The Role of Fin Geometry in Heat Sink Performance

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Agenda

- Introduction
- Assumptions
- Entropy Generation Rate
- Results and Discussion
- Conclusions
- Acknowledgment
Assumptions

- Isothermal fins with adiabatic tips
- Air flow normal to fin axis
- Steady, laminar and 2-D flow
- Incompressible fluid with constant properties
- Negligible radiation heat transfer
- No heat sources within the fin itself
- No contact resistance between fin and base surface
Entropy Generation Rate for a Pin-Fin

Bejan (1996):

$$\dot{S}_{gen} = \frac{Q^2 R_{th}}{T^2} + \frac{F_D U_\infty}{T}$$

$$R_{th} = \frac{1}{k A_c m \tanh(mH)}$$

$$F_D = C_D \left( \frac{1}{2} \rho U_\infty^2 \right) A_p$$

$$C_D = \frac{C_1}{\sqrt{Re_L}} + C_2 + \frac{C_3}{Re_L}$$
Dimensionless Entropy Generation Rate

\[ N_s = \frac{1}{Re_L \sqrt{C_5 Nu_L k_{eq} \tanh(\gamma \sqrt{C_6 Nu_L k_{eq}})}} + \frac{1}{2} C_D B \gamma Re_L^2 \]

\[ C_5 = \frac{P A_c}{L^3} \quad \text{and} \quad C_6 = \frac{P L}{A_c} \]

\[ Nu_L = \frac{h L}{k} \quad \text{and} \quad Re_L = \frac{U_\infty L}{\nu} \]
### Parameters for Different Geometries

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Plate</th>
<th>Circular</th>
<th>Square</th>
<th>Elliptical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>$L$</td>
<td>$d$</td>
<td>$s$</td>
<td>$2a$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$tL$</td>
<td>$\pi d^2/4$</td>
<td>$s^2$</td>
<td>$\pi a b$</td>
</tr>
<tr>
<td>$A_p$</td>
<td>$LH$</td>
<td>$dH$</td>
<td>$sH$</td>
<td>$2aH$</td>
</tr>
<tr>
<td>$P$</td>
<td>$2(L + t)$</td>
<td>$\pi d$</td>
<td>$4s$</td>
<td>$4aE(e)$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1.357</td>
<td>5.781</td>
<td>0</td>
<td>$\left(-4.1(0.67 - \exp(0.733\epsilon))\right)$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0</td>
<td>1.152</td>
<td>2</td>
<td>$1.1526\epsilon^{0.951}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0</td>
<td>1.26</td>
<td>0</td>
<td>$\frac{-8.5 + 9.92\epsilon\cdot4}{0.88\epsilon - 1 + \epsilon^4}$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0.75</td>
<td>0.593</td>
<td>0.102</td>
<td>$0.75 - 0.16\exp(-0.018\epsilon^{-3.1})$</td>
</tr>
<tr>
<td>$C_5$</td>
<td>$2\epsilon_1(1 + \epsilon_1)$</td>
<td>$\pi^2/4$</td>
<td>4</td>
<td>$\frac{\pi^4 \epsilon}{16E^2(e)}$</td>
</tr>
<tr>
<td>$C_6$</td>
<td>$2(1 + \epsilon_1)/\epsilon_1$</td>
<td>4</td>
<td>4</td>
<td>$16E^2(e)/\pi^2 \epsilon$</td>
</tr>
<tr>
<td>$n$</td>
<td>1/2</td>
<td>1/2</td>
<td>0.675</td>
<td>1/2</td>
</tr>
</tbody>
</table>
Effect of Axis Ratio on Drag Force

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Effect of Approach Velocity on Drag Force

Approach Velocity, $U_\infty$ (m/s)

Drag Force, $F_D$ (N)

- SPF
- CPF($\varepsilon=1.0$)
- EPF($\varepsilon=0.8$)
- EPF($\varepsilon=0.5$)
- EPF($\varepsilon=0.2$)
- RPF
Effect of Axis Ratio on Average Heat Transfer

![Graph showing the effect of axis ratio on Nusselt number.]

- RPF
- EPF
- CPF
- SPF

Nusselt Number, $Nu$

Axis Ratio, $\varepsilon$

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Effect of Approach Velocity on Average Heat Transfer

Entropy Generation Number, $N_s$

Approach Velocity, $U_\infty$ (m/s)

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Entropy Generation Rate

Approach Velocity, $U_\infty$ (m/s)

$N_{Sf}$
$N_{Sh}$

$N_s$

$N_{Sf}$
$N_{Sh}$

$N_{Sf}$
$N_{Sh}$

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Effect of Approach Velocity on Entropy Generation Rate

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Effect of Perimeter on Entropy Generation Rate

Entropy Generation Number, N

Perimeter, (m)

SPF
CPF(ε=1.0)
EPF(ε=0.8)
EPF(ε=0.5)
EPF(ε=0.2)
RPF

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Effect of Aspect Ratio on Entropy Generation Rate

![Graph showing the effect of aspect ratio on entropy generation number. The graph plots entropy generation number (Ns) against aspect ratio. Different curves represent various conditions: SPF, CPF(ε=1.0), EPF(ε=0.8), EPF(ε=0.5), and RPF. The y-axis ranges from 0.01 to 0.06, and the x-axis ranges from 0 to 10.](image)

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Conclusions

✓ Optimum dimensionless entropy generation rate exists for each geometry depending upon approach velocity, perimeter, and aspect ratio.

✓ Square geometry is the worst choice from the point of view of entropy generation rate.

✓ Circular geometry appears as the best for low approach velocities and small perimeters.
Conclusions

✓ Flat plate gives the best results for high approach velocities and large surface areas.
✓ Elliptical geometry offers high heat transfer rates and lower drag coefficients for medium approach velocities and larger surface areas.
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