# A COMPACT MODEL FOR SPHERICAL ROUGH CONTACTS

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# OVERVIEW

- Introduction
- Objectives
- Present Model
- General Pressure Distribution
- Dimensional Analysis
- Comparison with Experimental Data
- Conclusions

## Contact of Spheres

- elastic, smooth
  - Hertz (1881) theory of elastic contact of spheres

$$P_{Hz}\ \ r/a_{Hz} \qquad P_{0,Hz}\sqrt{1 - \ \ r/a_{Hz}\ \ ^{2}}$$

- elastic, rough spheres, elastic microcontacts
  - Greenwood and Tripp (1967)
- plastic, rough surfaces, plastic microcontacts
  - Mikic and Roca (1974)
- roughness parameter
  - Greenwood et al. (1984)

$$\alpha = \frac{\sigma\rho}{a_H^2} = \sigma\left(\frac{16\rho E'^2}{9F^2}\right)^{1/3}$$

# OBJECTIVES

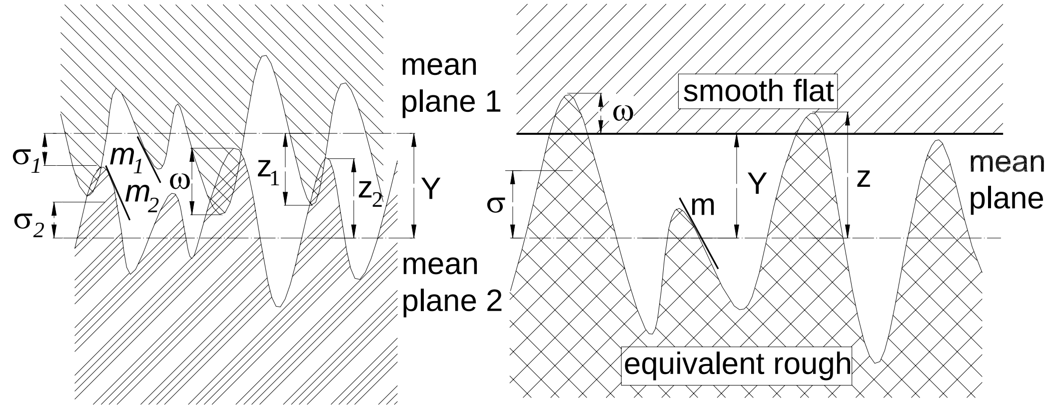
- develop model to predict the parameters of spherical contact:
  - pressure distribution,
  - elastic deformation,
  - compliance,
  - number of microcontacts,
  - size of the contact area

- derive simple correlations for determining contact parameters that can be used in other analyses such as thermal contact models

# COOPER MIKIC YOVANOVICH (CMY) (1969)

**conforming rough contacts**
**Plastic Model**

- Gaussian surfaces
- plastically deformed hemispherical asperities




a) section through two contacting surfaces

b) corresponding section through equivalent rough - smooth flat

- cross-level theory

$$Y/\sqrt{2}$$

$$\frac{A_r}{A_a} \qquad \frac{1}{2}\,\mathrm{erfc}$$

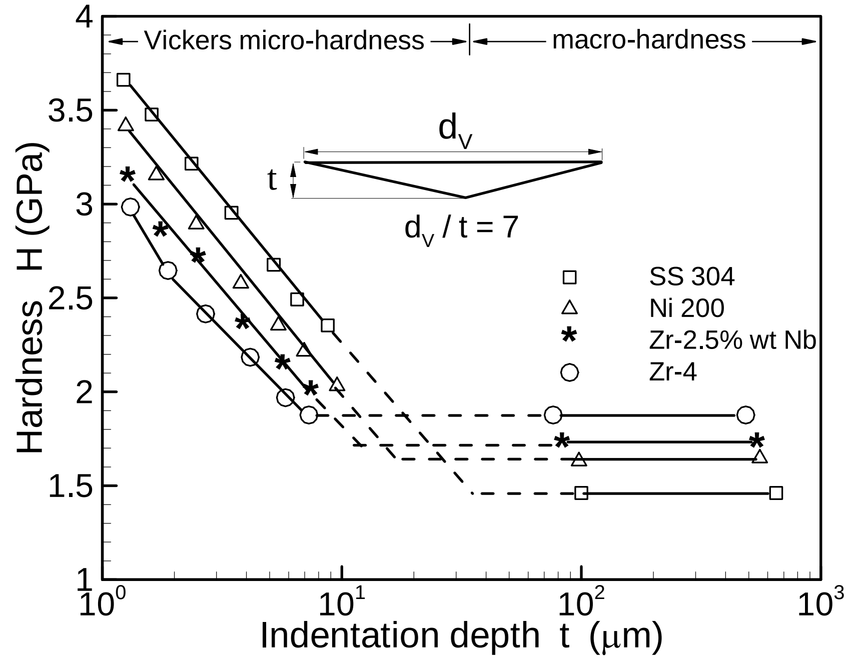
$$a_s \qquad \sqrt{\frac{8}{\;}} \quad \frac{\;}{m} \quad \exp \quad {}^{2} \quad \mathrm{erfc}$$

$$n_s \qquad \frac{1}{16} \quad \frac{m}{\;} \quad {}^{2}\, \frac{\exp\left(-2\;\;{}^{2}\right)}{\mathrm{erfc}} A_a$$

# MICROHARDNESS

## Hegazy (1985)

- microhardness may not be constant throughout the material as a result of machining process
- microhardness decreases with increasing depth of indentation until bulk hardness level

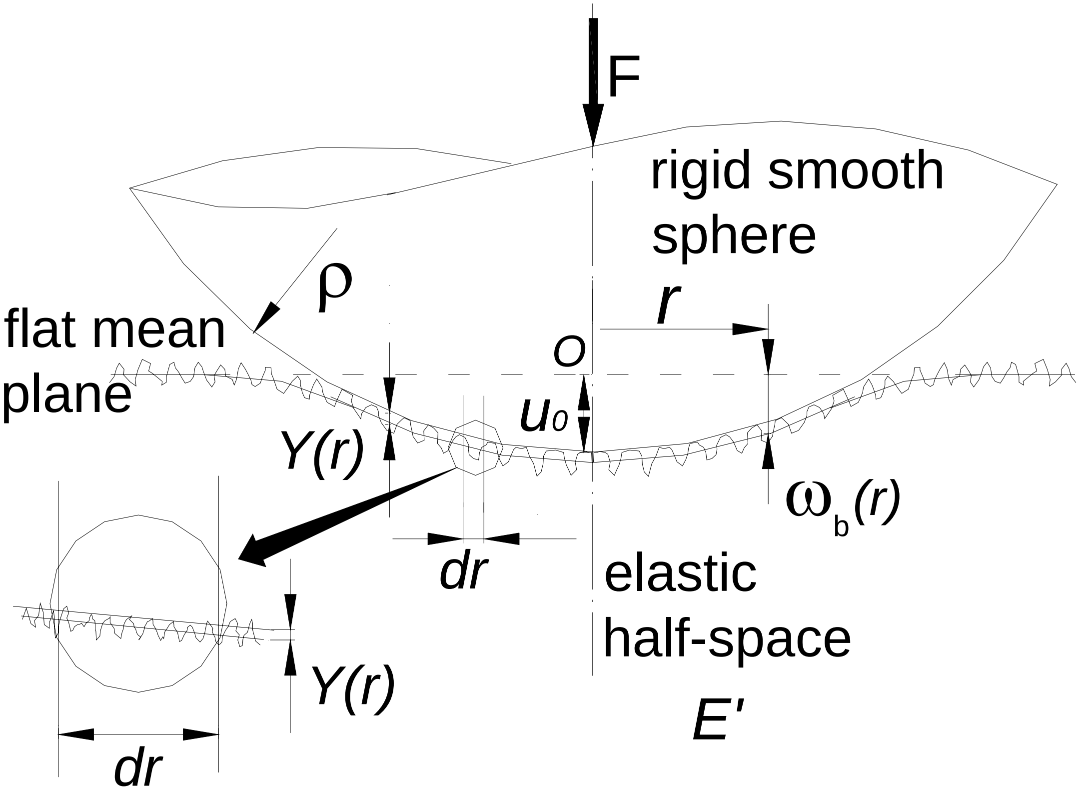



$$H_v \quad c_1 \; d_v'^{\;c_2}$$

$$d_v' \quad d_v/d_0$$

# PRESENT MODEL: ASSUMPTIONS

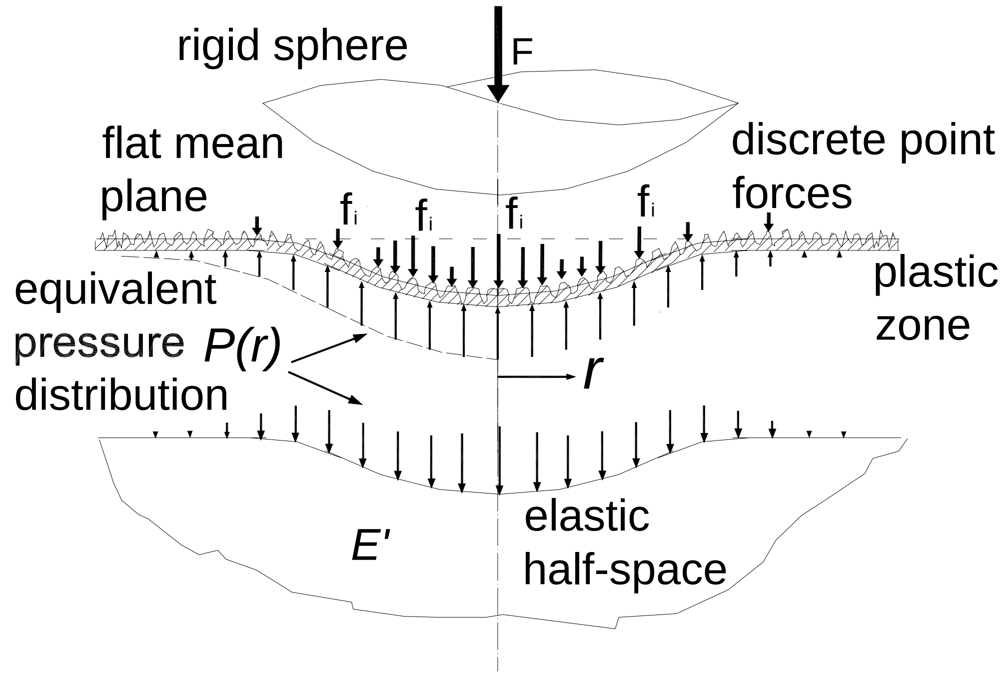
- spherical surfaces
- Gaussian asperity distribution
- plastic microcontacts
- elastic macrocontact




$$Y(r) = \omega_b(r) - u(r) = \omega_b(r) - u_0 + r^2/2\rho$$

# ASSUMPTIONS 2

- deformation of each asperity is independent of its neighbors
- no friction
- first loading cycle
- static contact



# RELATIONSHIPS FOR MECHANICAL MODEL

dimensionless local separation

$$\lambda(r) = Y(r) / \left(\sqrt{2}\sigma\right)$$

effective pressure distribution

$$P(r) = \frac{1}{2} H_{mic}(r)\ \mathrm{erfc}(\lambda(r))$$

elastic deformation of half-space due to applied P(r)

$$\omega_b(r) = \begin{cases} \dfrac{2}{E^*}\displaystyle\int_0^\infty P(s)\,ds & r = 0 \\ \dfrac{4}{\pi E^* r}\displaystyle\int_0^r s\,P(s)\,K\left(\dfrac{s}{r}\right) ds & r > s \\ \dfrac{4}{\pi E^*}\displaystyle\int_r^\infty P(s)\,K\left(\dfrac{r}{s}\right) ds & r < s \end{cases}$$

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

force balance

$$F = 2\pi \int_0^\infty P(r)\,r\,dr$$

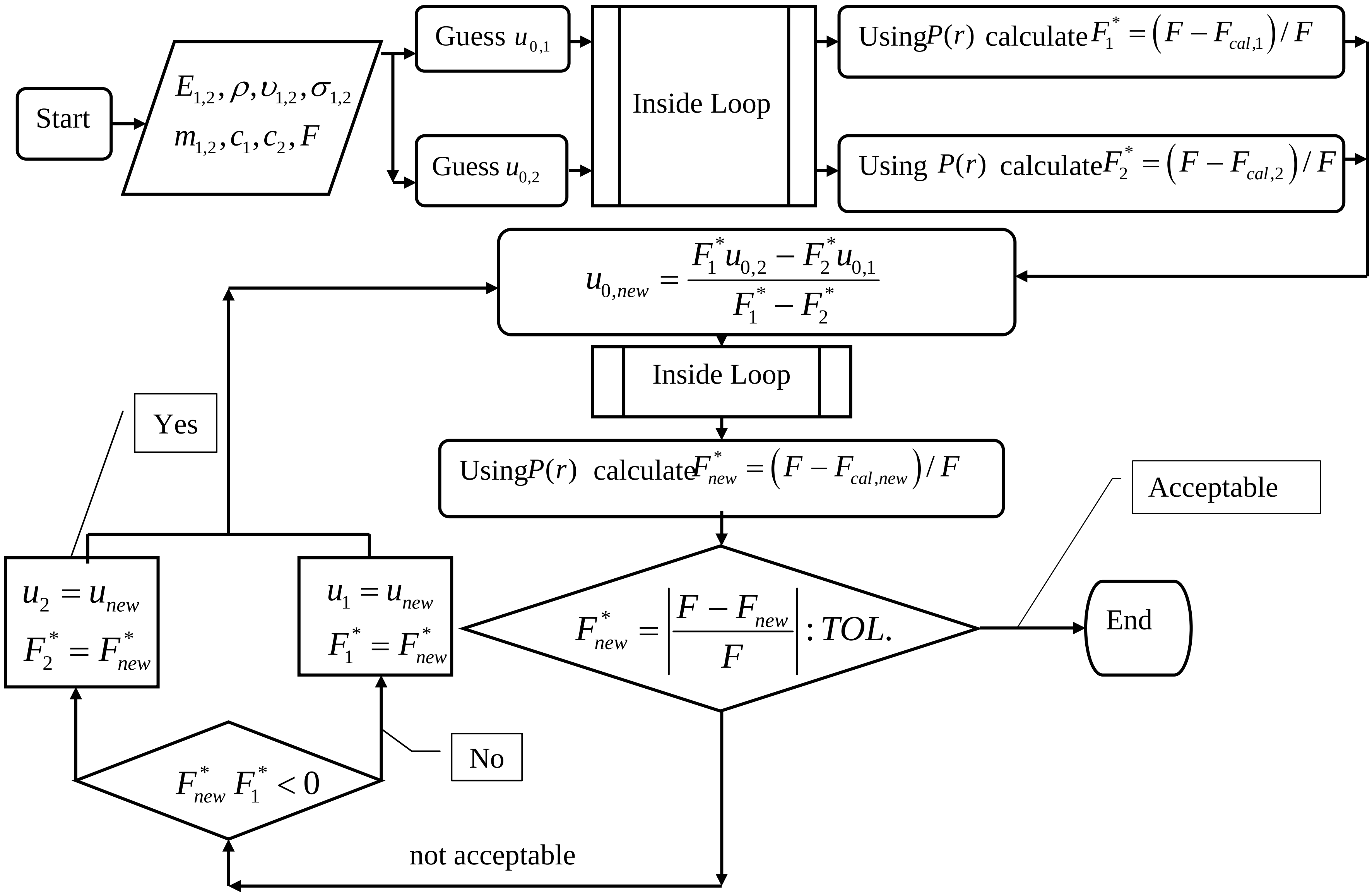
local microcontact radius

$$a_s(r) = \sqrt{\frac{8}{\pi}}\left(\frac{\sigma}{m}\right)\ \exp\left(\lambda^2(r)\right)\ \mathrm{erfc}\left(\lambda(r)\right)$$

local microhardness

$$H_{mic}(r) = c_1\left(\sqrt{2\pi}a_s(r)\right)^{c_2}$$

# NUMERICAL ALGORITHM – MAIN LOOP

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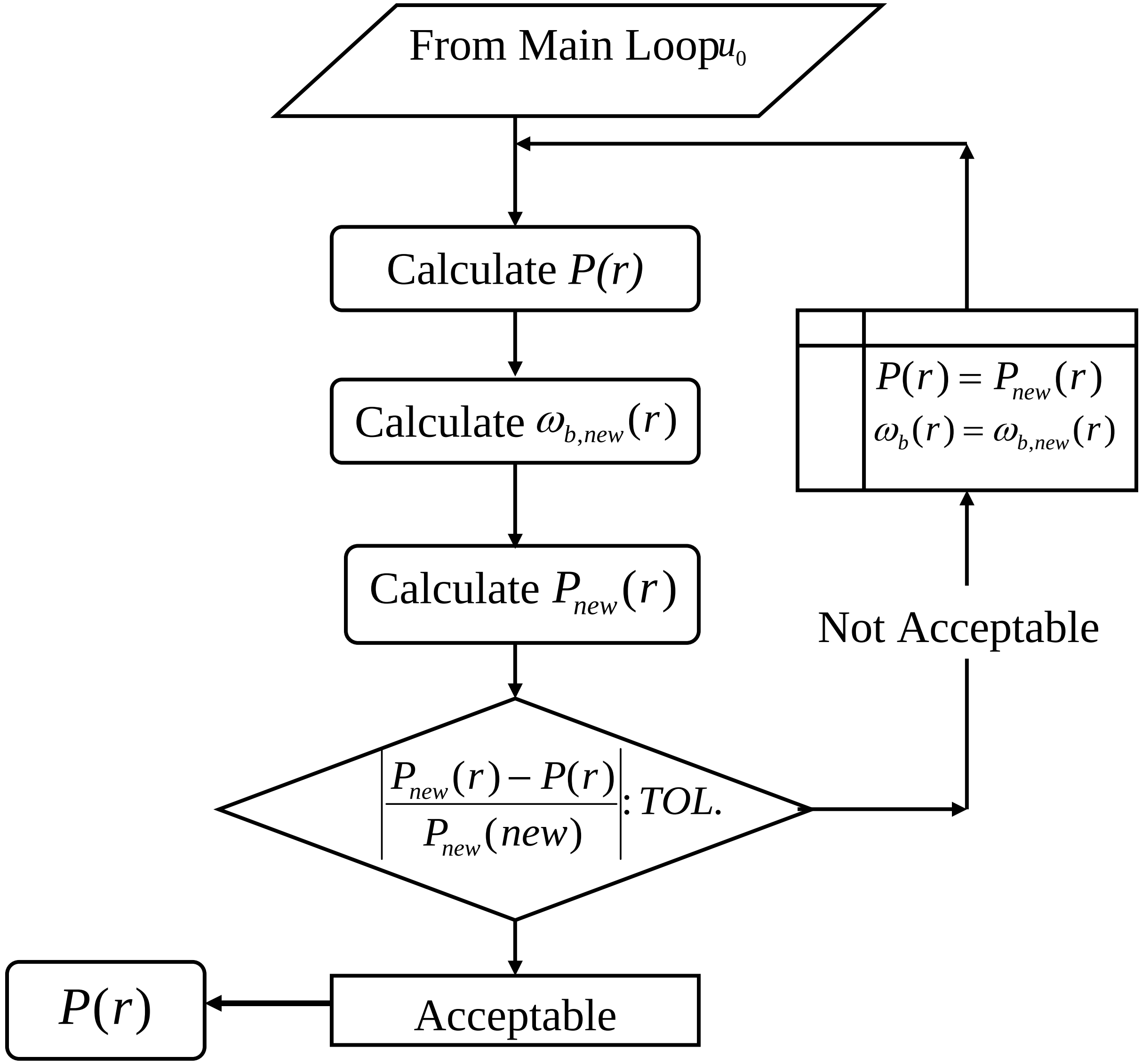
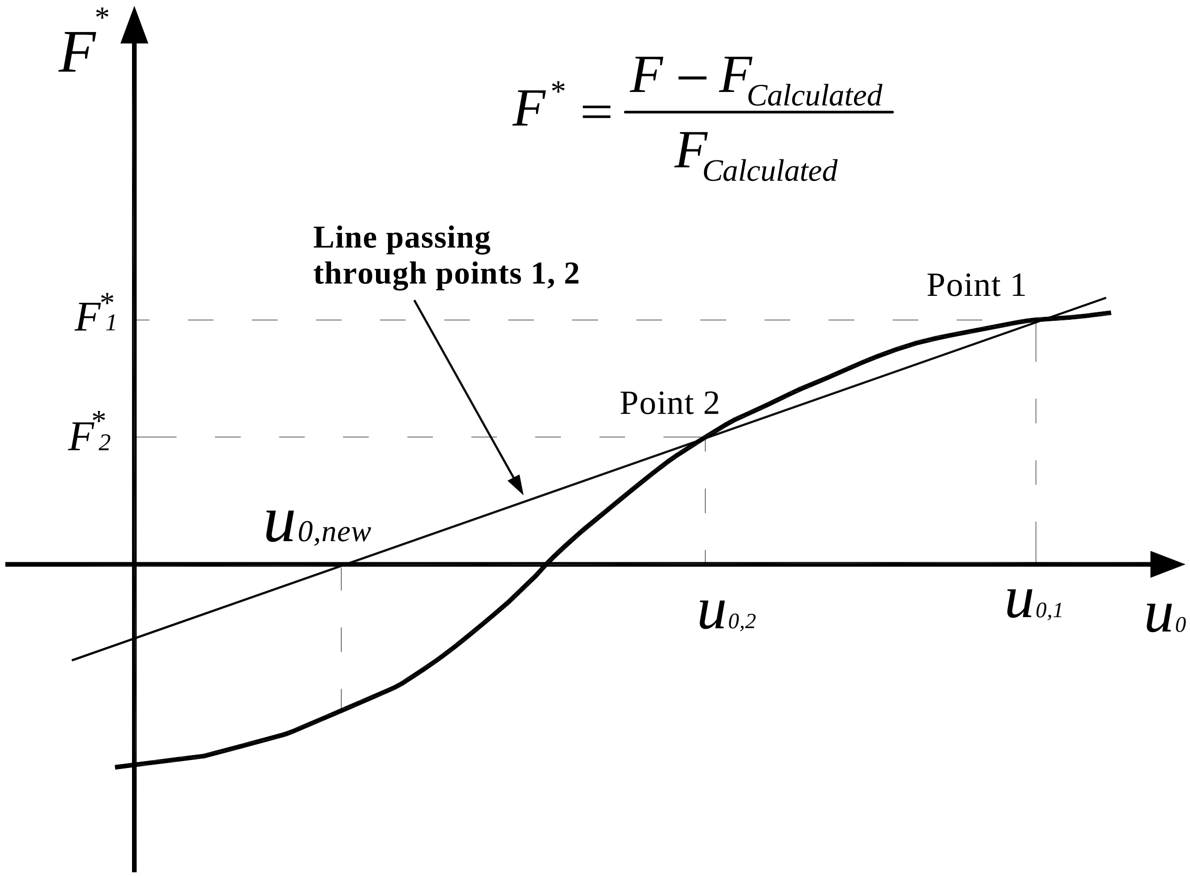
# NUMERICAL ALGORITHM – INSIDE LOOP

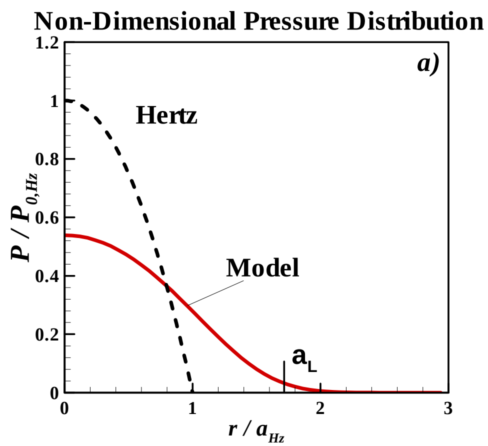


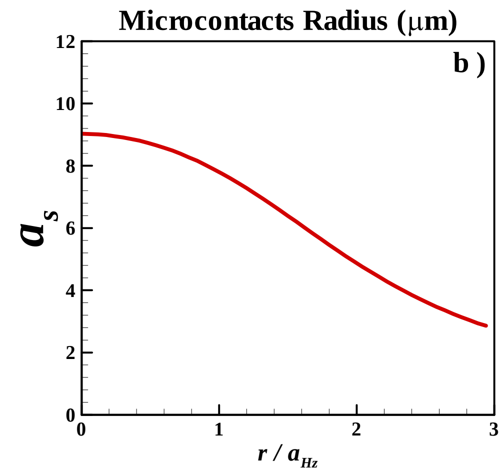
# SUCCESSIVE ITERATION

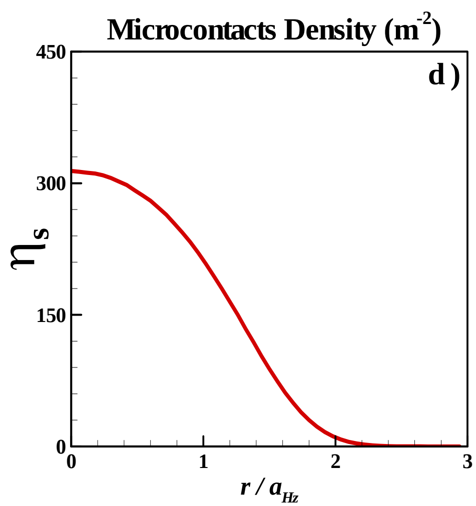


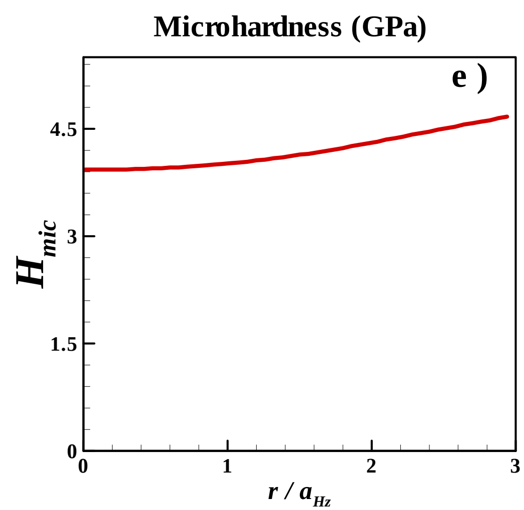
# OUTPUT PARAMETERS OF MODEL

| | | | | | |
|---|---|---|---|---|---|
| Radius of curvature | $\rho = 25(mm)$ | Roughness | $\sigma = 1.414(\mu m)$ | Force | $F = 50(N)$ |
| Surface slope | $m = 0.107$ | Young's modulus | $E_1 = E_2 = 204(GPa)$ | Poisson's | $\nu_1 = \nu_2 = 0.3$ |
| Microhardness | $c_1 = 6.27(GPa)$ | Microhardness | $c_2 = -0.15$ | Sample dia. | $b_L = 25(mm)$ |

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# GENERAL PRESSURE DISTRIBUTION

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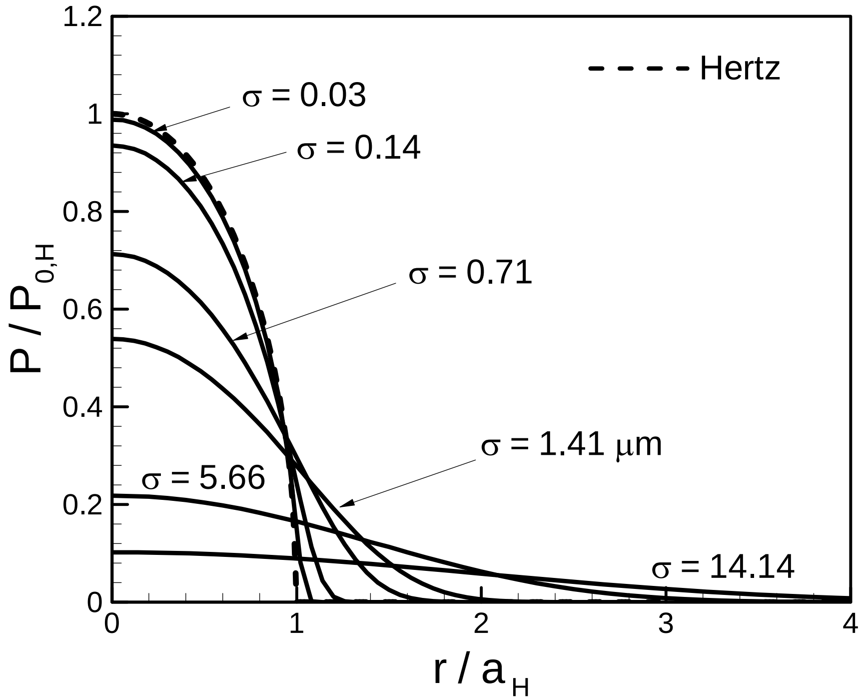
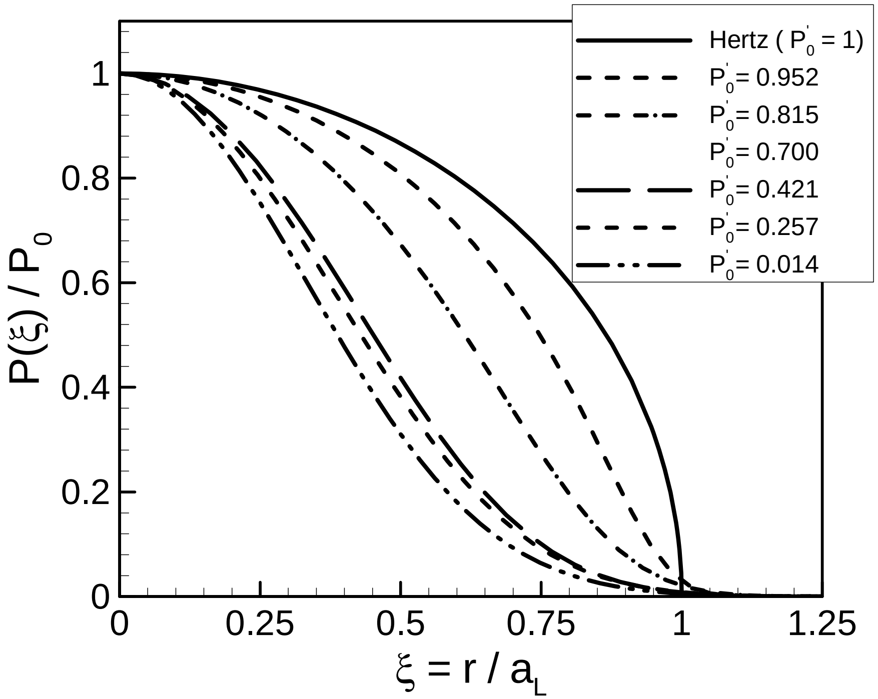



$$P(\xi) = P_0 \left(1 - \xi^2\right)^{\gamma}$$

$$\gamma = 1.5 \frac{P_0}{P_{0,H}} \left(\frac{a_L}{a_H}\right)^2 - 1$$

$$P_0 = (1 + \gamma) \frac{F}{\pi a_L^2}$$

Hertzian limit

$$P_H(r / a_H) = P_{0,H} \left(1 - (r / a_H)^2\right)^{\gamma}$$

$$\gamma_H = 0.5$$

$$P_{0,H} = \frac{1.5F}{\pi a_H^2}$$

# DIMENSIONAL ANALYSIS

- effective microhardness,

$$H_{mic} = \text{Const.}$$

- surface slope *m* is assumed to be a function of surface roughness, Lambert (1995)

$$m \qquad 0.076 \qquad ^{0.52}$$

- maximum contact pressure is a function of

$$P_0 \qquad P_0 \qquad , \quad , E', F, H_{mic}$$

- three non-dimensional parameters

| Parameter | Dimension |
|---|---|
| Effective elastic modulus, $E'$ | $ML^{-1}T^{-2}$ |
| Force, $F$ | $MLT^{-2}$ |
| Microhardness, $H_{mic}$ | $ML^{-1}T^{-2}$ |
| Radius of curvature, | $M$ |
| Roughness, | $M$ |
| Max. contact pressure, $P_0$ | $ML^{-1}T^{-2}$ |

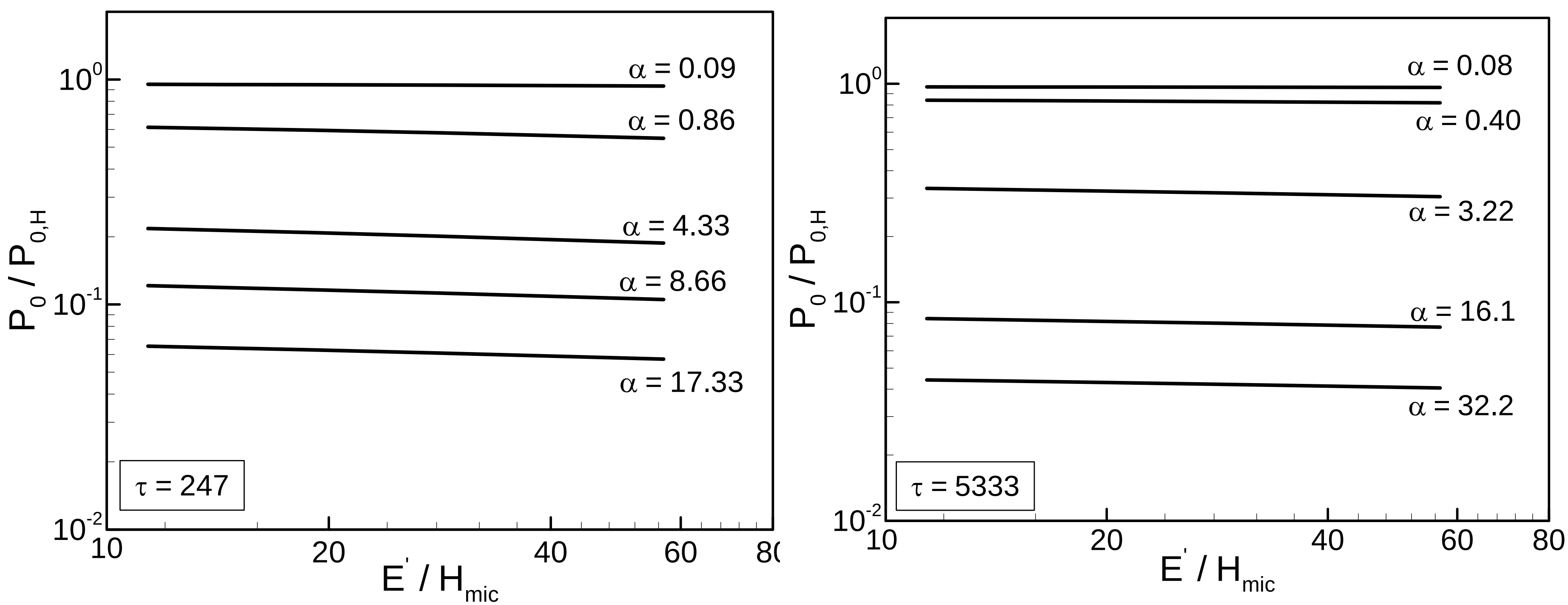
$$\alpha = \frac{\sigma}{\omega_{0,H}} \equiv \frac{\sigma\rho}{a_H^2}$$

$$\tau = \frac{\rho}{a_H}$$

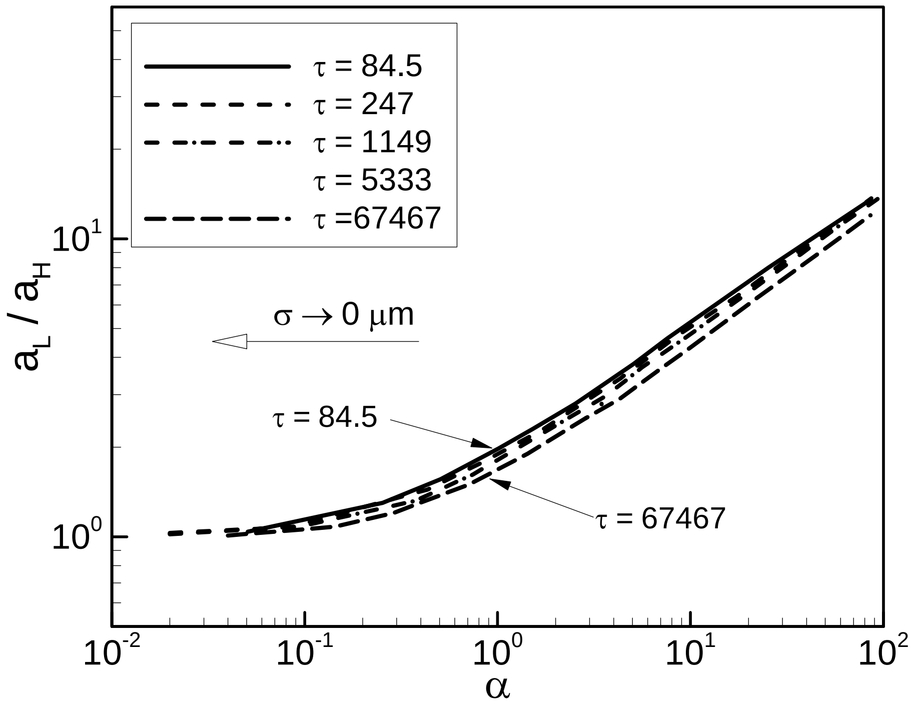
$$E \qquad \frac{E'}{H_{mic}}$$

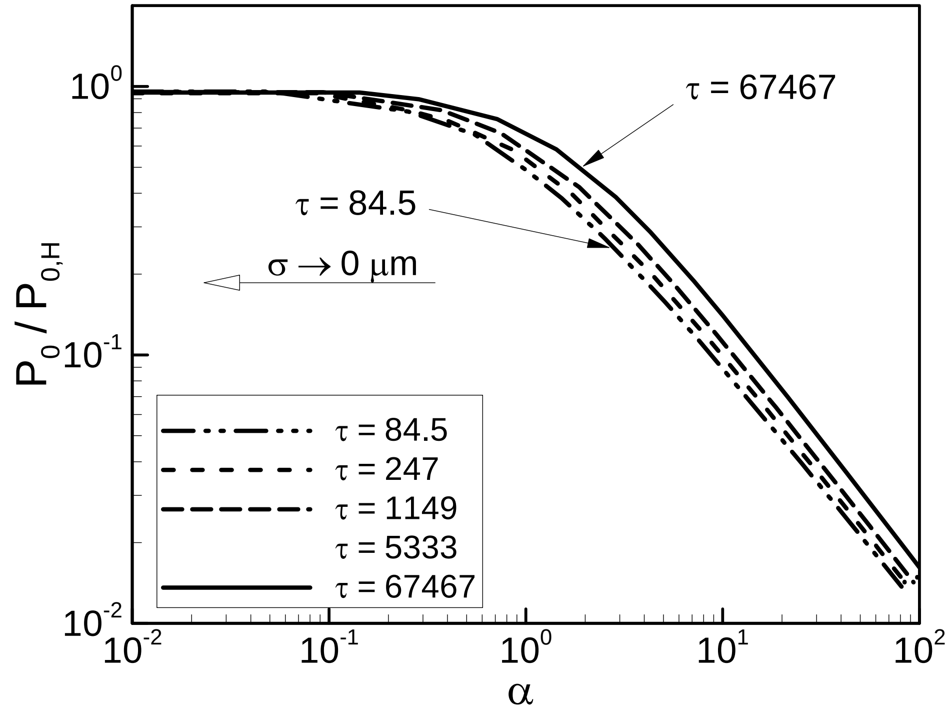
# EFFECT OF MICROHARDNESS PARAMETER

effect of microhardness parameter on the maximum contact pressure is small and therefore ignored.



# CORRELATIONS




$$P_0' = \frac{P_0}{P_{0,H}} = \frac{1}{1 + 1.37\alpha / \tau^{0.075}}$$

$$a_L' = \frac{a_L}{a_H} = \begin{cases} 1.605 / \sqrt{P_0'} & 0.01 \le P_0' \le 0.47 \\ 3.51 - 2.51 P_0' & 0.47 \le P_0' \le 1 \end{cases}$$

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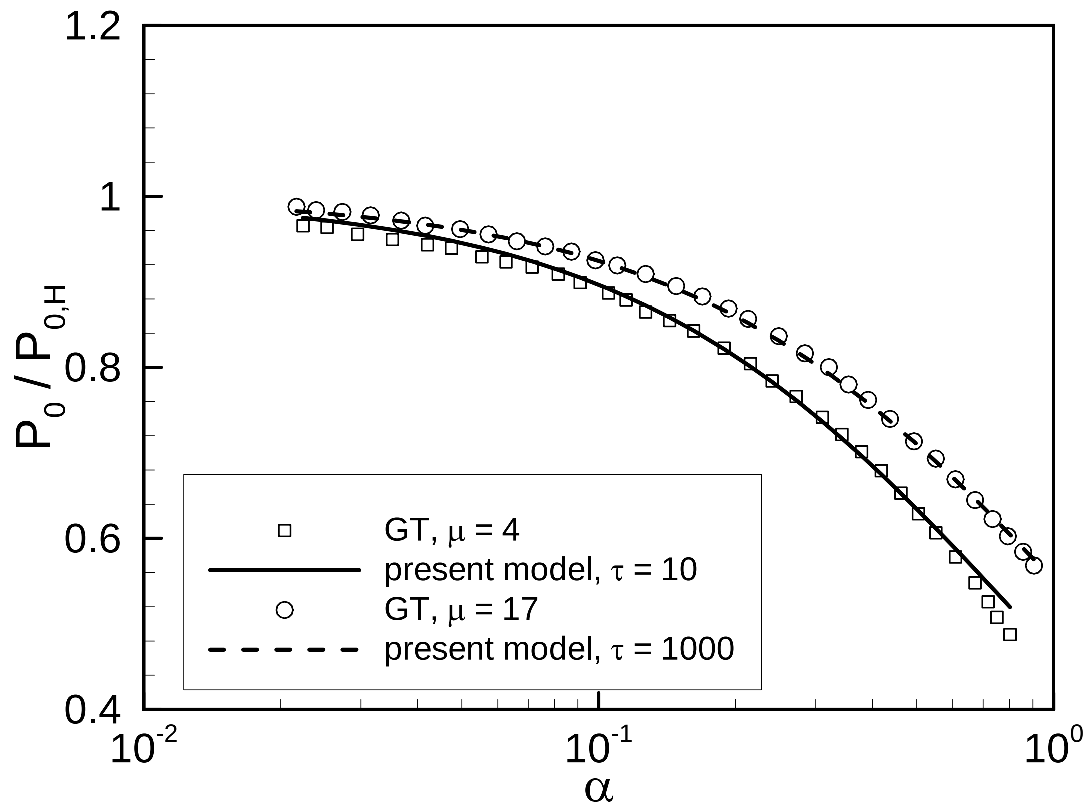
# COMPARISON WITH GT MODEL

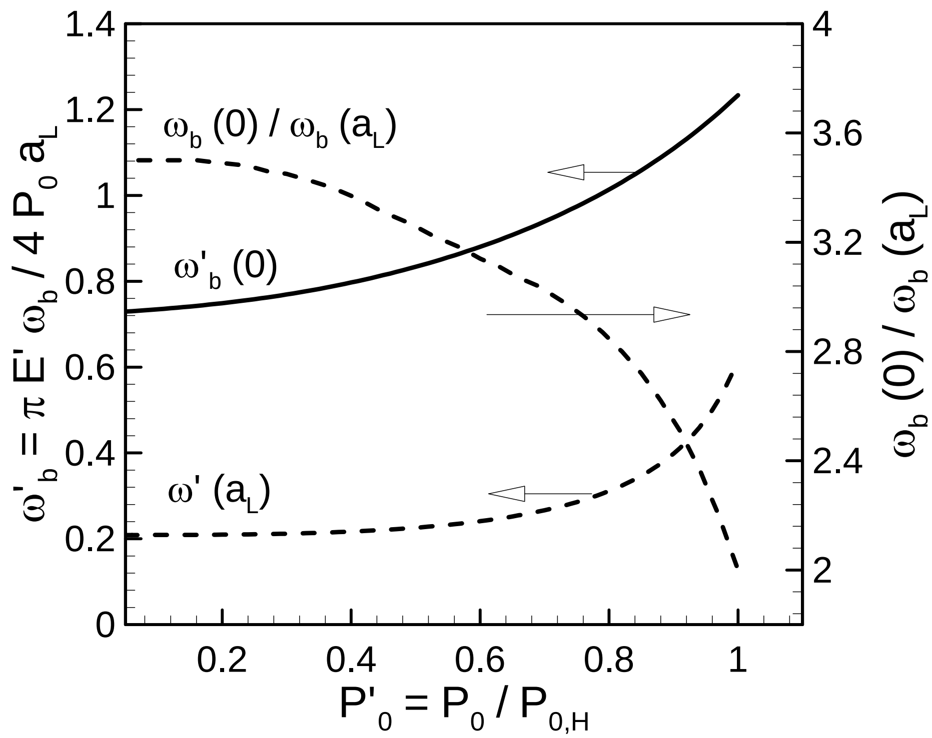
**Greenwood and Tripp (1967) disadvantages:**

- complex, requires computer programming and numerically intensive solutions
- $\beta$ and $\eta_s$ cannot be measured directly, sensitive to the surface measurements
- constant summit radius $\beta$ is unrealistic



# ELASTIC DEFORMATION OF HALF-SPACE

- using general pressure distribution, relationships are derived for:
  - elastic deformation of half-space
  - compact correlation is derived for compliance



# EXPERIMENTAL DATA

$\tau = \rho / a_H$

Kagami, Yamada, and Hatazawa (KYH) 1982

$16.8 \le \tau \le 187.4$

$\rho = 3.15$ mm, $0.082 \le \sigma \le 1.45$ $\mu$m, $0.19 \le F \le 88$ N

carbon steel spheres – carbon steel and copper flats

Greenwood, Johnson, and Matsubara (GJM) 1984

$31 \le \tau \le 170.8$

$\rho = 12.7$ mm, $0.19 \le \sigma \le 2.2$ $\mu$m, $4.8 \le F \le 779$ N

hard steel balls – hard steel flats

Tsukada and Anno (TA) 1979

$7.8 \le \tau \le 47.6$

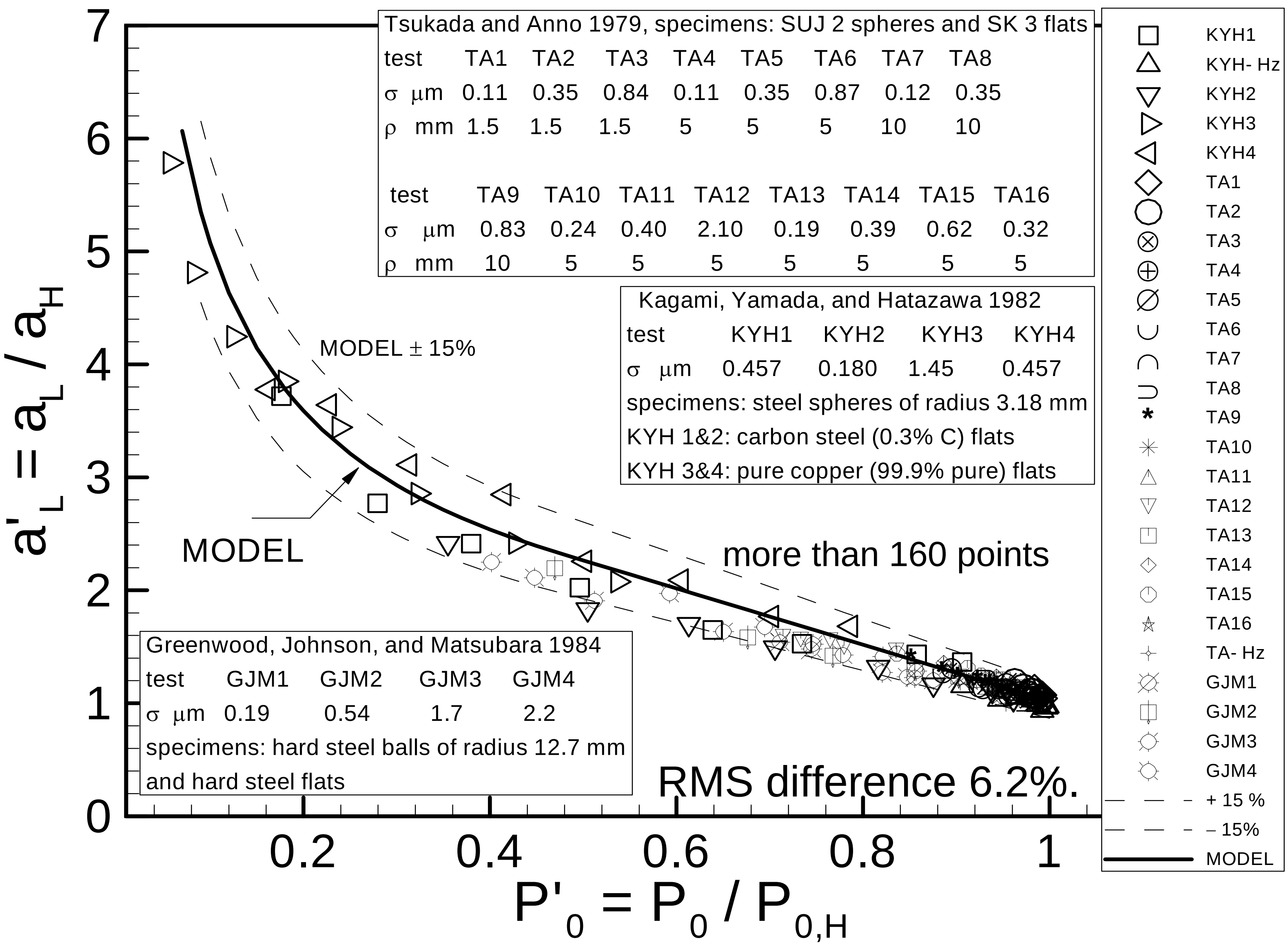
$\rho = 1.5, 5, 10$ mm, $0.11 \le \sigma \le 2.1$ $\mu$m, $23.5 \le F \le 1375$ N

SUJ 2 spheres – SK 3 flats

$10^{-3}$ $10^{-2}$ $10^{-1}$ $10^{0}$ $10^{1}$ $10^{2}$

$\alpha = \sigma \rho / a_H^2$

# COMPARISON WITH DATA: CONTACT RADIUS

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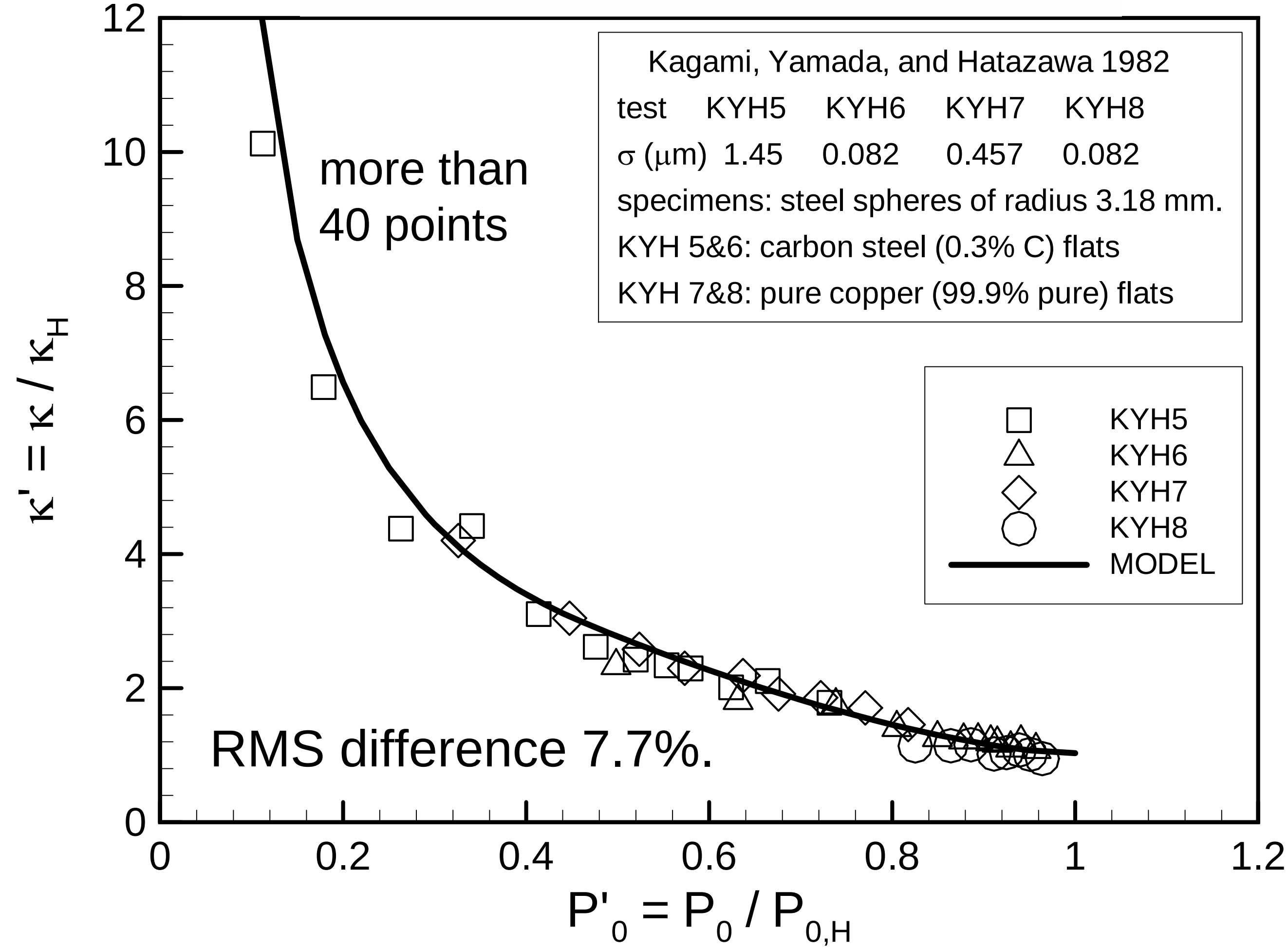
# COMPARISON WITH DATA: COMPLIANCE

$$\kappa' = \frac{\kappa}{\kappa_H} = 0.5\left(a'_L\right)^2 + \frac{8P'_0\, a'_L}{\pi^2\left[4.79 - 3.17\left(P'_0\right)^{3.13}\right]}$$



# SUMMARY AND CONCLUSIONS

- a general pressure distribution that encompasses all spherical rough contacts including Hertzian limit is proposed
- compact correlations for contact radius and compliance are proposed and validated with experimental data
- It is shown that the non-dimensional maximum contact pressure is the main parameter that controls the solution of spherical rough contacts

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