Models and Experiments for Laminar Natural Convection from Heated Bodies in Enclosures

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Outline

• Introduction and problem description
• Literature review and objectives
• Experimental measurements
• Model development and validation
• Summary and conclusions
Problem Definition

- Steady state, natural convection
- Non-intersecting inner and outer boundaries
- Isothermal boundary conditions, \( T_i > T_o \)

Geometry:
- Relative boundary size
  \[
  \sqrt{A_o} / \sqrt{A_i} = d_o / d_i \quad \text{(spheres)}
  \]
- Effective gap spacing
  \[
  \delta_e = (d_o - d_i) / 2 \quad \text{(spheres)}
  \]
- Eccentricity
  \[
  e_h = e_v = 0
  \]
Parameter Definitions

- Total heat transfer rate
  \[ Q = \iint_{A_i} -k \frac{\partial \theta}{\partial \hat{n}} \, dA_i , \quad \theta = T(\vec{r}) - T_b \]

- Non-dimensionalized by Nusselt number
  \[ Nu_{\sqrt{A_i}} = \frac{Q}{k \sqrt{A_i (T_i - T_o)}} = S^*_{\sqrt{A_i}} \text{ for } Ra \to 0 \]

- Rayleigh number
  \[ Ra_{\sqrt{A_i}} = \frac{g \beta (T_i - T_o) (\sqrt{A_i})^3}{\nu \alpha} \]
Literature Review

Experimental and Numerical Studies

• Concentric spherical enclosures
  ▪ Experimental data for high Rayleigh number, laminar boundary layer flow only
  ▪ All other data from numerical simulations

• Other enclosure geometries
  ▪ Spheres, cubes, cylinders
  ▪ Experimental and numerical data

• No experimental data for full range of Rayleigh including transition and diffusive limit
Literature Review

Correlations and Models

• Warrington & Powe (1985), Warrington et al. (1988)
  ▪ Correlation of data for variety of inner and outer shapes
  ▪ Effective gap spacing based on equivalent spheres
  ▪ Valid for laminar boundary layer flow only

• Raithby & Hollands (1975, 1985, 1998)
  ▪ Analytically based model for concentric spheres
  ▪ Series combination of resistances of conduction layers at inner and outer boundaries
  ▪ For other geometries, effective gap spacing of Warrington & Powe (1985) recommended
Objectives

- Experimental measurements:
  - Variety of geometries, spheres, cubes, cylinder, etc.
  - Wide range of $Ra_{\sqrt{A_i}}$
    - Laminar boundary layer convection (atmospheric pressure)
    - Diffusive limit (reduced pressure)

- Analytical modeling:
  - Full range of $Ra_{\sqrt{A_i}}$ from conduction to convection
  - Applicable to wide range of geometries
    - Inner and outer boundary shapes and orientation
    - Relative boundary sizes
  - Physically based analysis
Experimental Method

• Wide range of Rayleigh number by of fluid density through reduction in gas pressure (Saunders, 1936, Hollands, 1988)

• Assume ideal gas

\[ \rho = \frac{p}{RT_bZ} \Rightarrow Ra^{\sqrt{A_i} = \frac{g \beta (T_i - T_o)(\sqrt{A_i})^3 p^2 c_p}{R^2 T_b^2 k \mu Z^2}} \]

• Transient test method (Hollands, 1988)

  ▪ Assumes “quasi” steady conditions
  ▪ Fraction of the time required for steady state tests
Experimental Apparatus

- Spherical and cubical outer geometries
- Eleven different inner bodies
- Temperatures measured using T-type thermocouples
Experimental Apparatus

- All tests performed in vacuum chamber
- Enclosure walls cooled by cold plates
- Keithley 2700 data acquisition system
- Labview v.5.1 software
  - control of experiment
  - data acquisition and reduction
Model Development

- Assume linear superposition of diffusive and convective limits
- Convection-only data for $s_o/s_i = 2$ concentric cubes

\[ \text{Diffusive Limit} \]

\[ \text{Boundary Layer Asymptote} \]

\[ \text{Transition Flow Asymptote} \]

\[ Nu_{\text{conv}} = Nu \sqrt{A_i} - S^* \sqrt{A_i} \]
Model Summary

- Combination of three asymptotic solutions

\[ Nu_{\sqrt{A_i}} = S^*_{\sqrt{A_i}} + \left[ \left( \frac{1}{Nu_{tr}} \right)^2 + \left( \frac{1}{Nu_{bl}} \right)^2 \right]^{-1/2} \]

- \( S^*_{\sqrt{A_i}} = \) conduction shape factor
- \( Nu_{tr} = \) transition flow convection
- \( Nu_{bl} = \) laminar boundary layer flow convection

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**Conduction Shape Factor**

- **Linear superposition of two asymptotic solutions**
  
  \[ S^* \frac{A_i}{\delta_e} = \sqrt{\frac{A_i}{\delta_e}} + S^* \]
  
  \[ S^* \frac{A_i}{\delta_e} = \text{full space diffusive limit} \]
  
  \[ \sqrt{A_i/\delta_e} = \text{1D planar resistance} \]

- **Effective gap spacing from equivalent spherical shell**
  
  \[ \delta_e = \frac{d_o - d_i}{2} \]
  
  Inner surface area \[ d_i = \sqrt{\frac{A_i}{\pi}} \]
  
  Enclosed volume \[ d_o = \left[ 6 \left( V + \frac{\pi}{6} d_i^3 \right)/\pi \right]^{1/3} \]

- **Dimensionless conduction shape factor**
  
  \[ S^* \frac{A_i}{\delta_e} = \frac{2\sqrt{\pi}}{\left[ 1 + 6\sqrt{\pi} \left( V^{1/3}/\sqrt{A_i} \right)^3 \right]^{1/3}} + S^* \]
  
  \[ S^* \frac{A_i}{\delta_e} = 1 - \left[ 1 + 6\sqrt{\pi} \left( V^{1/3}/\sqrt{A_i} \right)^3 \right]^{1/3} \]
Boundary Layer Convection

- **Assumptions**
  - Laminar flow
  - $T_b$ uniform
  - Non-intersecting boundary layers

- **Series combination of resistances**
  
  $$ R_{conv} = R_i + R_o \quad R_i = \frac{T_i - T_b}{Q} \quad R_o = \frac{T_b - T_o}{Q} $$

- **Non-dimensionalize using Nusselt number**
  
  $$ Nu_i = \frac{1}{k \sqrt{A_i}} \frac{1}{R_i} \quad Nu_o = \frac{1}{k \sqrt{A_o}} \frac{1}{R_o} \quad Nu_{bl} = \frac{Nu_i}{1 + 1/\phi} $$

  $$ \phi = \frac{T_i - T_b}{T_b - T_o} = \frac{R_i}{R_o} = \frac{\sqrt{A_o} \cdot Nu_o}{\sqrt{A_i} \cdot Nu_i} $$
Boundary Layer Convection

- Convection modeled using Yovanovich [34] and Jafarpur [36]
  \[ Nu_{\sqrt{A}} = F(Pr) G_{\sqrt{A}} Ra_{\sqrt{A}}^{1/4} \]

- Laminar boundary layer convection asymptote
  \[ \frac{Nu_{bl}}{Nu_i} = \frac{F(Pr) G_{\sqrt{A_i}} Ra_{\sqrt{A_i}}^{1/4}}{1 + 1/\phi} \]
  \[ \frac{Nu_{bl}}{Nu_i} = \left( \frac{F(Pr) G_{\sqrt{A_i}} Ra_{\sqrt{A_i}}^{1/4}}{1 + (A_i/A_o)^{7/10} \left( G_{\sqrt{A_i}} / G_{\sqrt{A_o}} \right)^{4/5}} \right)^{5/4} \]
**Transition Flow**

- Boundary layers merge when $Ra < Ra_{cr}$
- Model as equivalent concentric spheres
- Three distinct regions are formed
- Central region
  - Radial conduction
  - Buoyancy induced flow
- For narrow gap spacing, $\delta << r_i$, temperature, velocity in central region

\[
T - T_b = -\frac{y}{\delta/2} (T_i - T_b), \quad T_b = \frac{T_i + T_o}{2}
\]

\[
u = \frac{g \beta (T_i - T_o) (\delta/2)^3}{12 \nu} \left[ \frac{y}{\delta/2} - \frac{y}{\delta/2} \right]
\]

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Transition Flow

- Enthalpy balance in top-end and bottom-end regions

\[
Q_{i,o} = \frac{\rho c_p W' g_e \beta (T_i - T_o)^2 \delta^3}{720 \nu}
\]

- Transition flow asymptote

\[
Nu_{tr} = \frac{\sqrt{2}}{360} \frac{\sqrt{A_i}}{L'} \left( \frac{\delta_{eff}}{\sqrt{A_i}} \right)^3 Ra \frac{\sqrt{A_i}}{L'}
\]

\[
\delta_{eff} = \text{gap spacing of equivalent cavity}
\]

\[
L' = \text{effective flow length}
\]
Validation: Concentric Cubes

\[ \frac{N_u}{A_i} \]

- Model
- Data

\[ s_o/s_i \]
- \{ 1.49 \}
- \{ 2 \}
- \{ 3.08 \}
- \{ 4.96 \}

\[ Ra_{\sqrt{A_i}} \]
Sphere in Cubical Enclosure

\[ \text{data } \Rightarrow \text{model } \Rightarrow \]

\[ \begin{align*}
\text{s}_o / d_i & \quad \{1.68, 2.23, 3.35\}
\end{align*} \]

\[ N_{u_{\sqrt{A_i}}} \]

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \]

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Cube in Spherical Enclosure
Cylinder in Cubical Enclosure

\[ \text{data} \Rightarrow \triangle \quad \text{model} \Rightarrow \ldots \} \text{cylinder in cubical enclosure} \]
Other Enclosure Geometries

- Oblate spheroid in spherical enclosure
- Cuboid in cubical enclosure

\[ N_u \sqrt{\bar{A}_i} \]

\[ R_a \sqrt{\bar{A}_i} \]
Summary and Conclusions

- Combined experimental / analytical study of natural convection heat transfer between heated body and cooled enclosure
- Experimental data for variety of enclosure configurations, dimensions
- Model developed based on combination of analytic, asymptotic relationships
  - Diffusive limit
  - Laminary boundary layer convection
  - Transition flow convection
- 2 – 6% RMS, 12% maximum difference between model and data