Review of Thermal Joint Resistance Models for Non-Conforming Rough Surfaces in a Vacuum

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CONTENTS

INTRODUCTION

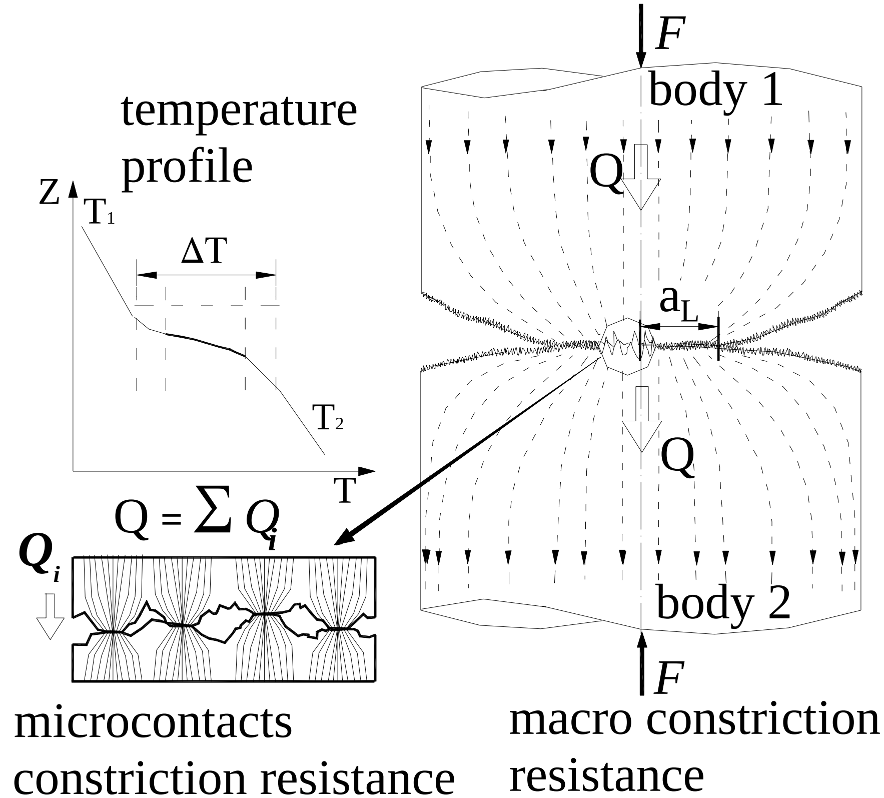
- conduction (microcontacts)
- conduction (interstitial fluid)
- radiation across the gap

- two resistances in series represent TCR in a vacuum, many researchers assumed

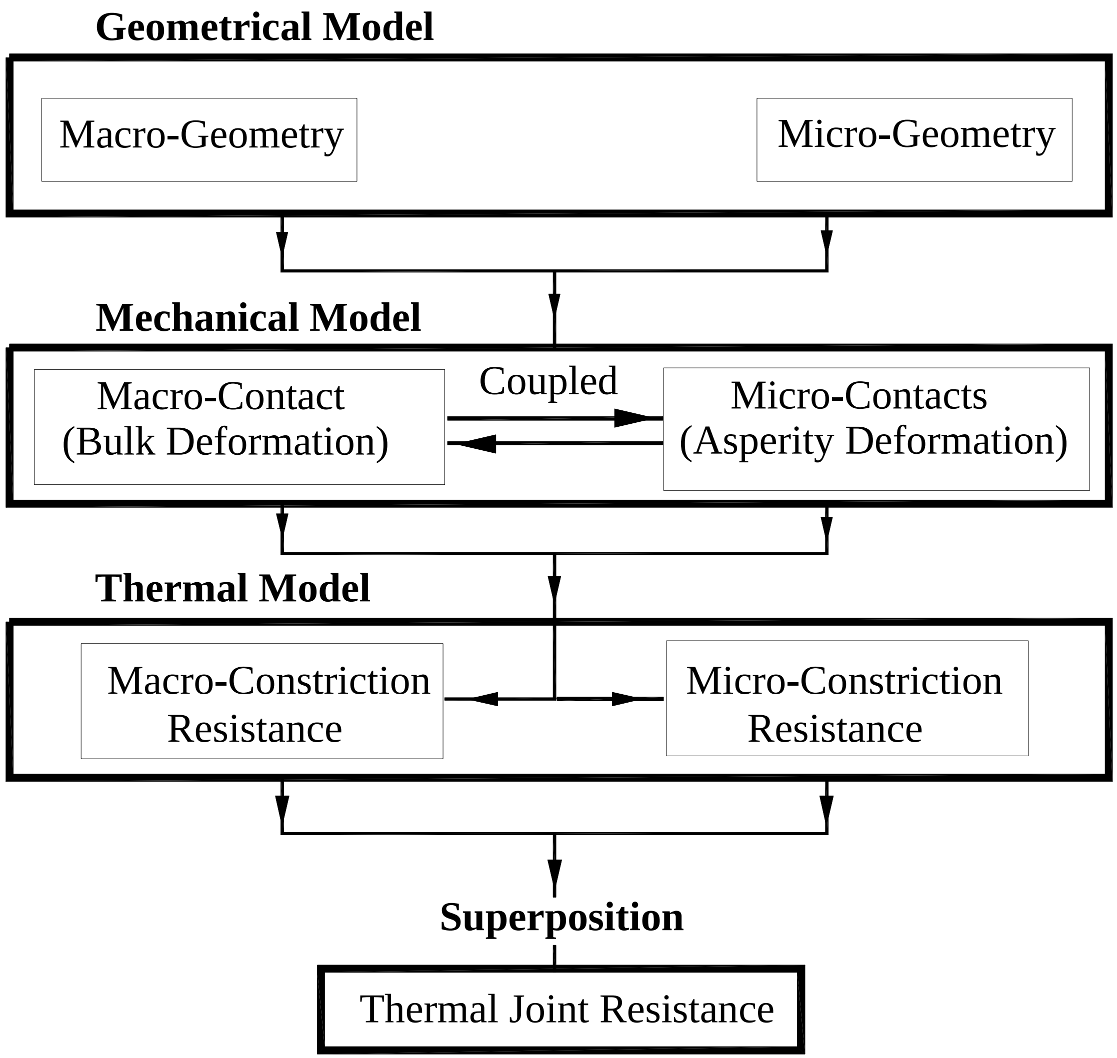
$$R_j = R_{mic} + R_{mac}$$

- spherical rough contact includes two problems:
  - micro scale problem
  - macro scale problem

- macrocontact area: region where microcontacts are distributed

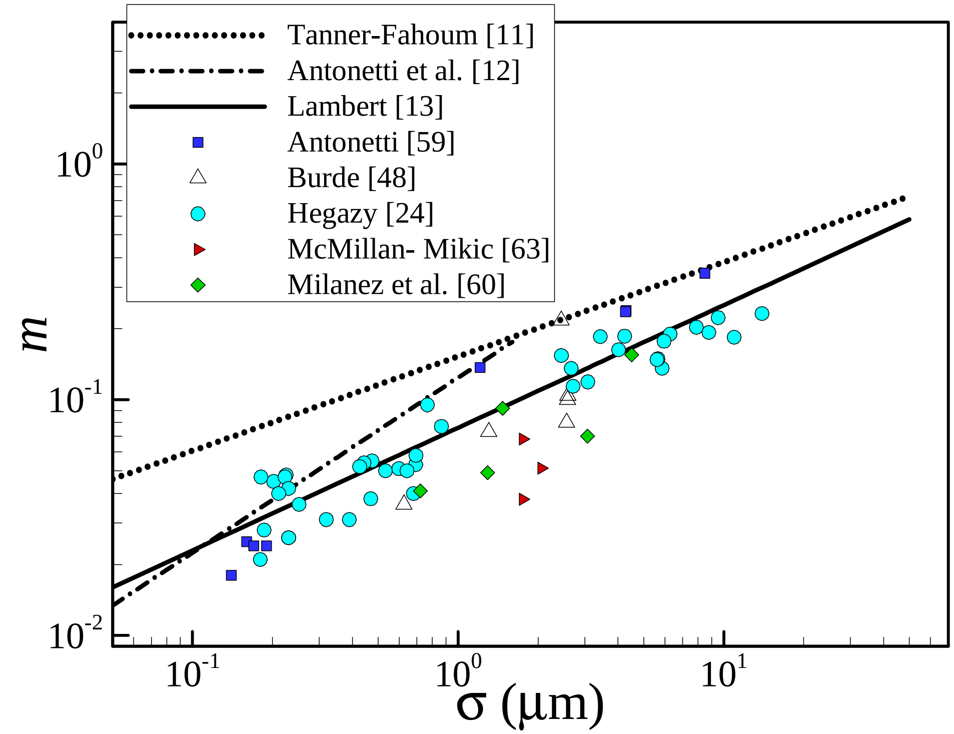


PROBLEM STATEMENT



ROUGH SURFACE PARAMETERS

- all solid surfaces are rough
- Gaussian rough surface
- approximate estimations of *m*



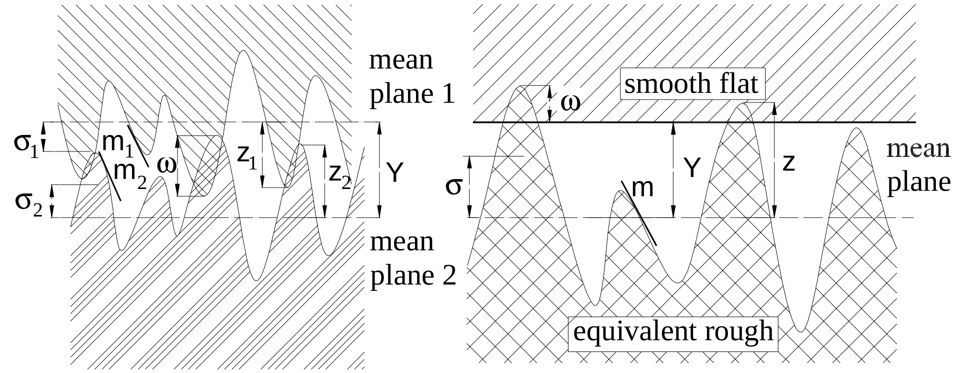
$$\sigma = R_q = \sqrt{\frac{1}{L}\int_0^L z^2(x)\,dx}$$

$$m = \frac{1}{L}\int_0^L \left|\frac{dz(x)}{dx}\right| dx$$

Reference	Correlation
Tanner and Fahoum	$m = 0.152\,\sigma^{0.4}$
Antonetti et al.	$m = 0.124\,\sigma^{0.743}, \quad \sigma \le 1.6(\mu m)$
Lambert	$m = 0.076\,\sigma^{0.52}$

EQUIVALENT ROUGH SURFACE

- equivalent surface circumvents problem of misalignment of contacting peaks, Francis (1977)
- equivalent surface of two Gaussian surfaces is itself Gaussian, Greenwood (1967)
- equivalent surface will be in general less anisotropic than the two contacting surfaces, Francis (1977)

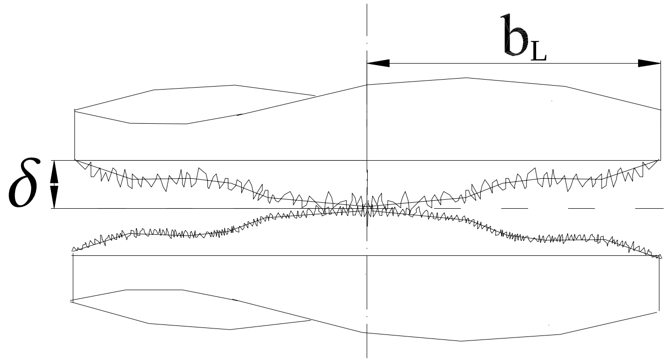


a) section through two contacting surfaces

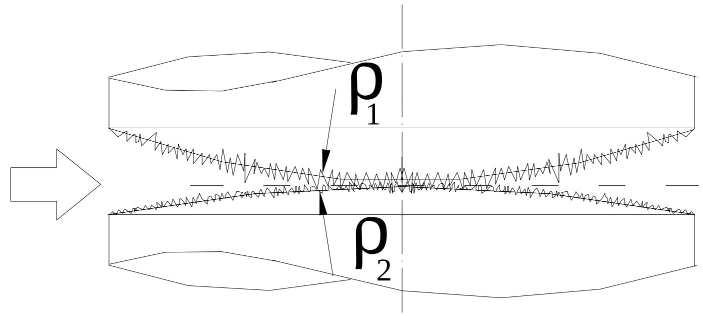
b) corresponding section through equivalent rough - smooth flat

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \quad \text{and} \quad m = \sqrt{m_1^2 + m_2^2}$$

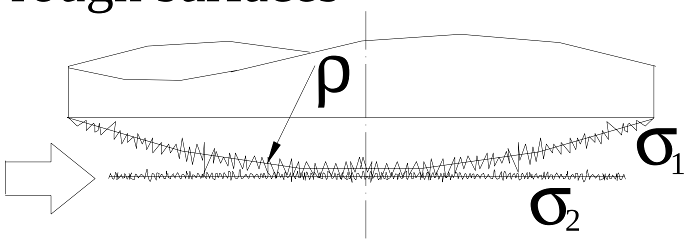
GEOMETRICAL MODELING



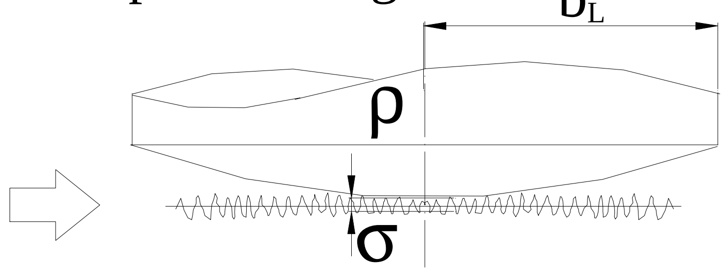
a) contact of non-conforming rough surfaces



b) contact of two rough spherical segments



c) rough sphere-flat contact, effective radius of curvature



d) equivalent sphere-flat contact, effective radius and roughness

$$\rho = \frac{b_L^2}{2\delta}$$

$$\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2}$$

MICROHARDNESS

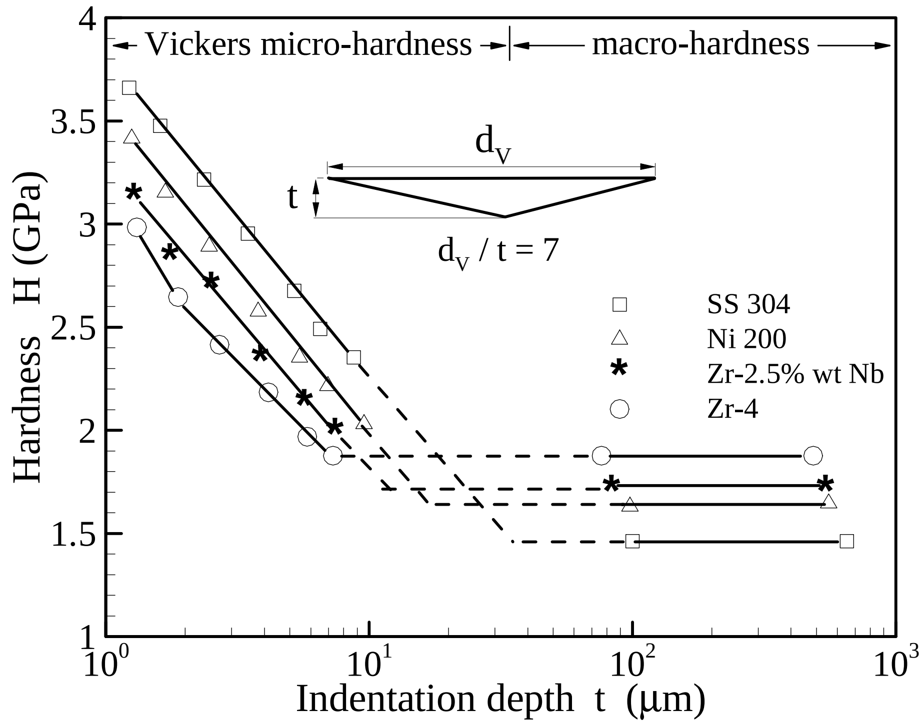
Hegazy (1985)

• effective microhardness is significantly greater than the bulk hardness

• microhardness decreases with increasing depth of the indenter until bulk hardness is obtained

Sridhar (1994)

• suggested empirical relations to estimate Vickers microhardness coefficients using bulk hardness



$$c_1 = H_{BGM}(4.0 - 5.77\kappa + 4.0\kappa^2 - 0.61\kappa^3)$$

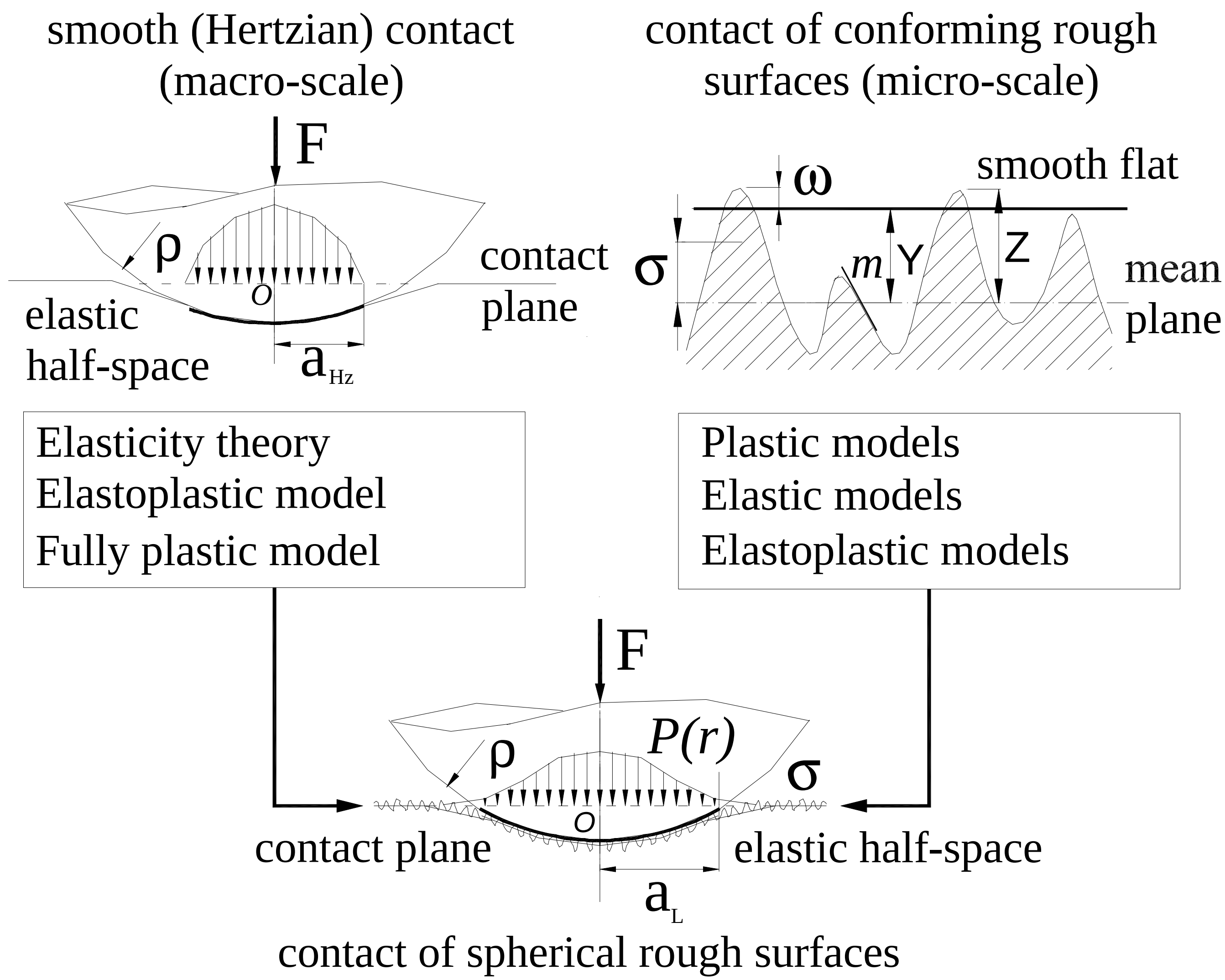
$$c_2 = -0.57 + 0.82\kappa - 0.41\kappa^2 + 0.06\kappa^3$$

$$\kappa = H_B/H_{BGM} \qquad 0.41 \le \kappa \le 2.39$$

$$H_v = c_1(d'_v)^{c_2}$$

$$d'_v = d_v/d_0$$

MECHANICAL ANALYSIS



MACROCONTACT PROBLEM

Hertz (1881) (elastic contact of smooth spheres)

• each body is modeled as elastic half-space loaded over small contact region

• strains are small; surfaces are frictionless

• pressure distribution assumed

$$P(r) = P_0 \sqrt{1 - (r/a_{Hz})^2}$$

Elastoplastic and Fully Plastic [Cavity model of Johnson (1985)]

• when yield point is exceeded plastic zone is small and fully contained by material which remains elastic

• contact load must be increased about 400 times from initial yielding to fully plastic flow, elastoplastic transition region is very long

• when the plastic deformation is severe, elastic deformation may be neglected

• Hardy et al. (1971) showed plastic flow leads to "flattening" of pressure distribution

MICROCONTACT

Common Assumptions

• contacting surfaces are rough, isotropic, with Gaussian asperity distribution

• microcontact are independent; interfacial force on microcontact acts normally (no friction)

• deformation mechanics (stress and displacement) are uniquely determined by shape of equivalent surface

Plastic Models

• Abott and Firestone (1933) model assumed asperities are "flattened" or, equivalently penetrate into the smooth surface without any change in shape of the part of surfaces not yet in contact

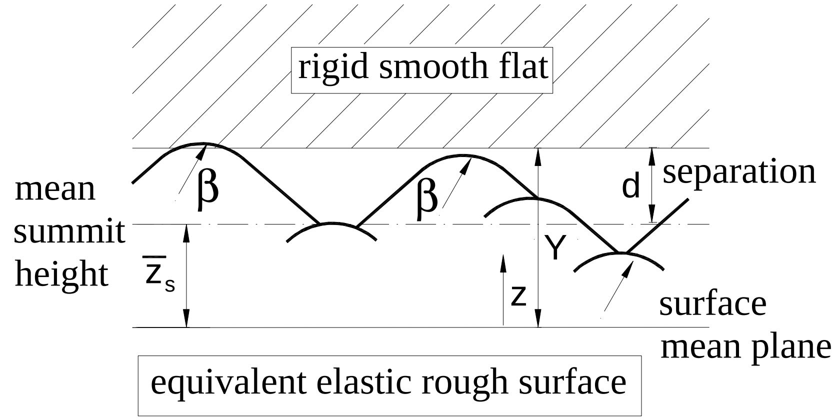
$$A_r/A_a = P_m/H_{mic}$$

• Tsukizoe and Kisakado (1965) and Cooper et al. (1969) derived relations for size and number of microcontacts

MICROCONTACT (CONT.'D)

Elastic Models, GW (1966)

- all summits have same radius of curvature; with Gaussian distribution
- distribution of summit heights is same as heights standard deviation, i.e., $\sigma_s = \sigma$
- summits deform elastically and Hertz theory applied for each individual summit



Elastoplastic Models

- Chang et al. (1987)
- Zhao et al. (2000)

MICROCONTACTS DEFORMATIONS

Plasticity Index

• GW "most surfaces have plasticity indices larger than 1.0, except for very smooth surfaces, asperities will flow plastically under the lightest loads"

$$\gamma_{GW} = (E'/H)\sqrt{\sigma/\beta}$$

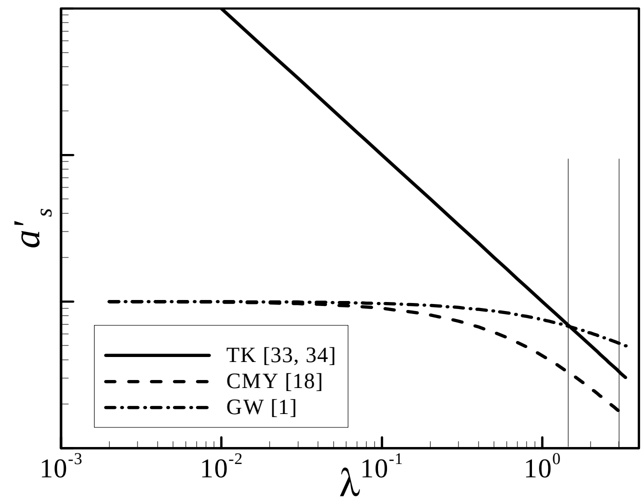
• Mikic (1974) reported mode of deformation, as stated by GW, depends only on material properties and shape of asperities, and it is not sensitive to pressure level

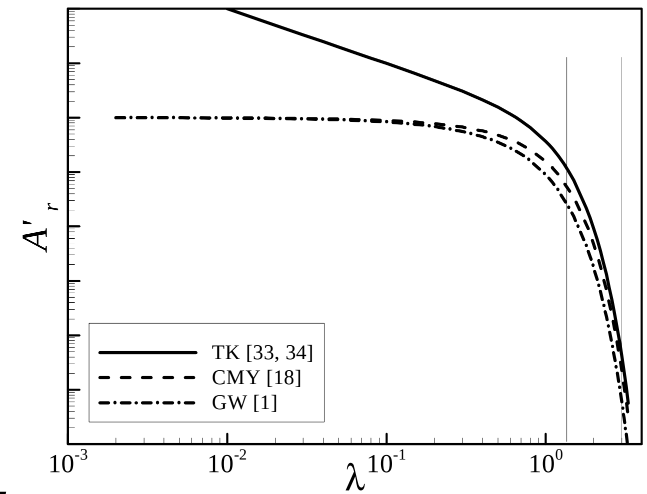
$$\gamma_{Mikic} = H_{mic}/E'm$$

$$\lambda = Y/\sqrt{2}\,\sigma$$

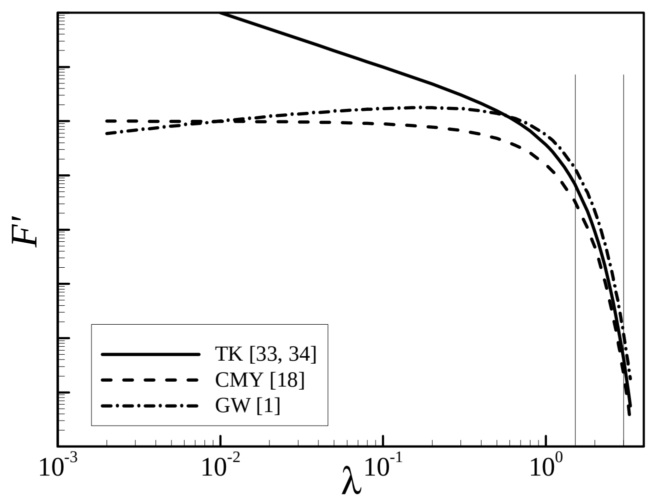
Model	a'_s	n'_s	F'
GW	$\sqrt{\exp(-\lambda^2)/\mathrm{erfc}(\lambda) - \sqrt{\pi}\,\lambda}$	$\mathrm{erfc}(\lambda)$	$\sqrt{\lambda}\,\exp\left(-\frac{\lambda^2}{2}\right)\left[(1+2\lambda^2)K_{\frac{1}{4}}\left(\frac{\lambda^2}{2}\right) - 2\lambda^2 K_{\frac{3}{4}}\left(\frac{\lambda^2}{2}\right)\right]$
TK	$1/\lambda$	$\lambda\exp(-\lambda^2)$	$\exp(-\lambda^2)/\lambda$
CMY	$\exp(\lambda^2)\mathrm{erfc}(\lambda)$	$\exp(-2\lambda^2)/\mathrm{erfc}(\lambda)$	$\mathrm{erfc}(\lambda)$

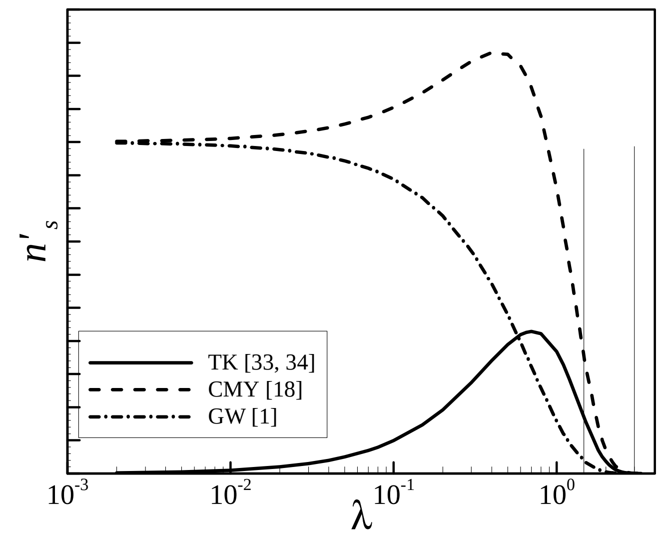
ELASTIC, PLASTIC MODEL TRENDS





$$\lambda = Y / \sqrt{2}\,\sigma$$





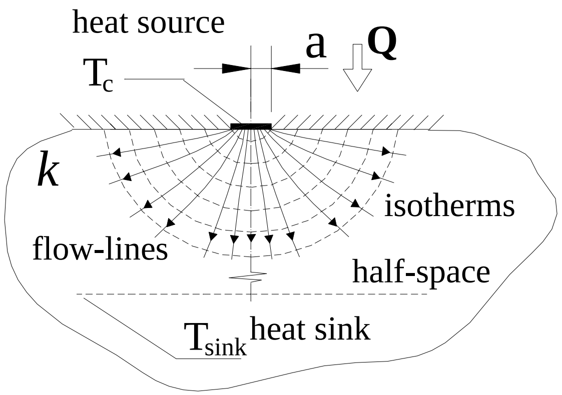
THERMAL ANALYSIS

Common Assumptions

- contacting solids are isotropic; thermophysical properties are constant
- contacting solids are thick relative to roughness or waviness scales
- surfaces are clean; contact is static
- radiation heat transfer is negligible
- microcontacts are circular; microcontacts are isothermal and flat
- steady-state heat transfer at microcontacts

SPREADING/CONSTRICTION RESISTANCE

TCR models assume a number of heat channels exist within macrocontact area



1. Heat Source on a Half-space

classical steady-state solutions are available for two boundary conditions;

- isothermal
- isoflux

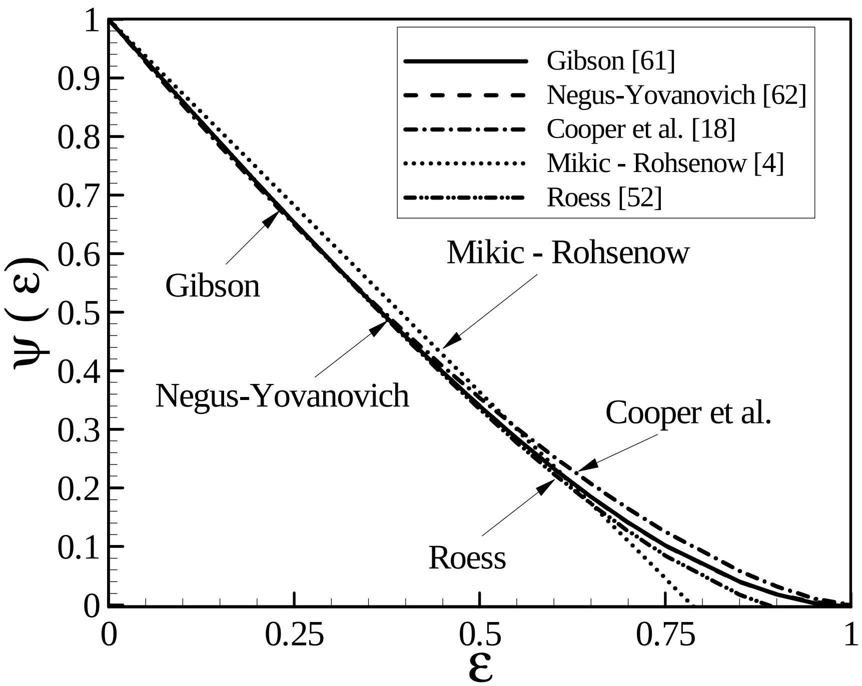
$$R_{s,isothermal} = 1/(4ka)$$

- difference between isoflux and isothermal heat sources is 8%

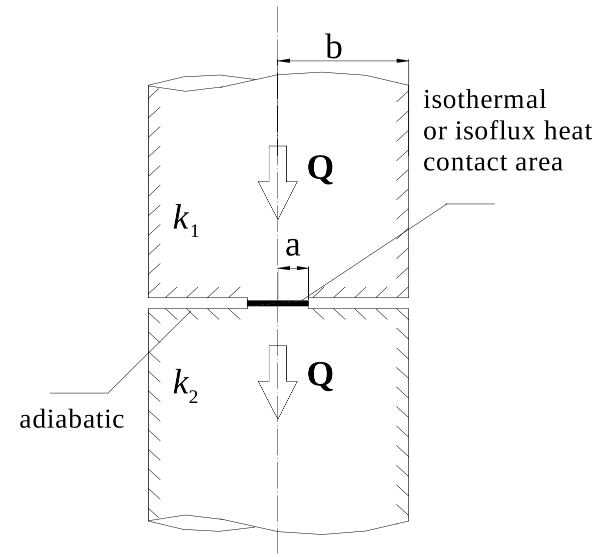
$$R_{s,isoflux} = 1.08 R_{s,isothermal}$$

SPREADING RESISTANCE 2

2. Flux Tube Solution



Reference	Correlation
Roess (1950)	$1 - 1.4093\varepsilon + 0.2959\varepsilon^3 + 0.0525\varepsilon^5 + 0.021041\varepsilon^7 + 0.0111\varepsilon^9 + 0.0063\varepsilon^{11}$
Mikic-Rohsenow (1966)	$1 - 4\varepsilon/\pi$
Cooper et al. (1969)	$(1-\varepsilon)^{1.5}$
Gibson (1976)	$1 - 1.4092\varepsilon + 0.3381\varepsilon^3 + 0.0679\varepsilon^5$
Negus-Yovanovich (1984)	$1 - 1.4098\varepsilon + 0.3441\varepsilon^3 + 0.0431\varepsilon^5 + 0.0227\varepsilon^7$



$$R_{two\ flux\ tubes} = \frac{\psi(\varepsilon)}{4k_1 a} + \frac{\psi(\varepsilon)}{4k_2 a} = \frac{\psi(\varepsilon)}{2k_s a}$$

NON-CONFORMING ROUGH TCR MODELS

Clausing and Chao (1963)

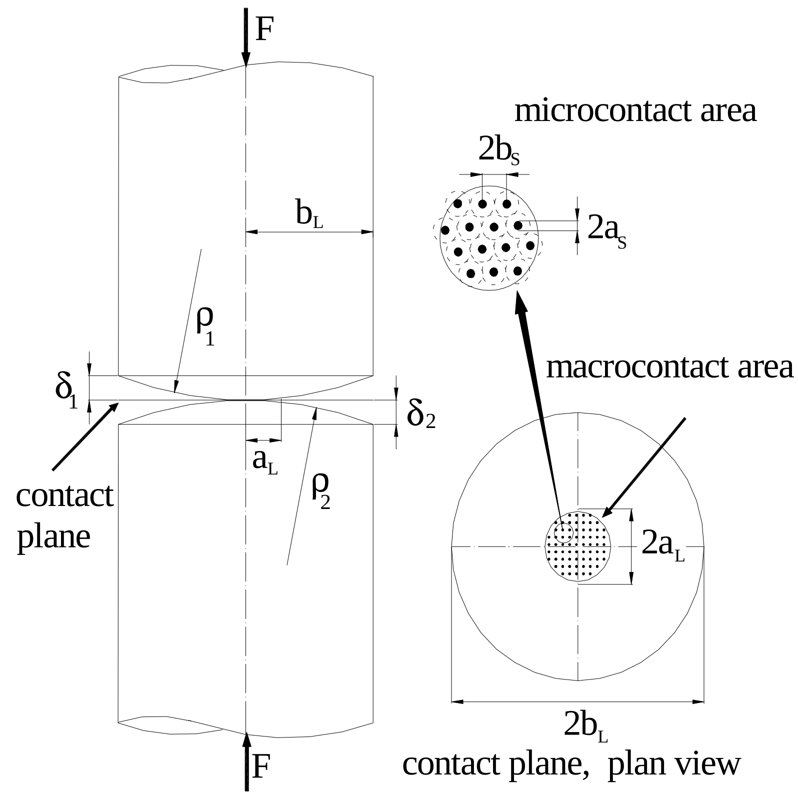
- plastic microcontacts; H_{mic} corrected by empirical factor to account for elastic deformation of asperities

- identical microcontacts uniformly distributed, in triangular array, over macrocontact region

- microcontacts considered as isothermal circular heat sources on a half-space

- average size of microcontacts a_s is independent of load and it is of same order of magnitude as surface roughness, i.e., $a_s = \sigma$

- neglecting effect of roughness on macrocontact, radius of macrocontact, a_L, obtained from Hertz theory



$$R_j = R_{mic} + R_{mac}$$

NON-CONFORMING ROUGH TCR MODELS 2

- **Mikic and Rohsenow (1966)** studied TCR for various types of surface waviness and conditions
- **Kitscha (1982)** and **Fisher (1985)** developed models similar to Clausing and Chao's model and experimentally verified their models for relatively small radii of curvature and different levels of roughness
- **Burde (1977)** derived expressions for size distribution, and number of microcontacts, which described the increase in macrocontact radius for increasing roughness
- **Lambert (1995)** studied TCR of two rough spheres in a vacuum

COMPARISON WITH DATA

- Clausing and Chao (1963)
- Yovanovich (1982)
- Lambert (1995)
- Yovanovich (1986) elasto-constriction approximation

Parameters
$57.3 \leq E' \leq 114.0\ (GPa)$
$16.6 \leq k_s \leq 75.8\ (W/mK)$
$0.12 \leq \sigma \leq 13.94\ (\mu m)$
$0.04 \leq m \leq 0.34\ (-)$
$0.013 \leq \rho \lessapprox 120\ (m)$

Ref.	Researcher	Specimen Material(s)
A	Antonetti (1983)	Ni 200
B	Burde (1977)	SPS 245, Carbon Steel
F	Fisher (1985)	Ni 200, Carbon Steel
H	Hegazy (1985)	Ni 200 SS 304 Zircaloy4 Zr-2.5%wt Nb
K	Kitscha (1982)	Steel 1020,Carbon Steel
M	Milanez et al. (2003)	SS 304

TCR LIMITS

Non-Conforming Rough Surface Model

$$R_j = R_{mic} + R_{mac}$$

Elastoconstriction Model

$$\sigma \rightarrow 0$$

$$a_L \rightarrow a_{Hz}$$

$$R_j \rightarrow R_{mac}$$

$$R_{mac} = \frac{\psi(a_{Hz}/b_L)}{2k_s a_{Hz}}$$

Conforming Rough Surface Model

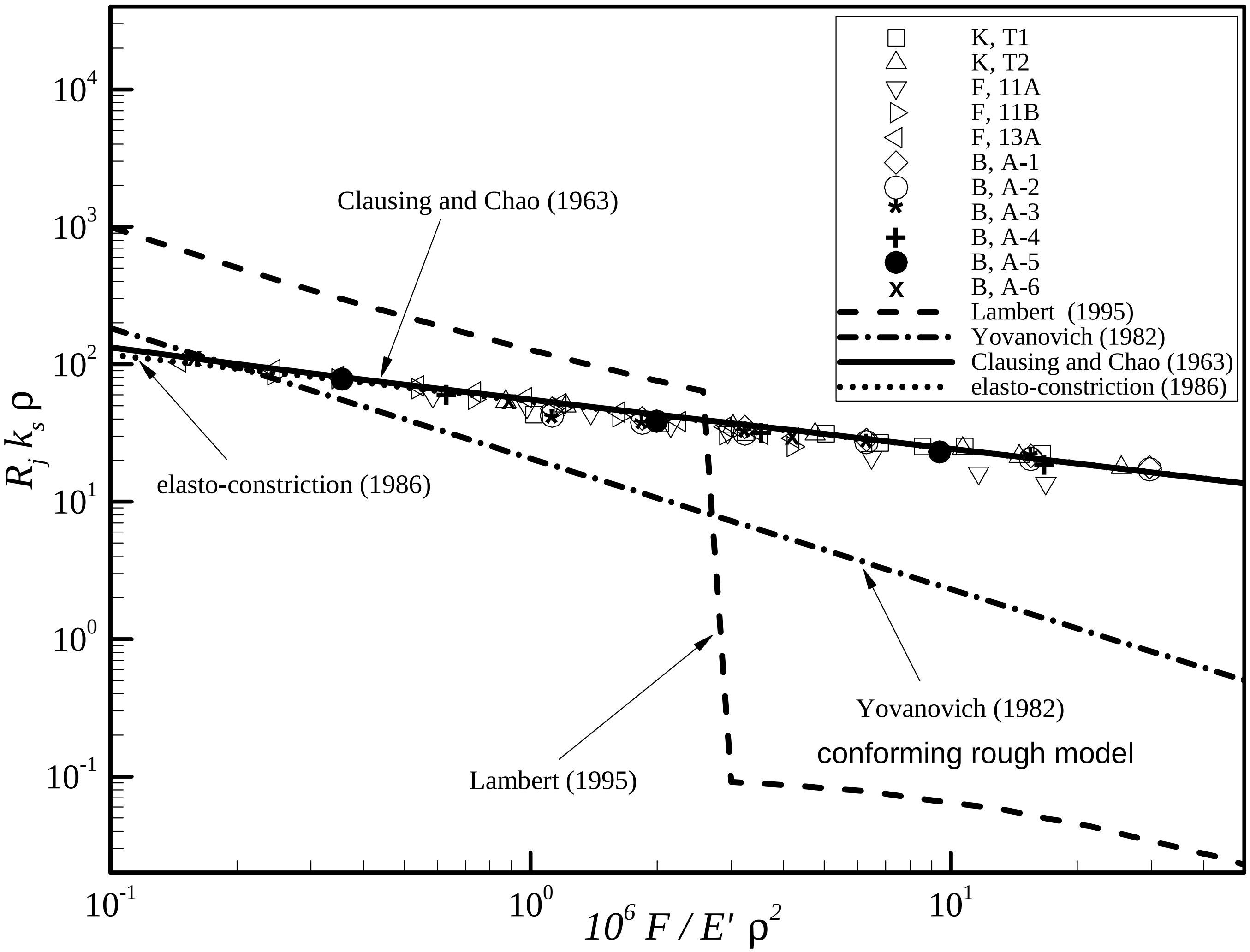
$$\rho \rightarrow \infty$$

$$P \rightarrow F/\pi b_L^2$$

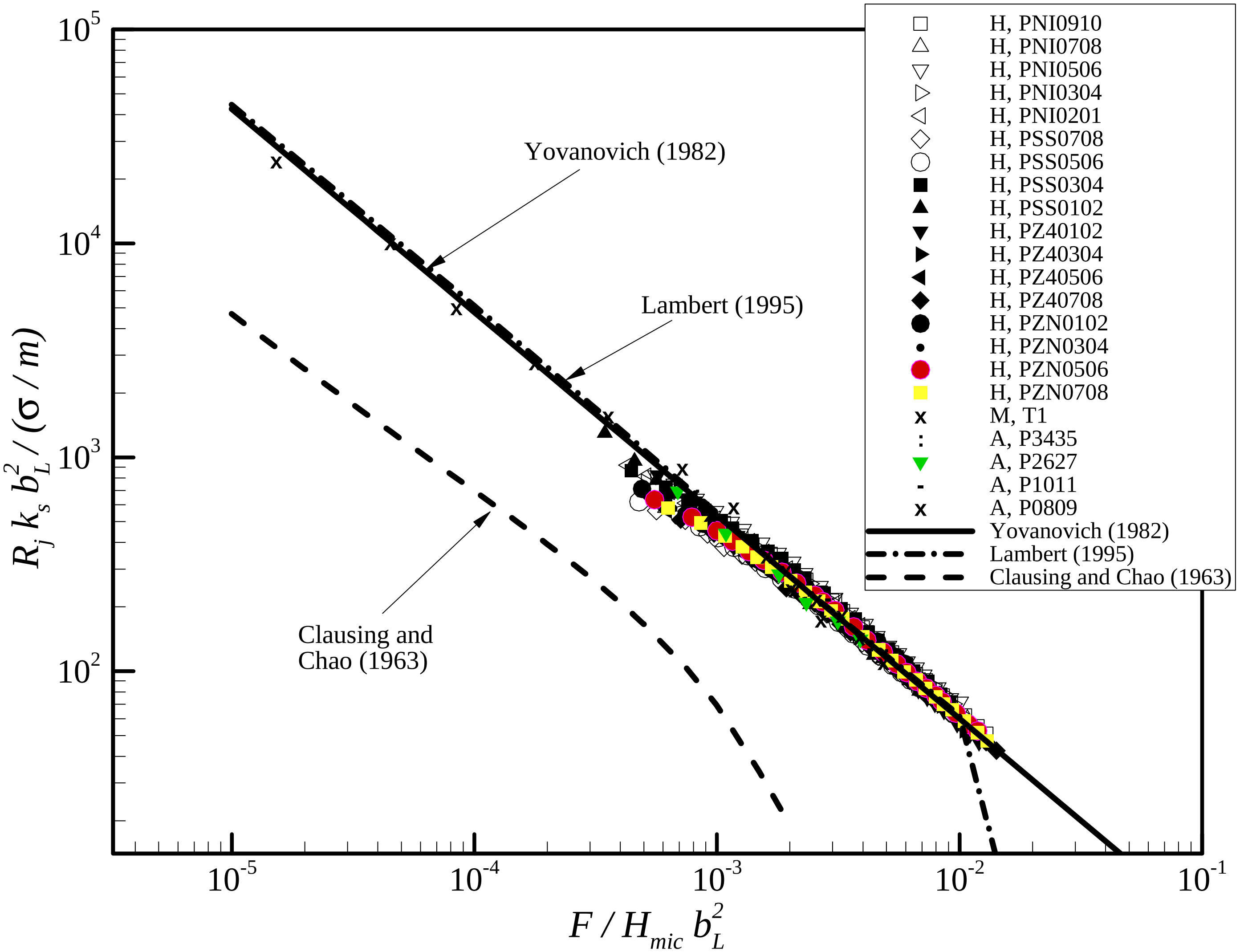
$$R_j \rightarrow R_{mic}$$

$$R_{mic} = \frac{(\sigma/m)}{1.25 A_a k_s (P/H_c)^{0.95}}$$

ELASTOCONSTRICTION LIMIT



CONFORMING ROUGH LIMIT



SUMMARY AND CONCLUSIONS

- TCR modeling consists of three analyses: geometrical, mechanical, and thermal; each one includes a macro and micro part
- recommended empirical correlations, *m*, were summarized and compared with experimental data. Uncertainty of correlations is high
- a set of scale relationships were derived for contact parameters for GW elastic, CMY and TK plastic conforming rough models. It was graphically shown that their trends are similar
- trends of contacting rough surfaces was determined essentially by surface statistical characteristics. Also combination of plastic and elastic modes would introduce no new features

SUMMARY AND CONCLUSIONS 2

- common assumptions of existing thermal analyses were summarized

- existing correlations for flux tube resistance were compared; it was shown all correlations show good agreement for the applicable range

- experimental data of many researchers were summarized and grouped into two limiting cases:
    - conforming rough $\rho \rightarrow \infty$
    - elasto-constriction $\sigma \rightarrow 0$
- data were non-dimensionalized and compared with TCR models at limiting cases

- no existing theoretical model covers both limiting cases

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