COMPACT ANALYTICAL MODELS FOR EFFECTIVE THERMAL CONDUCTIVITY OF ROUGH SPHEROID PACKED BEDS

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OVERVIEW

• Introduction

• Motivations and Objectives

• Conduction Through Contact Spots

• Conduction Through Interstitial Gas

• Present Model

• Comparison with Experimental Data

• Conclusions
INTRODUCTION

• high ratio of solid surface area to volume.

• packed beds applications:
  – Catalytic reactors, heat recovery systems, heat exchangers, heat storage systems, and insulators

• regular packing: Simple Cubic (SC), Body Center Close (BCC), and Face Center Close (FCC)
MOTIVATIONS AND OBJECTIVES

• existing models can be categorized into:
  – numerical (FEM) models:
    • Buonanno et al.: time consuming, B.C. must be fed into the code for thermal contact resistance
  – analytical models:
    • Slavin et al.: a point contact between spheres assumed
    • Ogniewicz & Yovanovich and Turyk & Yovanovich: limited to smooth spheres

• develop compact models for determining effective thermal conductivity that account for:
  – roughness
  – gas rarefaction effect
  – contact load
  – gas temperature and pressure
REGULAR PACKED BED ARRANGEMENTS

- solid fraction is defined

\[ \varepsilon = \frac{V_s}{V} \]

<table>
<thead>
<tr>
<th>Packing</th>
<th>Solid fraction</th>
</tr>
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<tbody>
<tr>
<td>SC</td>
<td>0.524</td>
</tr>
<tr>
<td>BCC</td>
<td>0.680</td>
</tr>
<tr>
<td>FCC</td>
<td>0.740</td>
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</tbody>
</table>

Simple Packing (SP) \( \varepsilon = 0.524 \)

Body Center Close (BCC) \( \varepsilon = 0.680 \)

Face Center Close (FCC) \( \varepsilon = 0.740 \)
HEAT TRANSFER MECHANISMS IN PACKED BEDS

two main paths for transferring thermal energy in packed beds are:

• conduction through microcontacts
• heat transfer through interstitial gas
thermal joint resistance network components:

- macro-constriction, $R_L$
- micro-constriction, $R_s$
- microgap resistance, $R_g$
- macrogap, $R_G$

$$R_j = \left[ \frac{1}{\left( \frac{1}{R_s} + \frac{1}{R_g} \right)^{-1} + \frac{1}{R_L \cdot R_G}} \right]^{-1}$$
CONDUCTION THROUGH CONTACT SPOTS

• macro-constriction resistance, \( R \), Bahrami et al. [17]

\[
P(\xi) = P_0 \left(1 - \xi^2\right)^y
\]

\[
P'_0 = \frac{P_0}{P_{0,H}} = \frac{1}{1 + 1.37 \alpha \tau^{-0.075}}
\]

\[
\alpha = \frac{\sigma \rho}{a_H^2} \text{ and } \tau = \frac{\rho}{a_H}
\]

\[
a'_L = \begin{cases} 
1.605 / \sqrt{P'_0} & 0.01 \leq P'_0 \leq 0.47 \\
3.51 - 2.51 P'_0 & 0.47 \leq P'_0 \leq 1
\end{cases}
\]

\[
R_L = \frac{1}{2k_s a_L}
\]

• micro-constriction resistance, \( R_s \), Bahrami et al. [14]

\[
R_s = \frac{0.565 H^* (\sigma / m)}{k_s F}
\]
Conduction regimes in a gas layer between two parallel plates:
- continuum
- temperature-jump or slip
- transition
- free-molecular

**Microgap resistance**, $R_g$, Bahrami et al. [26]

$$R_g = \frac{\sqrt{2} \sigma a_2}{\pi k_g a_L^2 \ln \left(1 + \frac{a_2}{a_1 + M / \sqrt{2} \sigma} \right)}$$

$a_1 = \text{erf}^{-1} \left( \frac{2 P_0}{H'} \right)$ and $a_2 = \text{erf}^{-1} \left( \frac{0.03 P_0}{H'} \right) - a_1$

**Macrogap resistance**, $R_G$, Bahrami et al. [25]

$$2\pi k_g R_G = \frac{1}{S \ln \left( \frac{S - B}{S - A} \right) + B - A}$$

$A = \sqrt{\rho^2 - a_L^2}$, $B = \sqrt{\rho^2 - b_L^2}$, $S = \rho - \omega_0 + M$
CONDUCTION IN BASIC CELLS

steps to determine the bed conductivity:

• calculate the relation between the apparent load and contact load
• break up the unit cell into contact regions
• calculate the thermal joint resistance of a contact region
• determine the effective conductivity

\[ k_c = \frac{L_c}{R_c A_c} \]
Kitscha and Yovanovich (1974) SC basic cell data

- Carbon steel sphere of radius 12.7 mm
- Flat steel 1020, \( \sigma = 0.13 \, \mu m \), \( b_L = 12.7 \, mm \)
- Air, \( Pr = 0.70 \), \( \gamma_g = 1.4 \), \( \alpha_T = 0.87 \), \( \Lambda_0 = 64 \, nm \)
- \( k_g \) (W/m K) = 0.0021 + 8 \times 10^{-5} T (K)

- Argon, \( Pr = 0.67 \), \( \gamma_g = 1.67 \), \( \alpha_T = 0.90 \), \( \Lambda_0 = 66.6 \, nm \)
- \( k_g \) (W/m K) = 0.0159 + 4 \times 10^{-6} T (K)
COMPARISON WITH EXPERIMENTAL DATA

SC packed bed, Buonanno et al. (2003) data

Buonanno et al. [3] data, SC packing
100Cr6 stainless steel spheres of radius 19.05 mm
\( k_s = 60 \text{ W/mK} \)
\( E_s = 200 \text{ GPa}, \nu_s = 0.3 \)
\( H_{mic} = 8.32 \text{ GPa} \)
air at 1 atm pressure, \( T = 20 ^\circ \text{C} \)
\( k_g = 0.027 \text{ W/mK}, Pr = 0.7, \gamma_g = 1.4 \)
\( \alpha_T \approx 0.78 \)
\( F_c = 0.983 \text{ N} \)

Buonanno et al. [5] data, SC packing
100Cr6 stainless steel spheres of radius 19.05 mm
\( k_s = 60 \text{ W/mK} \)
\( E_s = 200 \text{ GPa}, \nu_s = 0.3 \)
\( H_{mic} = 8.32 \text{ GPa} \)
air at 1 atm pressure, \( T = 20 ^\circ \text{C} \)
\( k_g = 0.027 \text{ W/mK}, Pr = 0.7, \gamma_g = 1.4 \)
\( F_c = 0.983 \text{ N} \)

atmospheric air
SUMMARY AND CONCLUSIONS

- compact models are proposed for determining effective thermal conductivity in regularly packed beds, SC and FCC arrangements

- present model accounts for thermophysical properties of spheres and gas, load, roughness, gas temperature and pressure, and gas rarefaction effects

- the present model is compared against experimental data, both SC and FCC, over a variety of packed bed conditions and good agreement is observed
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