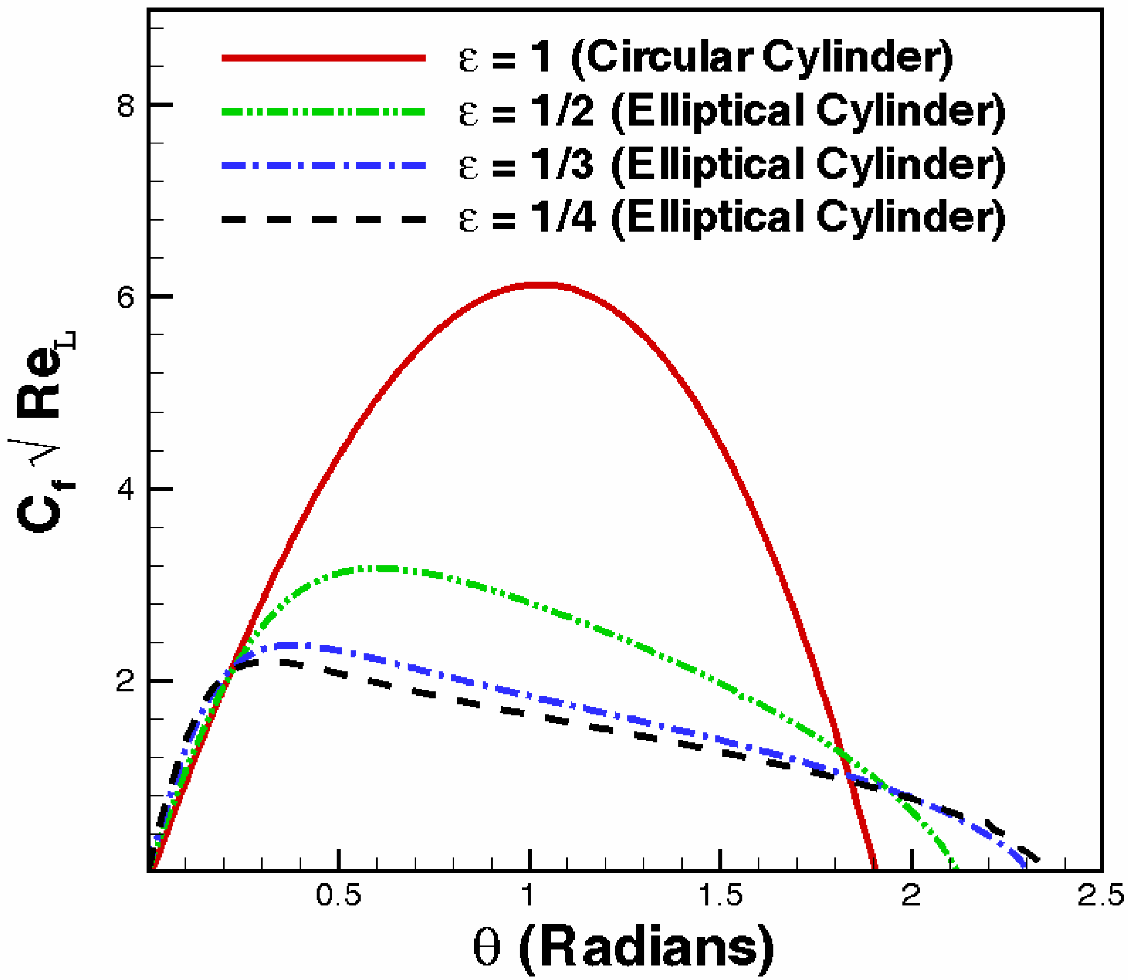
when $\epsilon \rightarrow 1$

$$\frac{Nu_D}{Re_D^{1/2} Pr^{1/3}} = \begin{cases} 0.5930 & \text{UWT} \\ 0.6321 & \text{UWF} \end{cases}$$

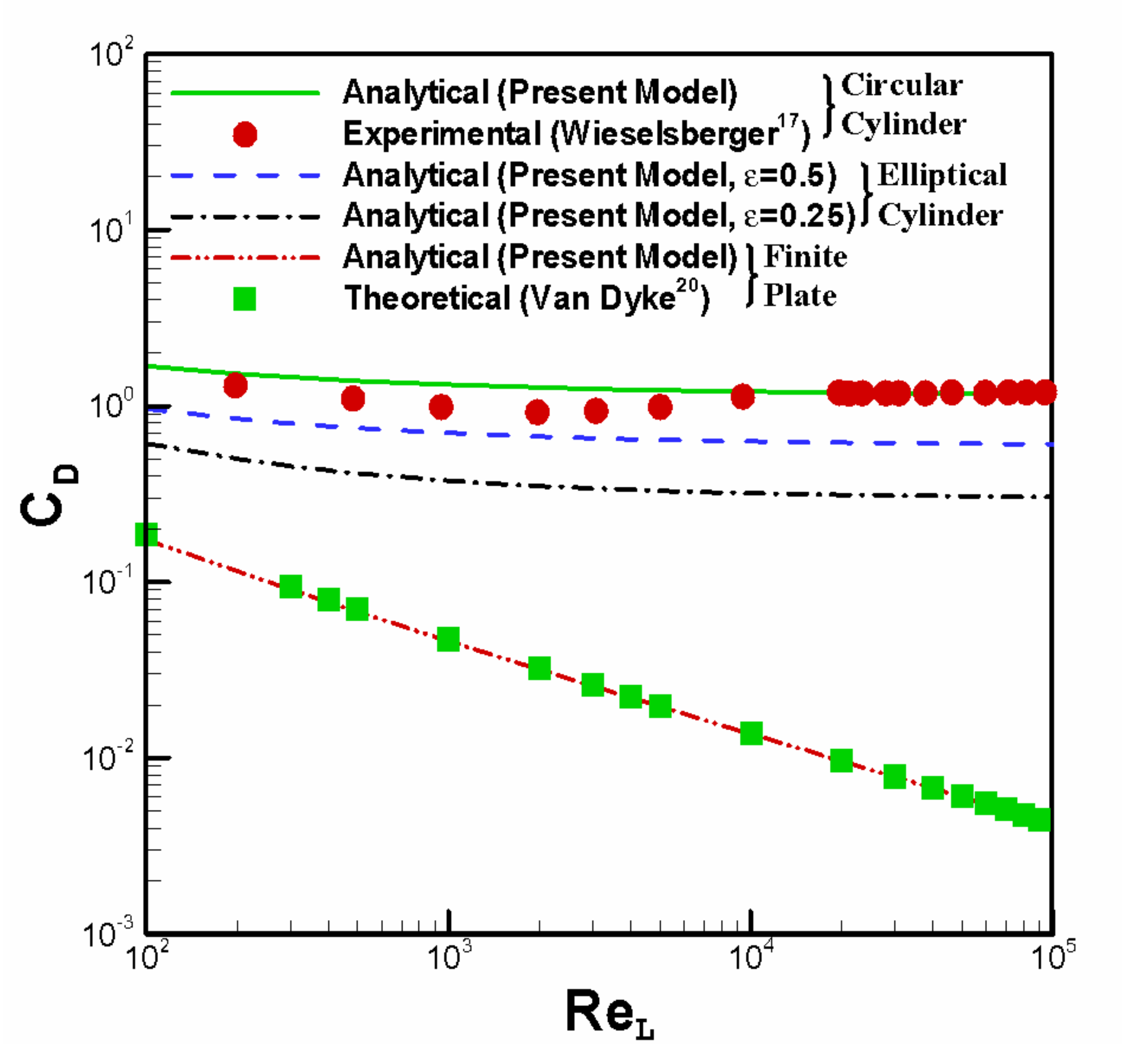
when $\epsilon \rightarrow 0$

$$\frac{Nu_L}{Re_L^{1/2} Pr^{1/3}} = \begin{cases} 0.750 & \text{UWT} \\ 0.912 & \text{UWF} \end{cases}$$

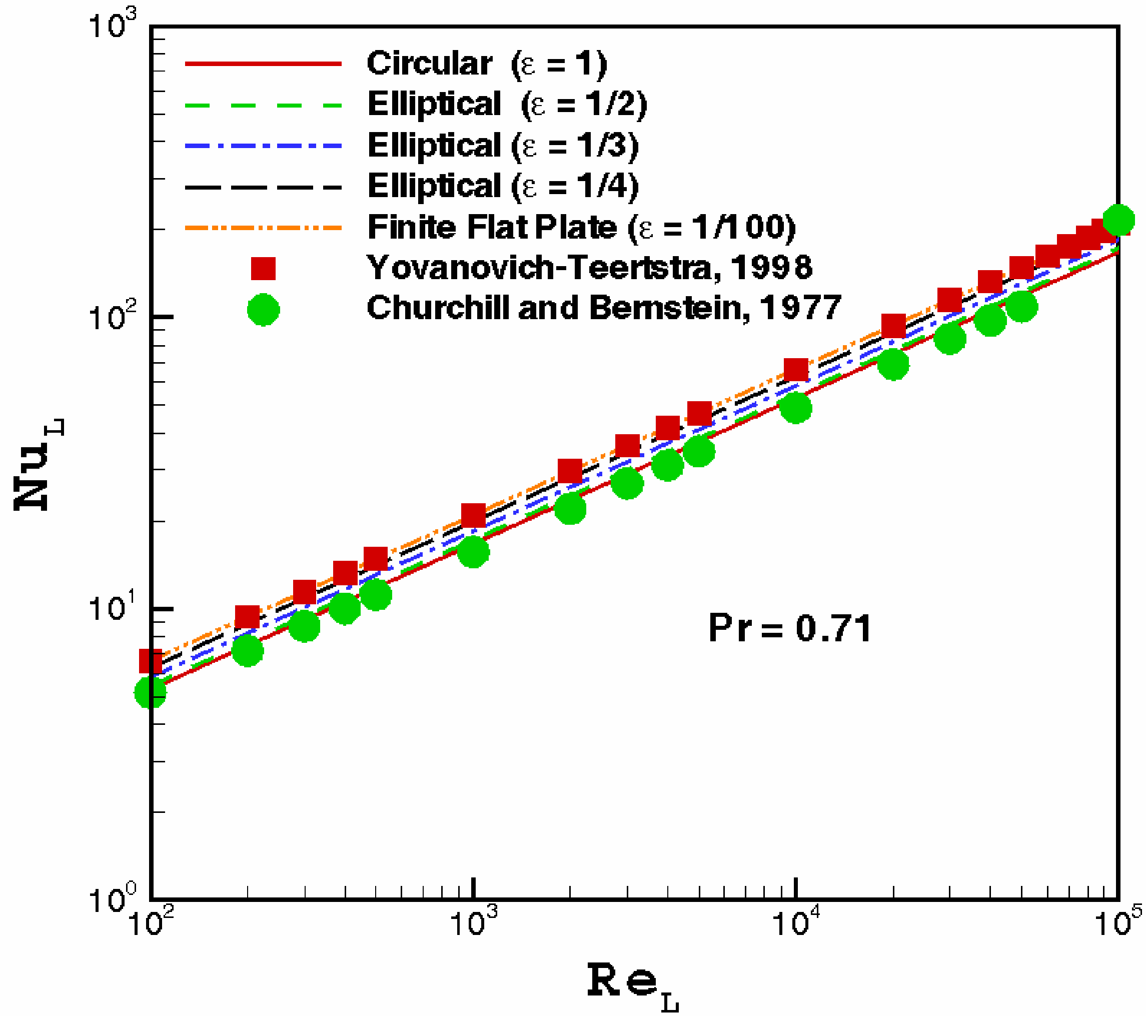
Local Shear Stress



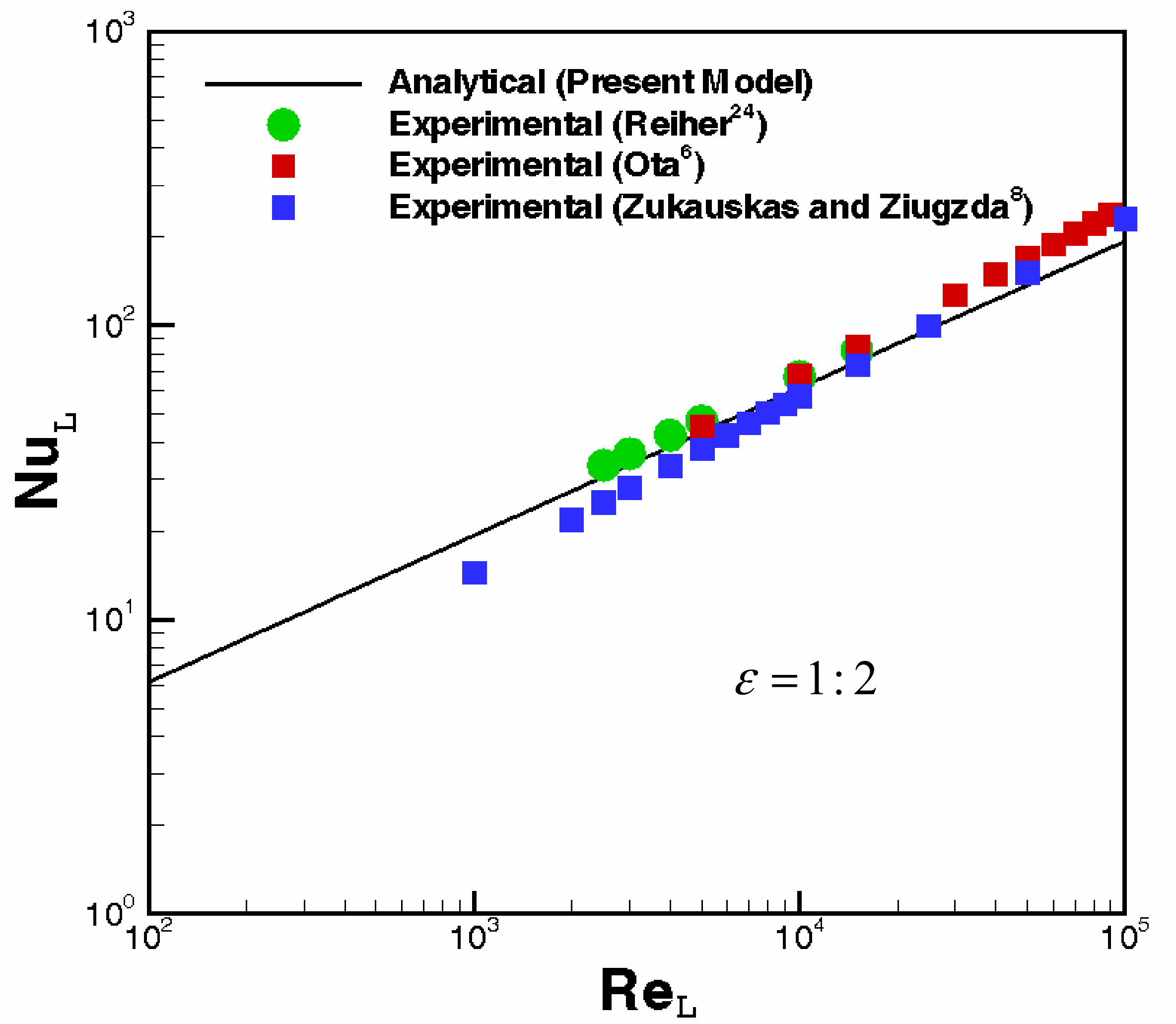
Total Drag Coefficient with Reynolds Number



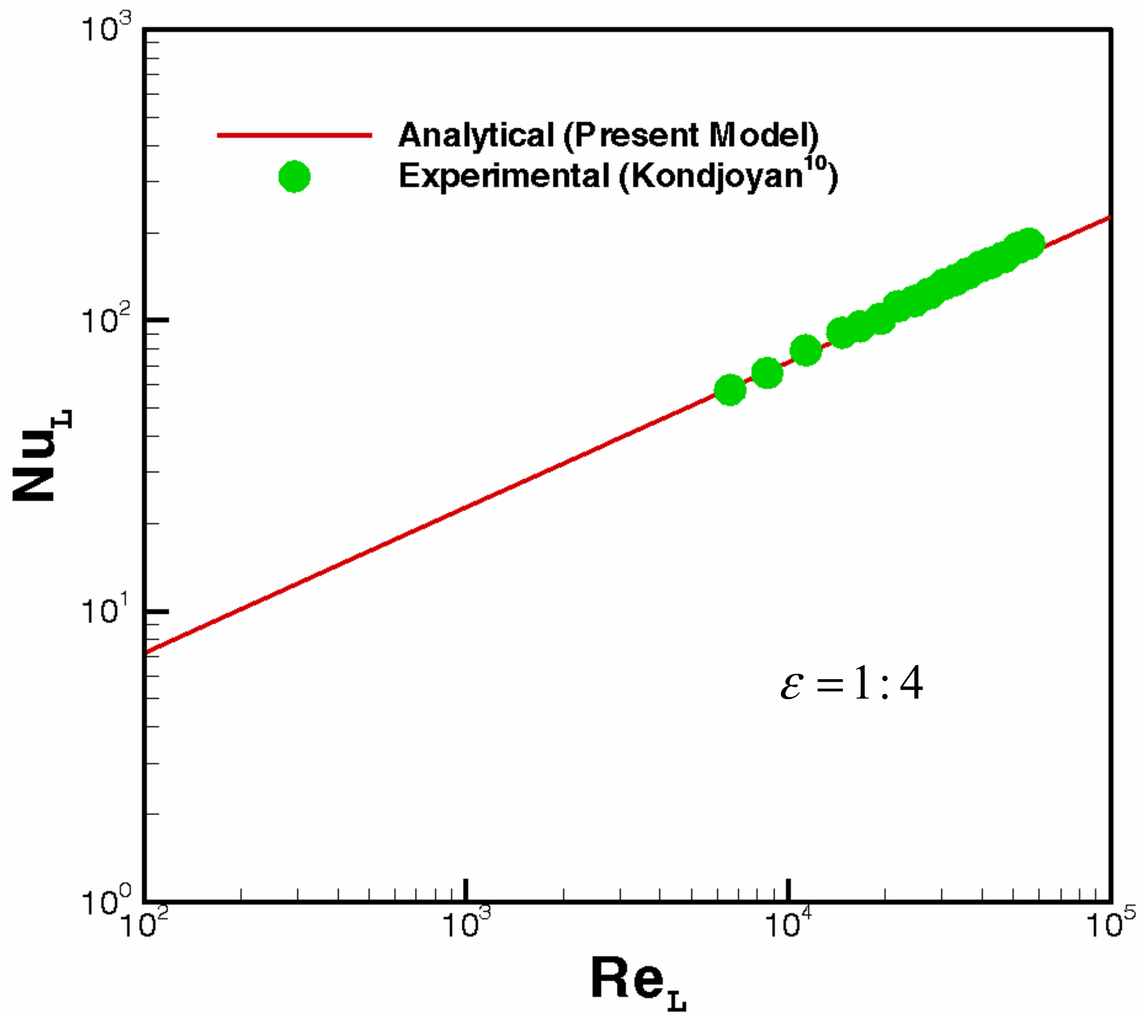
Average Nusselt Number with Reynolds Number



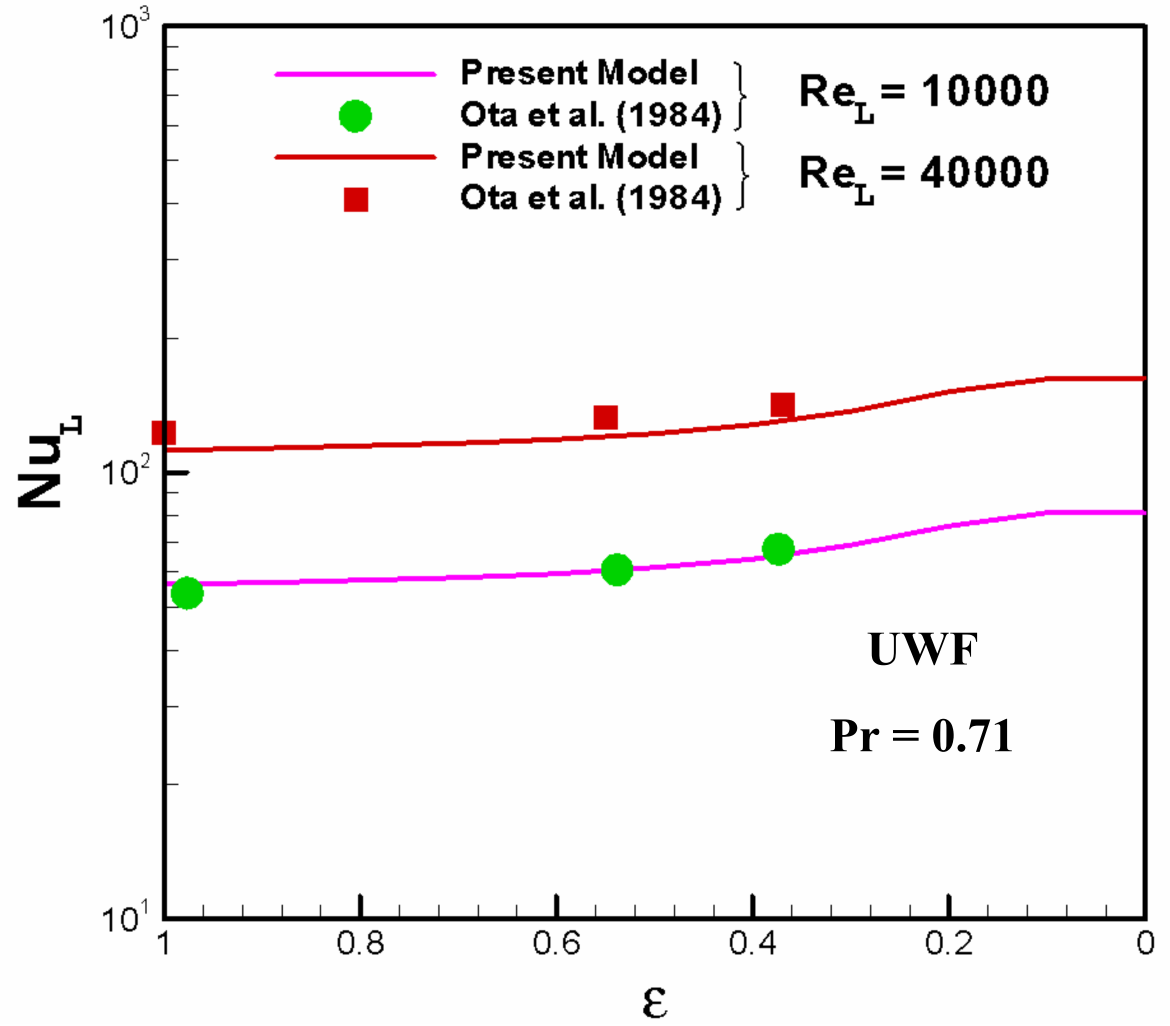
Average Nusselt Number with Reynolds Number



Average Nusselt Number with Reynolds Number



Average Nusselt Number with Axis Ratio



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Summary and Conclusions

- Approximate analytical method gives:
  - One model for drag coefficient
  - Two models for heat transfer coefficient
- These models can be used for:
  - Laminar range ($40 \le Re_D \le 10^5$)
  - Large Prandtl numbers ($Pr \ge 0.71$)
  - Any axis ratio ($0 \le \varepsilon \le 1$)

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